Multi-spacecraft analysis methods in space plasma: Towards a new era of multi-scale science

Alexandros Chasapis,
LASP, University of Colorado Boulder
LPC2E, CNRS
Contact: alexandros.chasapis@lasp.colorado.edu

Recap

What can we do with multipoint measurements:

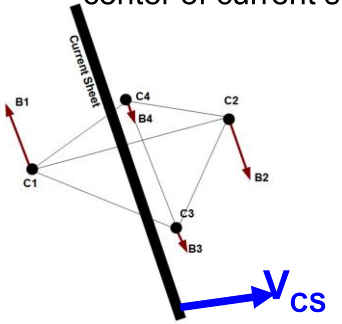
- Estimate 2-point increments $|\overrightarrow{\Delta \mathrm{B}}_{ij}(t)| = |\vec{\mathrm{B}}_i(t) - \vec{\mathrm{B}}_j(\mathrm{t})|$

- Partial Variance of Increments: Statistical detection of structures & current sheets $PVI(t) = \frac{|\overrightarrow{\Delta \mathrm{B}}(t)|}{\sqrt{< |\overrightarrow{\Delta B}(t)|^2 >_T}}$

- Higher-order statistics: Structure functions & Kurtosis $k(\tau) = \frac{< (\delta B_\tau)^4 >_t}{< (\delta B_\tau)^2 >_t^2}$

- Determine orientation of structures using the timing method $\begin{pmatrix} \bar{r}_{12} \\ \bar{r}_{13} \\ \bar{r}_{14} \end{pmatrix} \frac{1}{V_{CS}} \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} t_{12} \\ t_{13} \\ t_{14} \end{pmatrix}$

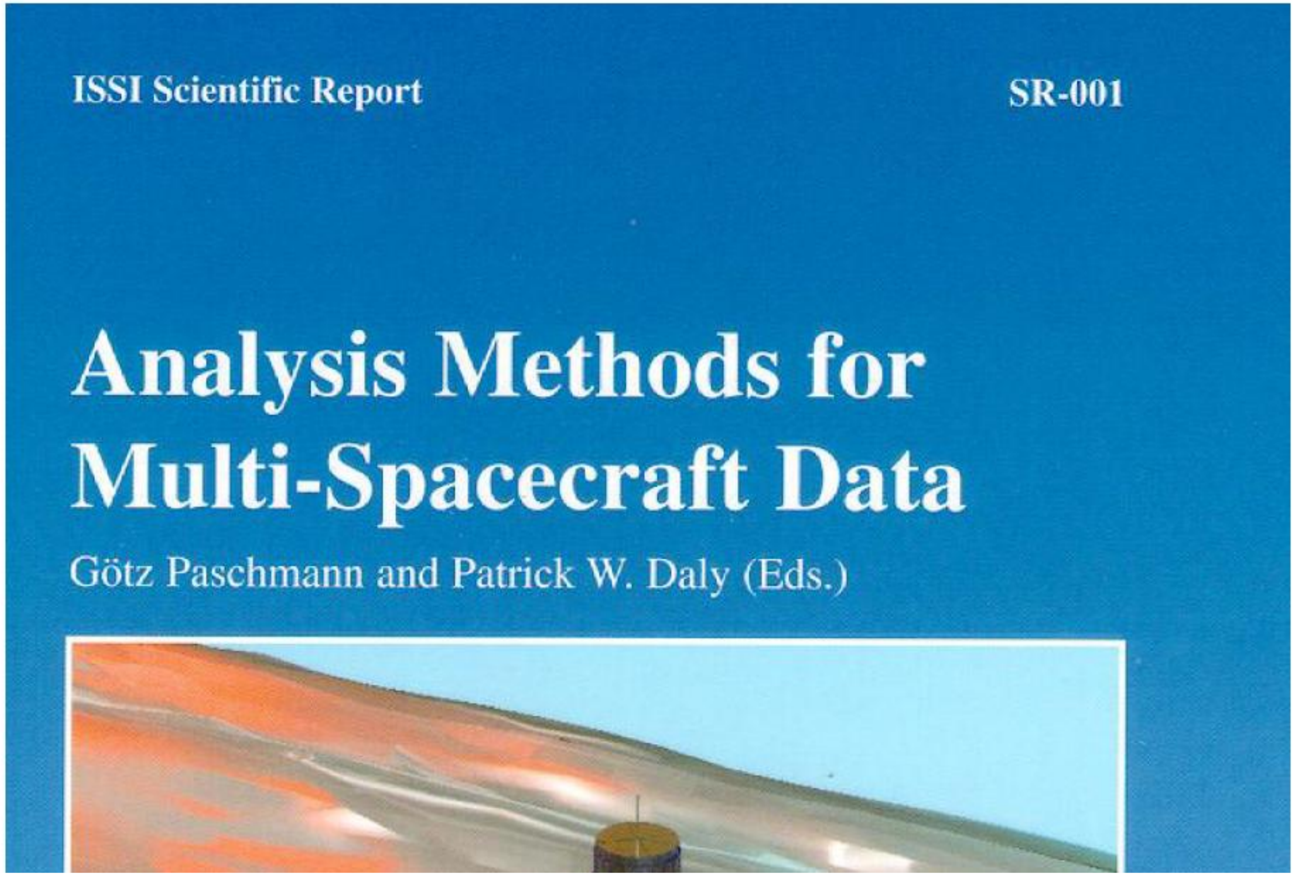
center of current sheet

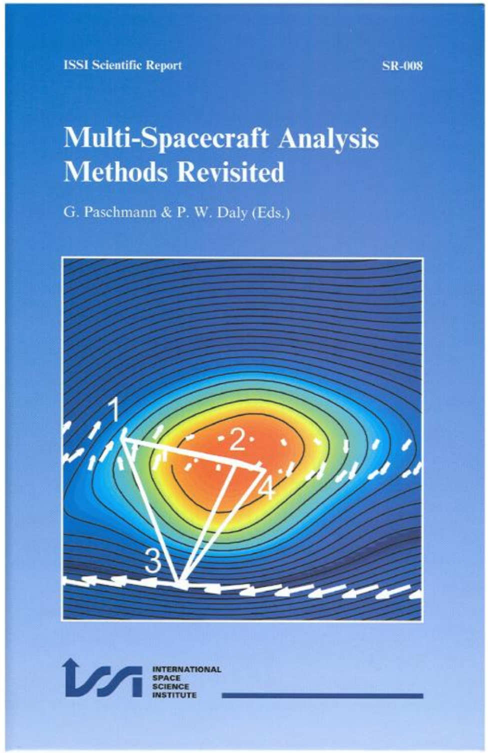


Part II: Estimating Gradients
The Curlometer Technique

The Book

Analysis Methods for Multi-Spacecraft Data / Götz Paschmann and Patrick Daly (eds.).
ISSI Scientific Reports Series, ESA/ISSI, Vol. 1. ISBN 1608-280X, 1998



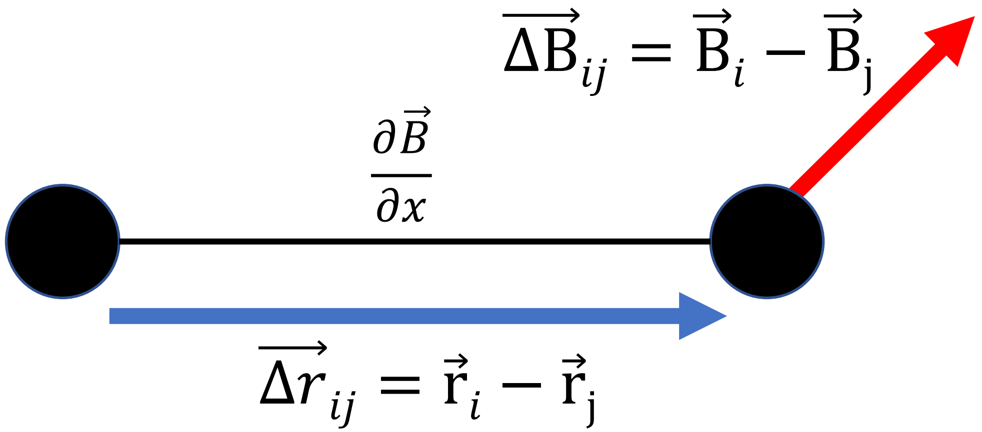


Estimating Gradients

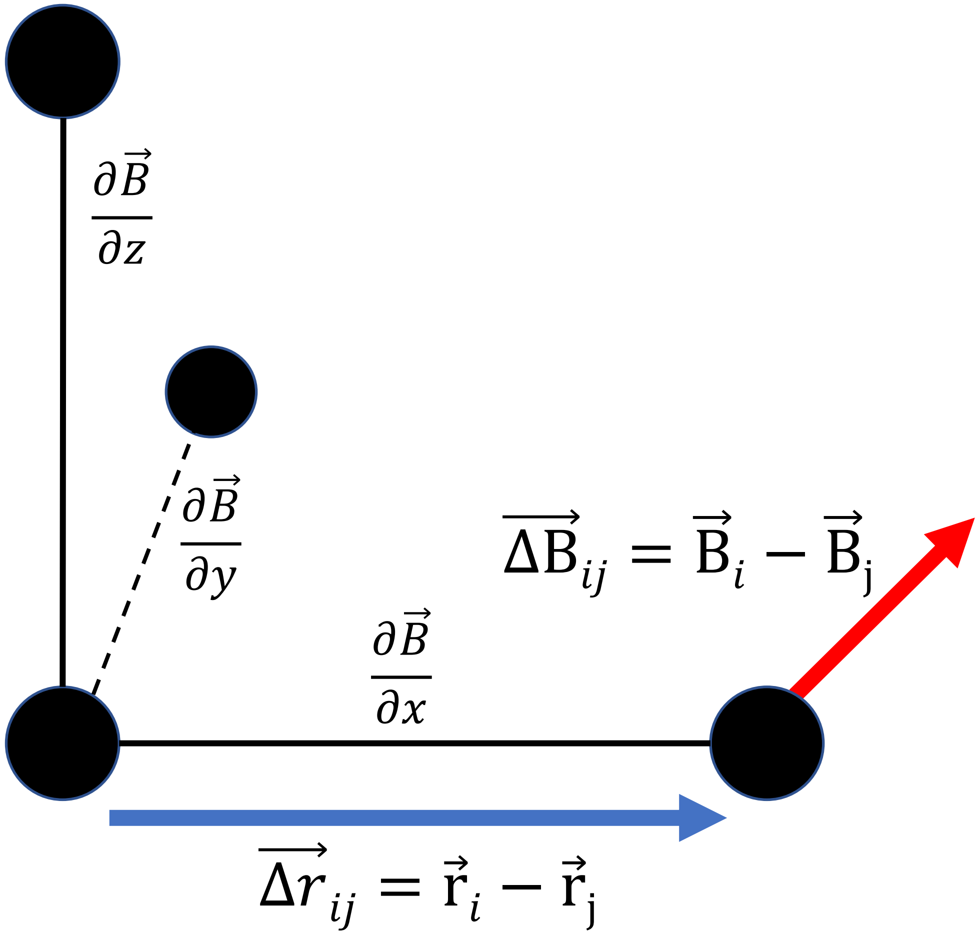
We can use 2-point measurements to get an estimate of the gradient along the direction of the spacecraft separation:

$$\frac{\Delta\vec{B}}{\Delta r_j} \sim \frac{\partial\vec{B}}{\partial r_j}$$

*like a finite difference or 1st order Taylor approx



Estimating Gradients



We can use 2-point measurements to get an estimate of the gradient along the direction of the spacecraft separation:

$$\frac{\Delta \vec{B}}{\Delta r_j} \sim \frac{\partial \vec{B}}{\partial r_j}$$

*like a finite difference or 1st order Taylor approx

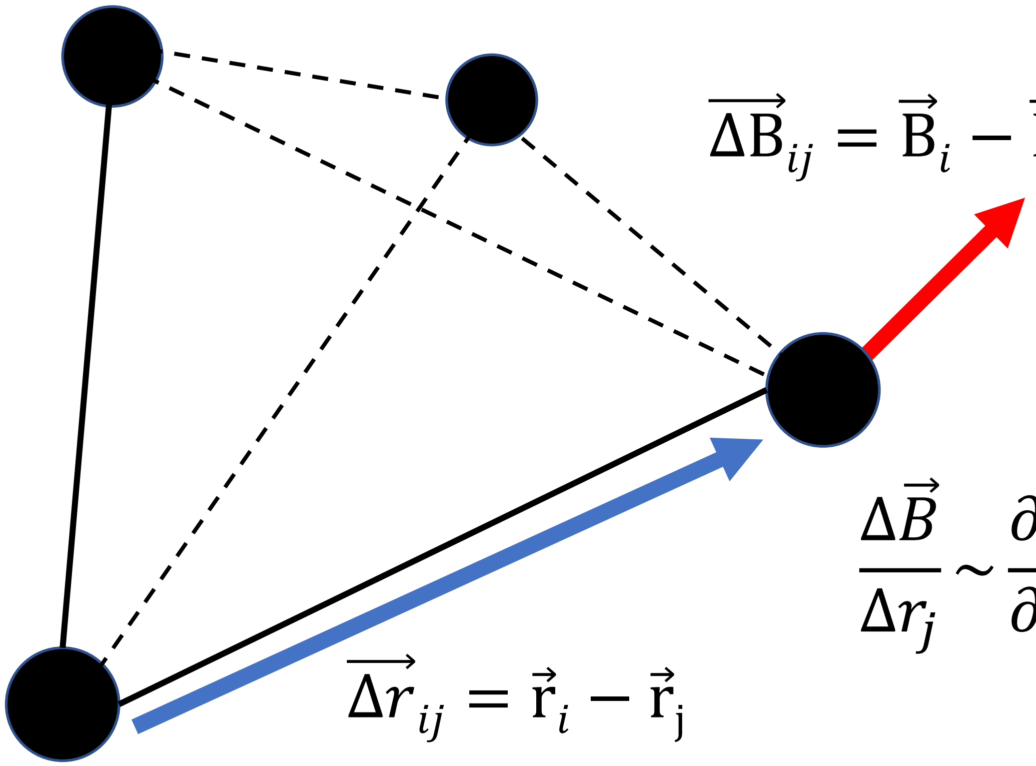
4 spacecraft give us 3 the gradients in independent directions:

$$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$$

We can use that to calculate any 3-d vector quantity of a field: grad,curl,divergence

$$\nabla \times \vec{B} = \vec{J}$$

Curlometer Technique

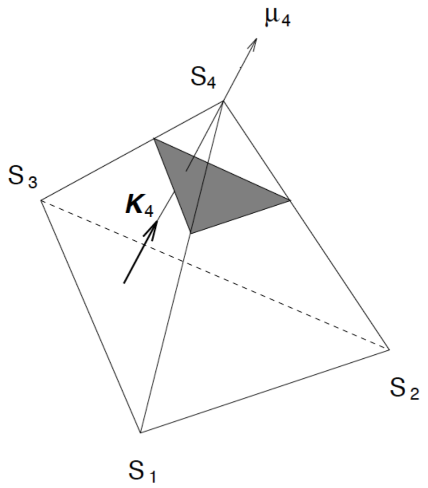


$$\overrightarrow{\Delta B}_{ij} = \vec{B}_i - \vec{B}_j$$

$$\frac{\Delta \vec{B}}{\Delta r_j} \sim \frac{\partial \vec{B}}{\partial r_j}$$

$$\overrightarrow{\Delta r}_{ij} = \vec{r}_i - \vec{r}_j$$

Expression of vector gradients in non-orthonormal system:



Reciprocal vectors:

$$\boldsymbol{k}_4 = \frac{\boldsymbol{r}_{12} \times \boldsymbol{r}_{13}}{\boldsymbol{r}_{14}.(\boldsymbol{r}_{12} \times \boldsymbol{r}_{13})}$$

Gradients:

$$\boldsymbol{LG}[u] = \sum_{\alpha=1}^{4} \boldsymbol{k}_\alpha u_\alpha$$

$$\mathrm{LG}[\boldsymbol{v}] = \sum_{\alpha=1}^{4} \boldsymbol{k}_\alpha \, \boldsymbol{v}_\alpha^T$$

Estimating Gradients

We can use that to calculate any 3-d vector quantity of a field: gradients, curl, divergence

- Curl of B (Current density) $\nabla \times \vec{B} = \vec{J}$
- Divergence of B (should be zero) $\nabla \cdot \vec{B} = 0$
- Magnetic field gradient tensor terms $\partial_i B_j$
- Velocity gradients $\partial_i v_j$
- Stress tensor $-(\boldsymbol{P} \cdot \nabla) \cdot \boldsymbol{u} = -p\theta - \Pi_{ij} D_{ij}$

Estimating Accuracy

Magnetic field:
Curl of B: Current density

$$\nabla \times \vec{B} = \vec{J}$$

- Independent measurement derived from plasma instruments

$$\vec{J} = q(n_p \vec{v}_p - n_e \vec{v}_e)$$

- Divergence of B

$$\nabla \cdot \vec{B} = 0$$

Test of the linearity assumption

Velocity: More complicated
No independent measurement to compare against

- Roberts et al. JGR 2023 provides uncertainty estimates

Sources of Uncertainty

- Assumption of linearity
- Measurement errors
- Spacecraft formation quality

Sources of Uncertainty

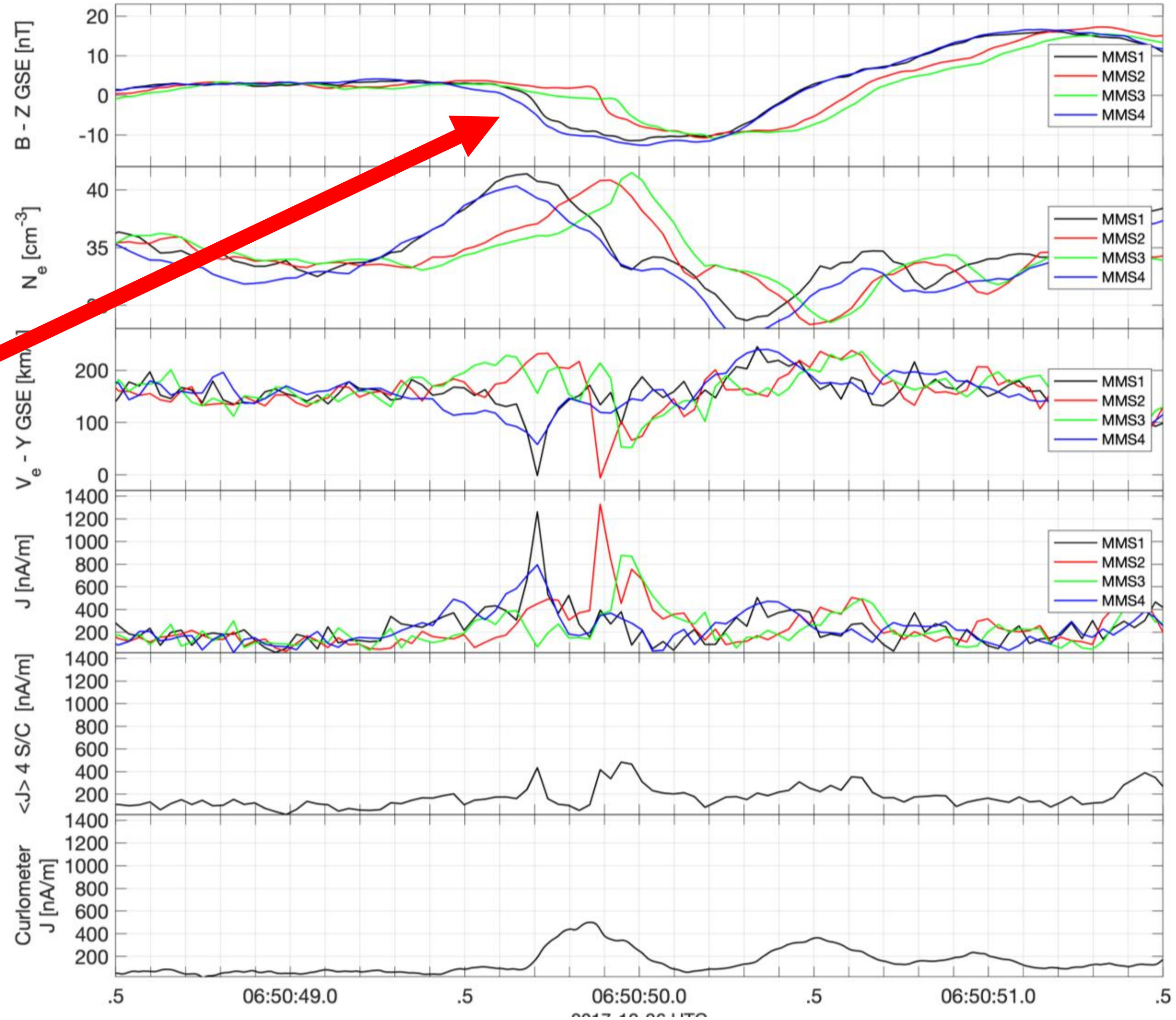
- **Assumption of linearity**
- Measurement errors
- Spacecraft formation quality

Sources of Uncertainty

- **Assumption of linearity**
- Measurement errors
- Spacecraft formation quality

$$\frac{\Delta \vec{B}}{\Delta r_j} \sim \frac{\partial \vec{B}}{\partial r_j}$$

Gradient at scale smaller than the spacecraft separation is under-sampled



Sources of Uncertainty

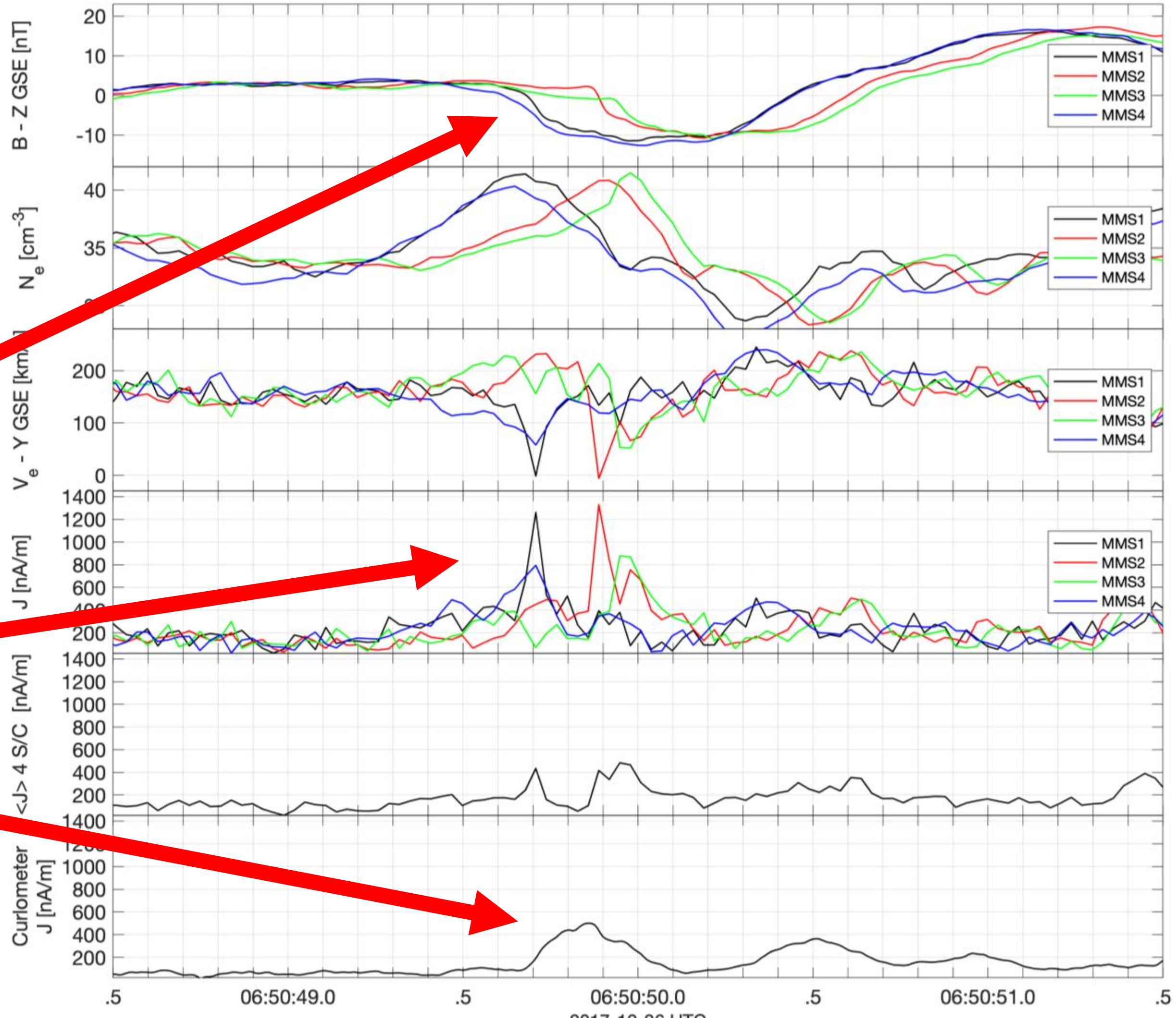
- **Assumption of linearity**
- Measurement errors
- Spacecraft formation quality

$$\frac{\Delta \vec{B}}{\Delta r_j} \sim \frac{\partial \vec{B}}{\partial r_j}$$

Gradient at scale smaller than the spacecraft separation is under-sampled

Curlometer provides a "volume-averaged" value

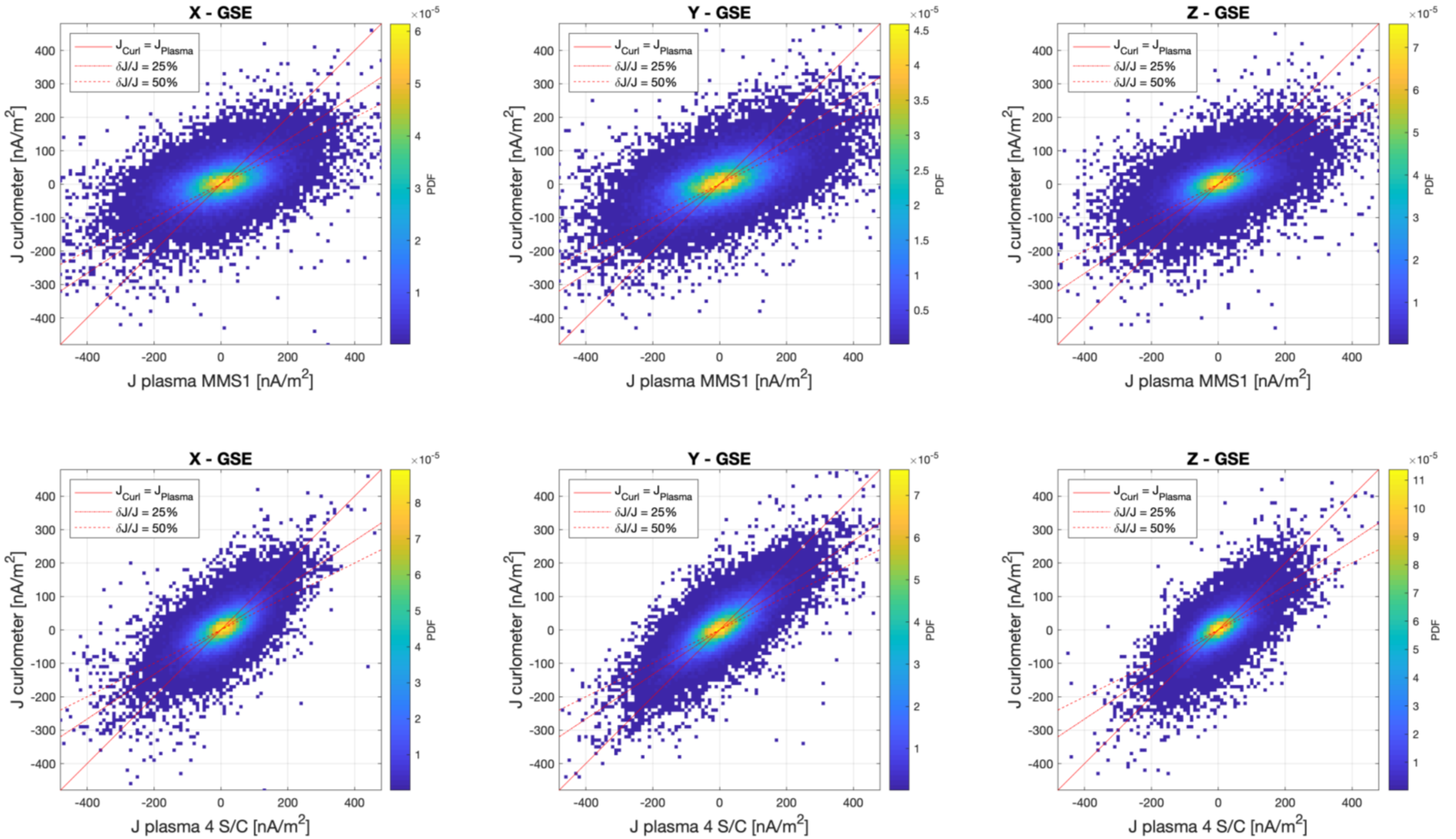
Current magnitude underestimated



Sources of Uncertainty

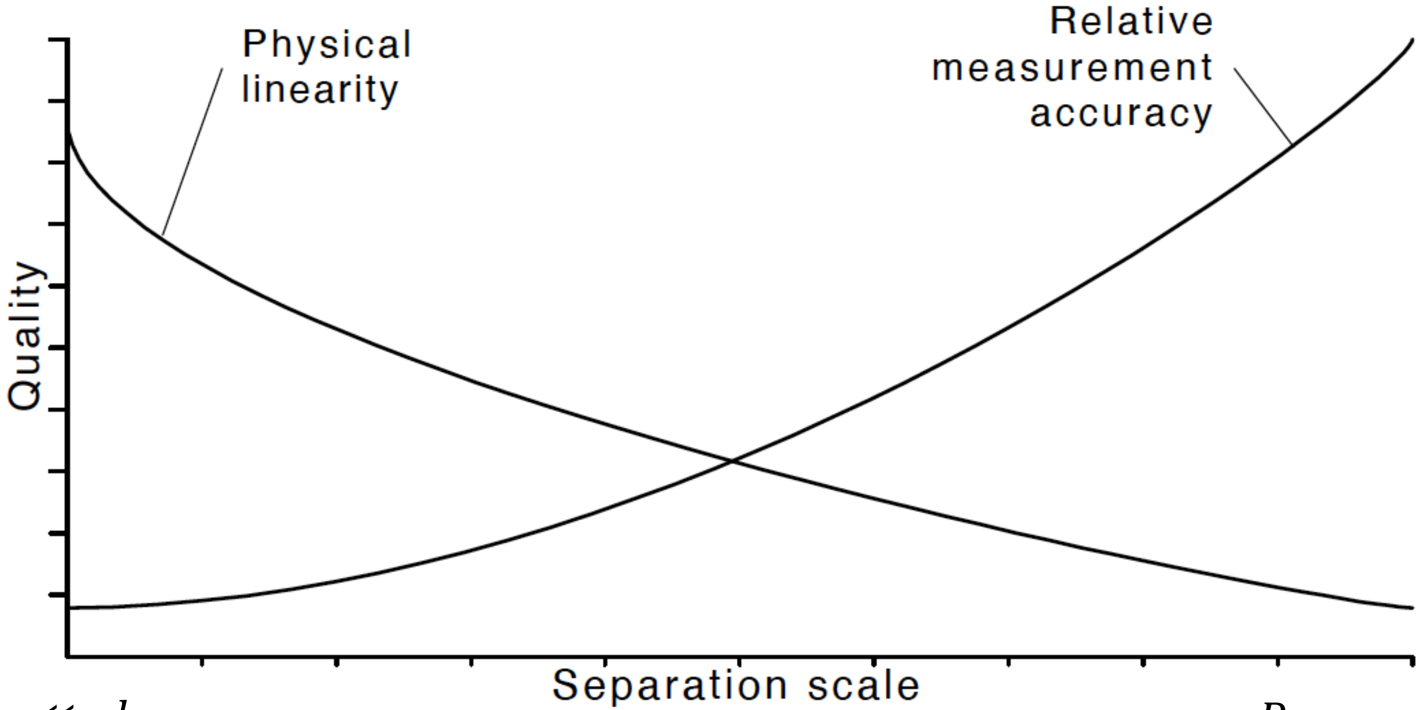
- **Assumption of linearity**
- Measurement errors
- Spacecraft formation quality

- Curlometer provides a "volume-averaged" value
- Current magnitude systematically underestimated
- Agreement when compared to 4-spacecraft average



Sources of Uncertainty

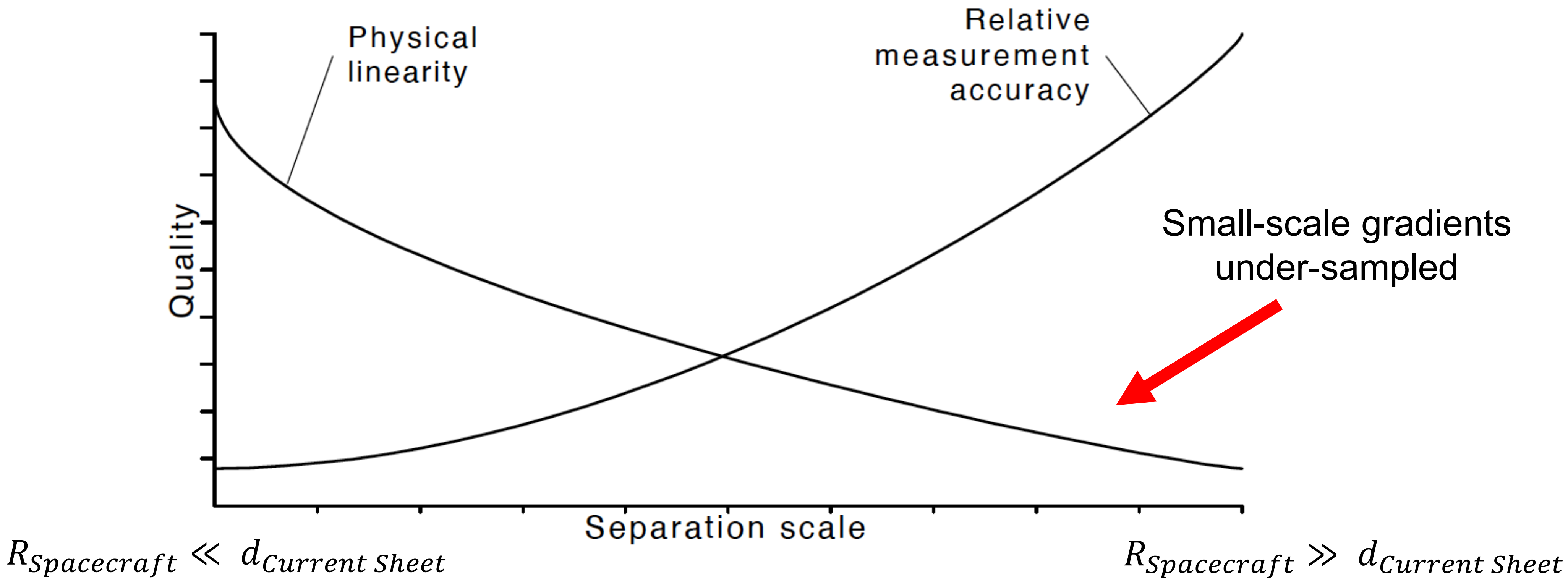
- **Assumption of linearity**
- Measurement errors
- Spacecraft formation quality



$R_{Spacecraft} \ll d_{Current\ Sheet}$ $R_{Spacecraft} \gg d_{Current\ Sheet}$

Sources of Uncertainty

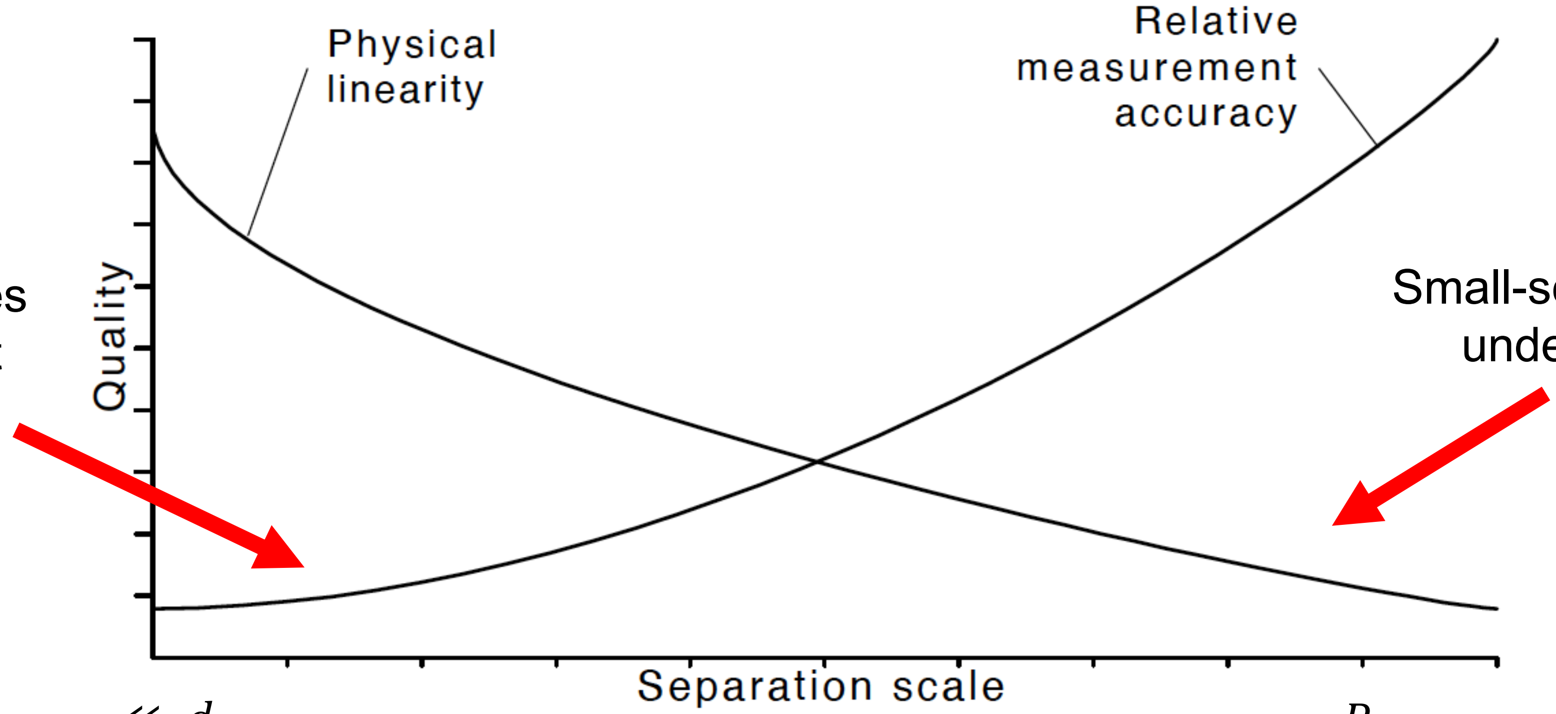
- **Assumption of linearity**
- Measurement errors
- Spacecraft formation quality



Sources of Uncertainty

- **Assumption of linearity**
- Measurement errors
- Spacecraft formation quality

Gradients at larger scales impacted by instrument errors



Small-scale gradients under-sampled

$R_{Spacecraft} \ll d_{Current\ Sheet}$ $R_{Spacecraft} \gg d_{Current\ Sheet}$

Sources of Uncertainty

- Assumption of linearity
- **Measurement errors**
- Spacecraft formation quality

Sources of Uncertainty

- Assumption of linearity
- **Measurement errors**
- Spacecraft formation quality

Propagation of measurement errors in underlying quantities used for the estimate will impact the accuracy of the curlometer

Ballpark values (pessimistic)

- Errors in field measurements:
  Magnetic field ~0.05 nT
  Proton & Electron Velocity ~ 5-10km/s

- Errors in spacecraft position accuracy (~0.5km)

$$\frac{\delta J}{|\boldsymbol{J}_{\mathrm{av}}|} = F_B \frac{\delta B}{\Delta B} + F_S \frac{\delta r}{\Delta r}$$

Reciprocal vectors:

$$\boldsymbol{k}_4 = \frac{\boldsymbol{r}_{12} \times \boldsymbol{r}_{13}}{\boldsymbol{r}_{14}.(\boldsymbol{r}_{12} \times \boldsymbol{r}_{13})}$$

Gradients:

$$\boldsymbol{LG}[u] = \sum_{\alpha=1}^{4} \boldsymbol{k}_\alpha u_\alpha$$

$$\mathrm{LG}[\boldsymbol{v}] = \sum_{\alpha=1}^{4} \boldsymbol{k}_\alpha \, \boldsymbol{v}_\alpha^T$$

Sources of Uncertainty

- Assumption of linearity
- Measurement errors
- **Spacecraft formation quality**

Sources of Uncertainty

- Assumption of linearity
- Measurement errors
- **Spacecraft formation quality**

Since the curlometer provides a “volume-averaged” estimate, we need the spacecraft formation to be as close to a regular tetrahedron for best results

The quality of the spacecraft formation can be quantified with two parameters: **Planarity & Elongation**

Chapters 12 & 13 of Analysis Methods for Multi-Spacecraft Data,
Paschmann & Daly, ISSI Scientific Report, 2000

Sources of Uncertainty

- Assumption of linearity
- Measurement errors
- **Spacecraft formation quality**

Since the curlometer provides a “volume-averaged” estimate, we need the spacecraft formation to be as close to a regular tetrahedron for best results

The quality of the spacecraft formation can be quantified with two parameters: **Planarity & Elongation**

Step 1: Calculate the volumetric tensor

$$R = \frac{1}{N} \sum_{\alpha=1}^{N} (\boldsymbol{r}_\alpha - \boldsymbol{r}_b)(\boldsymbol{r}_\alpha - \boldsymbol{r}_b)^T$$

Step 2: Get the eigen values

$$\begin{aligned} a &= \sqrt{R^{(1)}} \\ b &= \sqrt{R^{(2)}} \\ c &= \sqrt{R^{(3)}} \end{aligned}$$

Step 3: Calculate the formation quality parameters

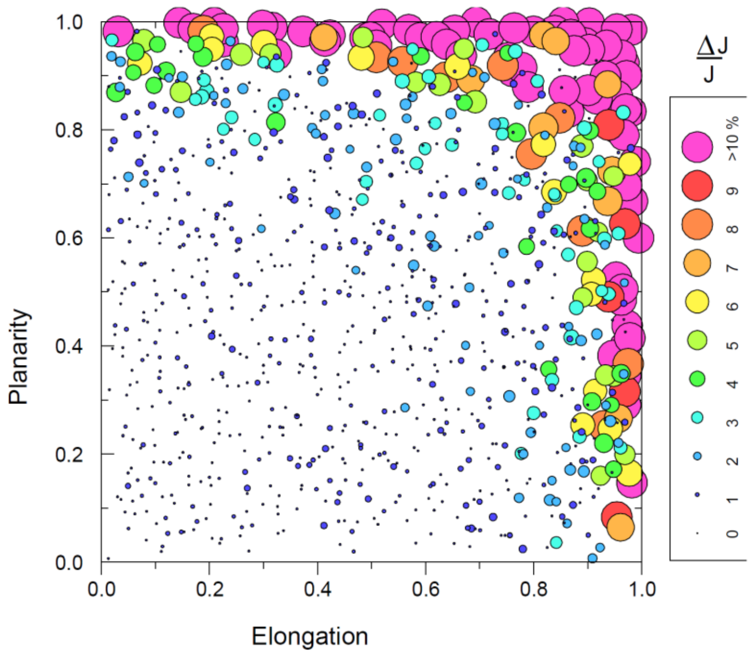
- **characteristic size** $L = 2\,a$
- **elongation** $E = 1 - (b/a)$
- **planarity** $P = 1 - (c/b)$

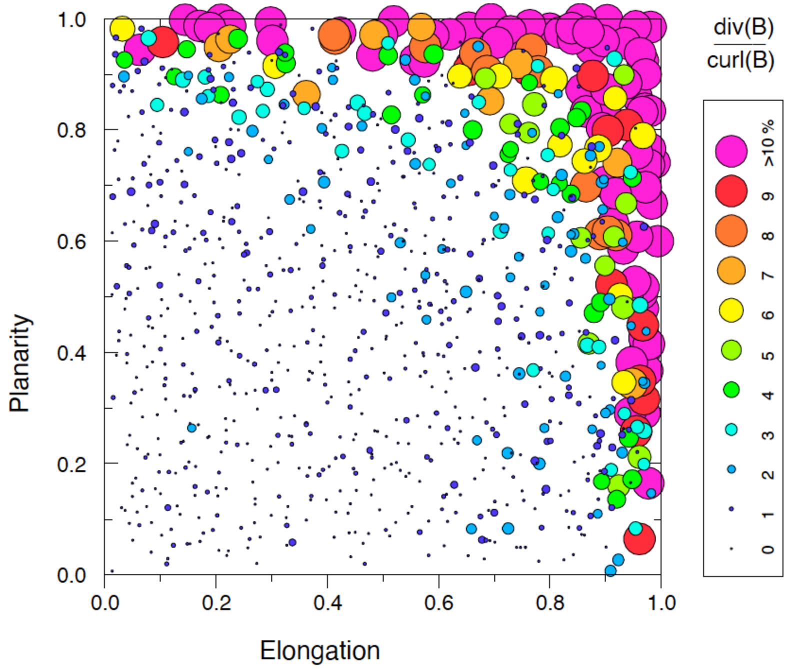
Chapters 12 & 13 of Analysis Methods for Multi-Spacecraft Data, Paschmann & Daly, ISSI Scientific Report, 2000

Sources of Uncertainty

- Assumption of linearity
- Measurement errors
- **Spacecraft formation quality**

Impact of formation quality based on models





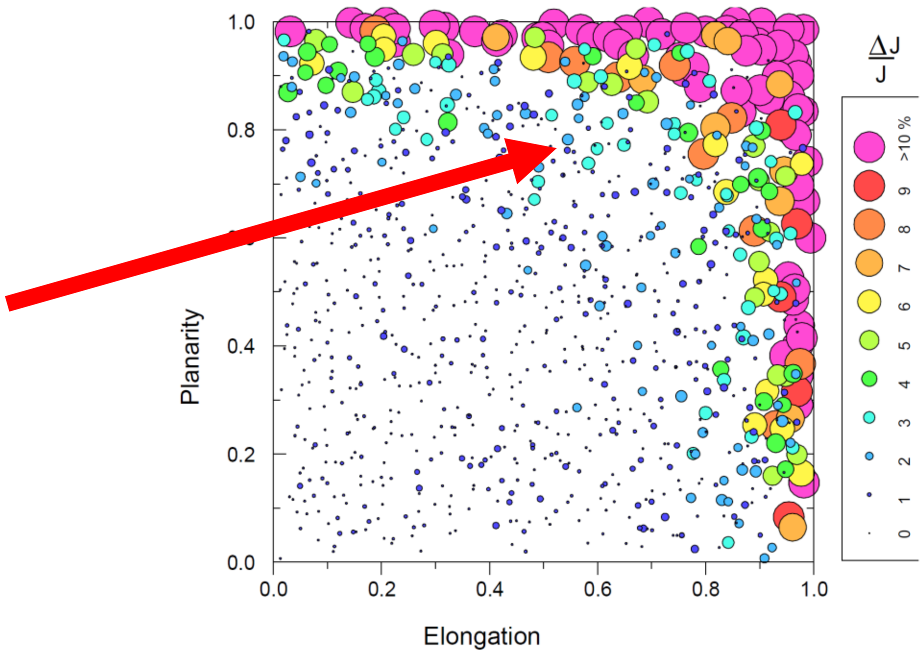
From: Analysis Methods for Multi-Spacecraft Data,
Paschmann & Daly, ISSI Scientific Report, 2000

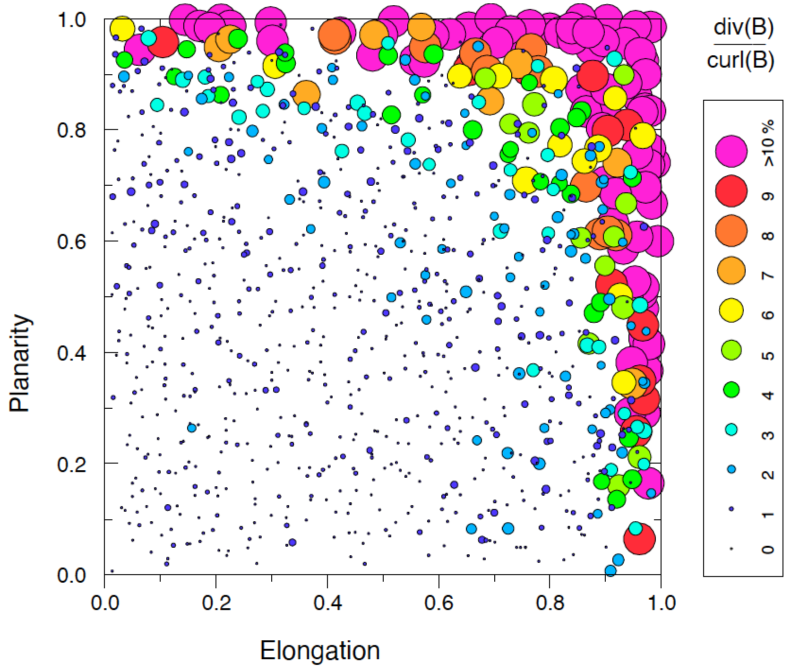
Sources of Uncertainty

- Assumption of linearity
- Measurement errors
- **Spacecraft formation quality**

The further away from a regular tetrahedron, the worse the estimate gets

Impact of formation quality based on models





From: Analysis Methods for Multi-Spacecraft Data,
Paschmann & Daly, ISSI Scientific Report, 2000

Estimating Accuracy

Magnetic field:
Curl of B: Current density

$$\nabla \times \vec{B} = \vec{J}$$

- Independent measurement derived from plasma instruments
  (less accurate, single point, lower time resolution)

- Divergence of B

$$\nabla \cdot \vec{B} = 0$$

Test of the linearity assumption

Velocity: More complicated
No independent measurement to compare against

- Roberts et al. JGR 2023 provides uncertainty estimates

Evaluating with current data

We can use the MMS burst-resolution data to evaluate the quality of the curlometer under different conditions and formation parameters

- Recent campaigns provided a large volume of burst data under a wider range formation quality parameters and plasma conditions

- We can directly compare the 4-spacecraft gradient estimates with the current density derived from plasma data

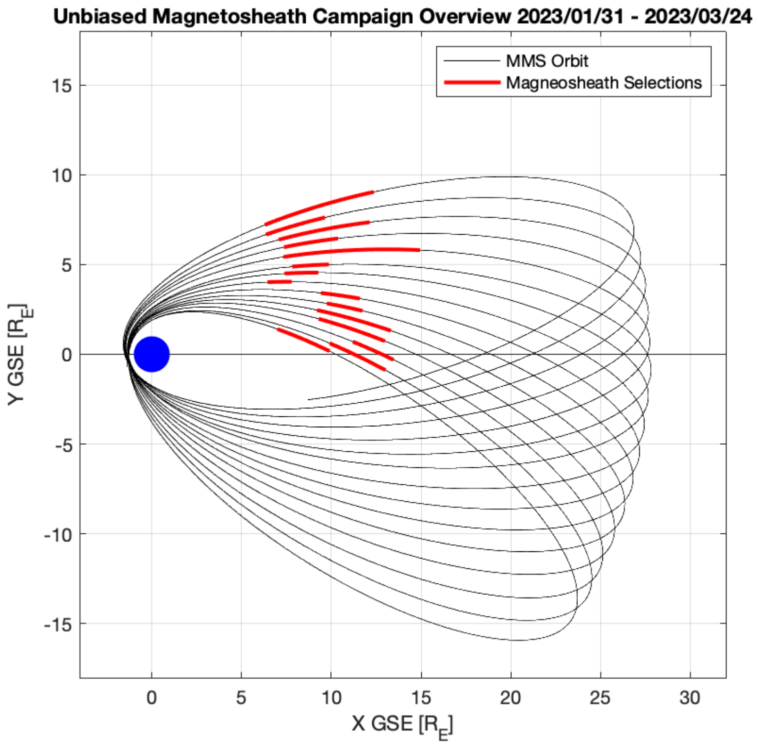
Evaluating with current data

We can use the MMS burst-resolution data to evaluate the quality of the curlometer under different conditions and formation parameters

- Recent campaigns provided a large volume of burst data under a wider range formation quality parameters and plasma conditions

- We can directly compare the 4-spacecraft gradient estimates with the current density derived from plasma data

2023 MMS unbiased magnetosheath campaign

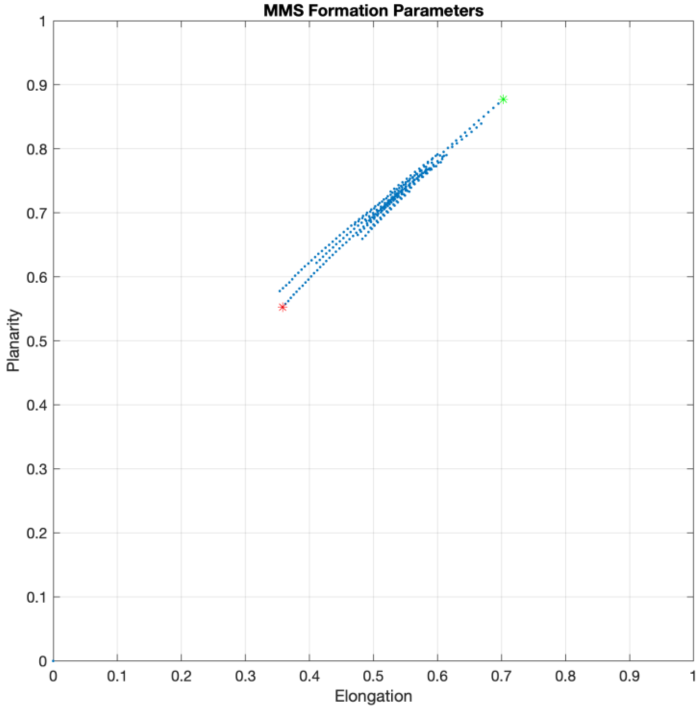


Evaluating with current data

We can use the MMS burst-resolution data to evaluate the quality of the curlometer under different conditions and formation parameters

- Recent campaigns provided a large volume of burst data under a wider range formation quality parameters and plasma conditions
- We can directly compare the 4-spacecraft gradient estimates with the current density derived from plasma data

Formation quality parameters during the 2023 MMS unbiased magnetosheath campaign

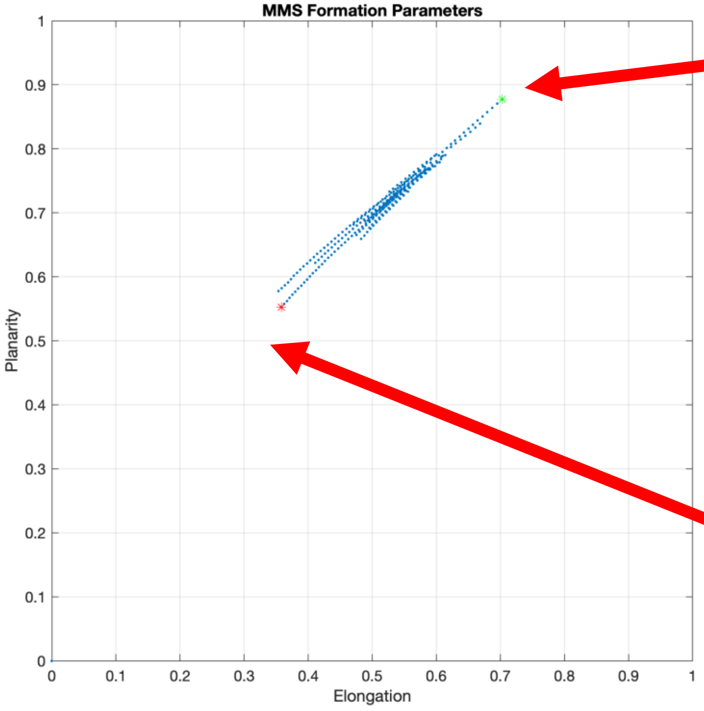


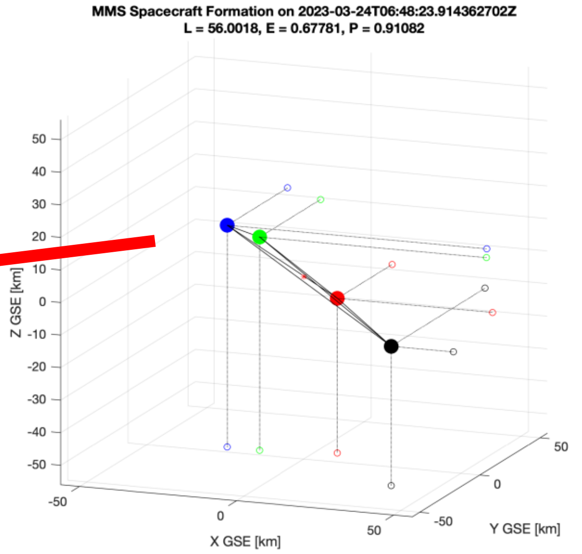
We can use the MMS burst-resolution data to evaluate the quality of the curlometer under different conditions and formation parameters

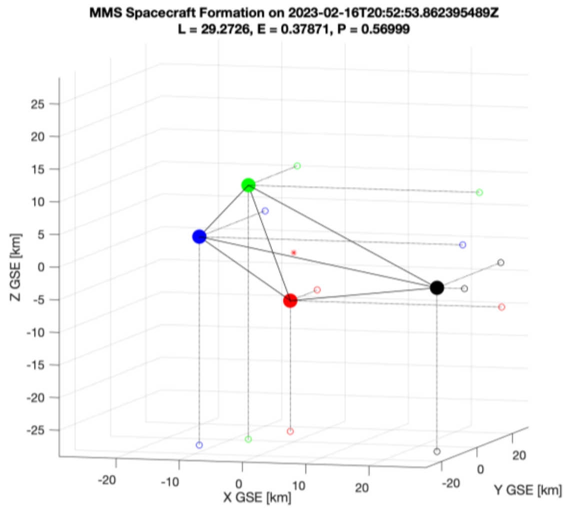
- Recent campaigns provided a large volume of burst data under a wider range formation quality parameters and plasma conditions

- We can directly compare the 4-spacecraft gradient estimates with the current density derived from plasma data

Formation quality parameters during the 2023 MMS unbiased magnetosheath campaign







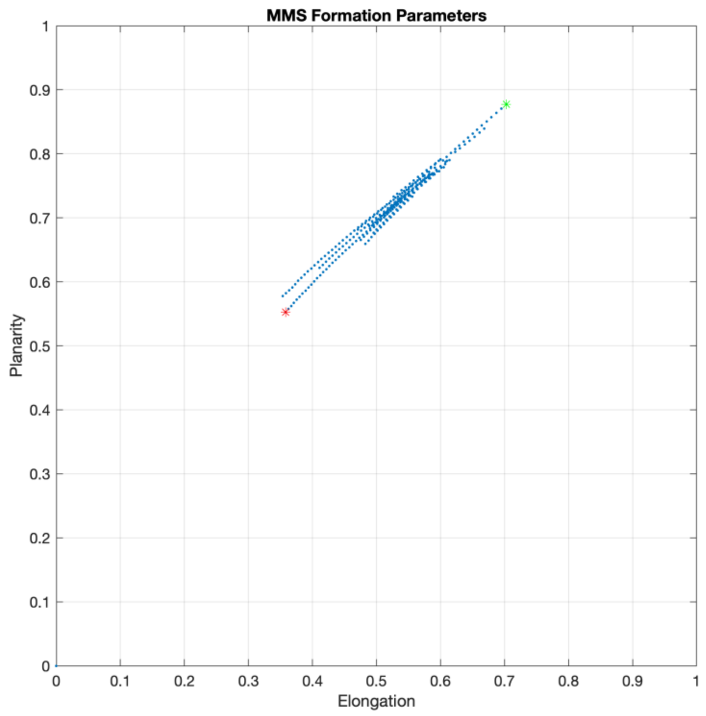
Evaluating with current data

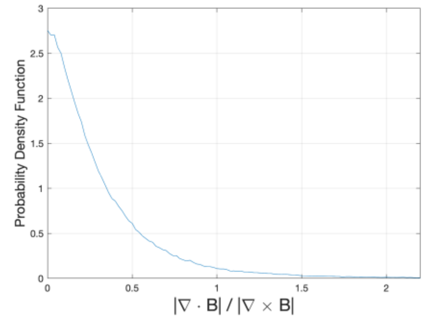
We can use the MMS burst-resolution data to evaluate the quality of the curlometer under different conditions and formation parameters

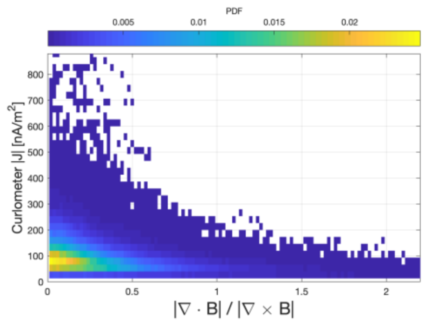
- Recent campaigns provided a large volume of burst data under a wider range formation quality parameters and plasma conditions

- We can directly compare the 4-spacecraft gradient estimates with the current density derived from plasma data

Formation quality parameters during the 2023 MMS unbiased magnetosheath campaign





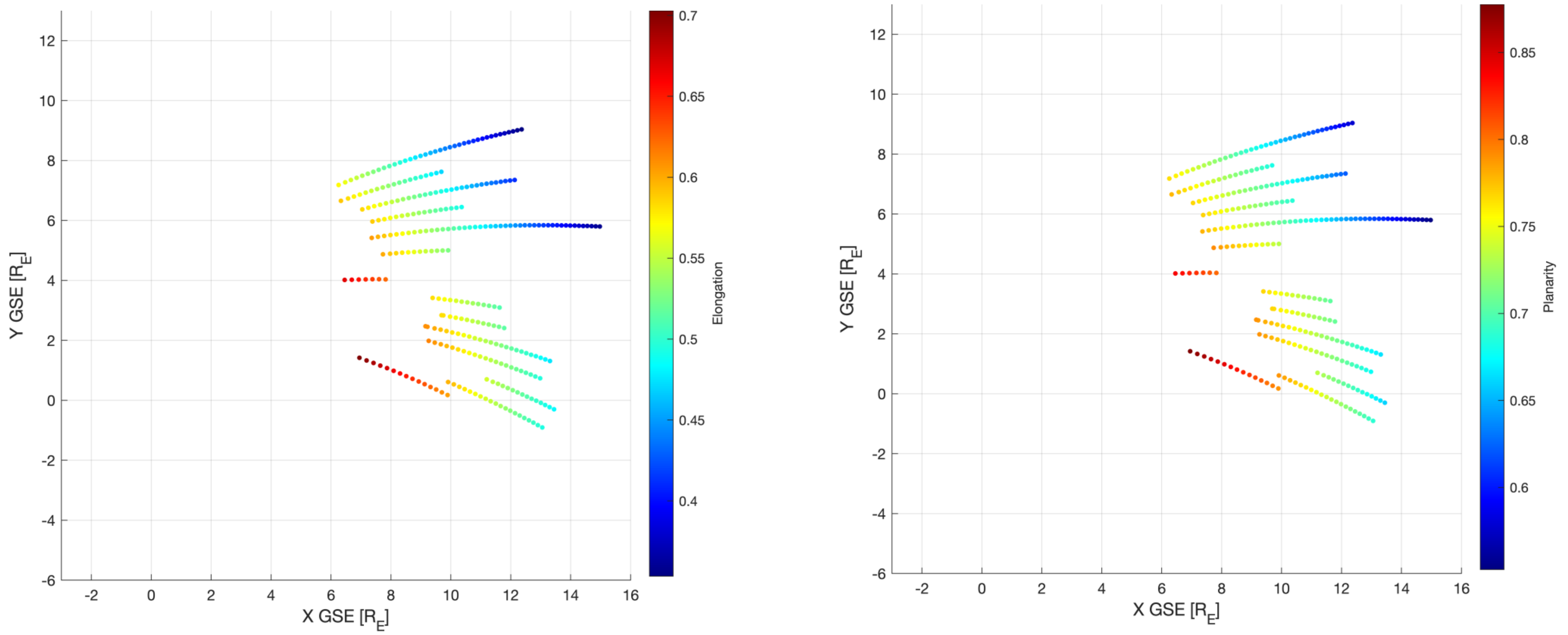


Evaluation of linearity assumption

$$\nabla \cdot \vec{B} = 0$$

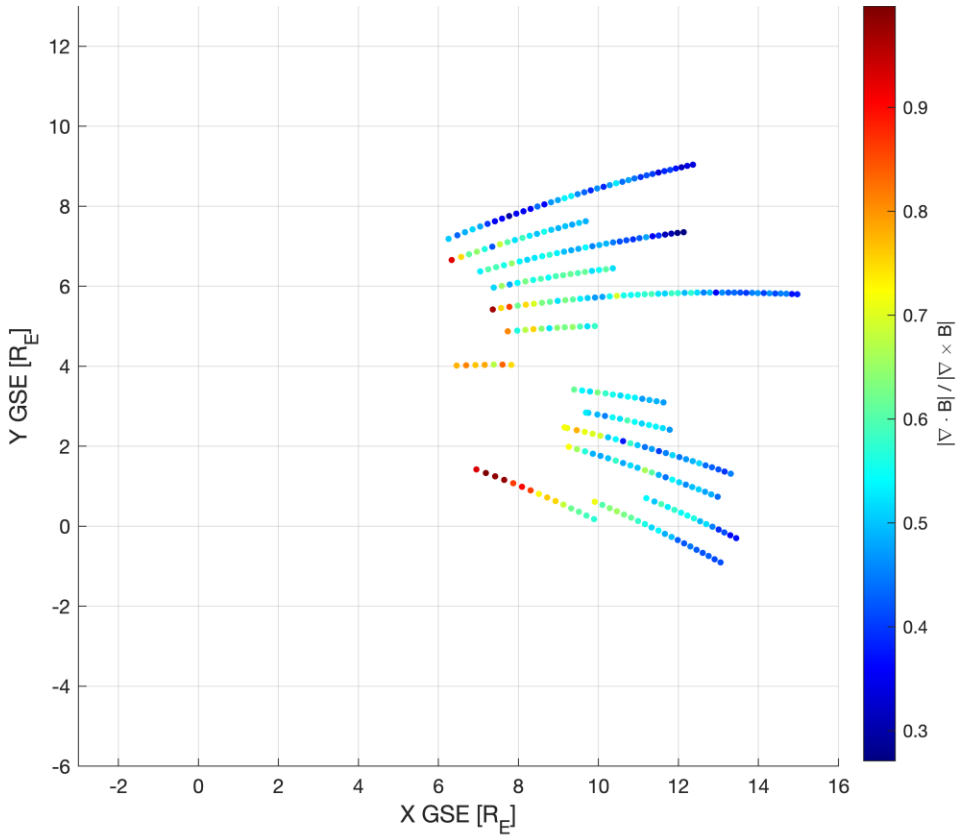
Evaluating with current data

Formation quality parameters during the 2023 MMS unbiased magnetosheath campaign

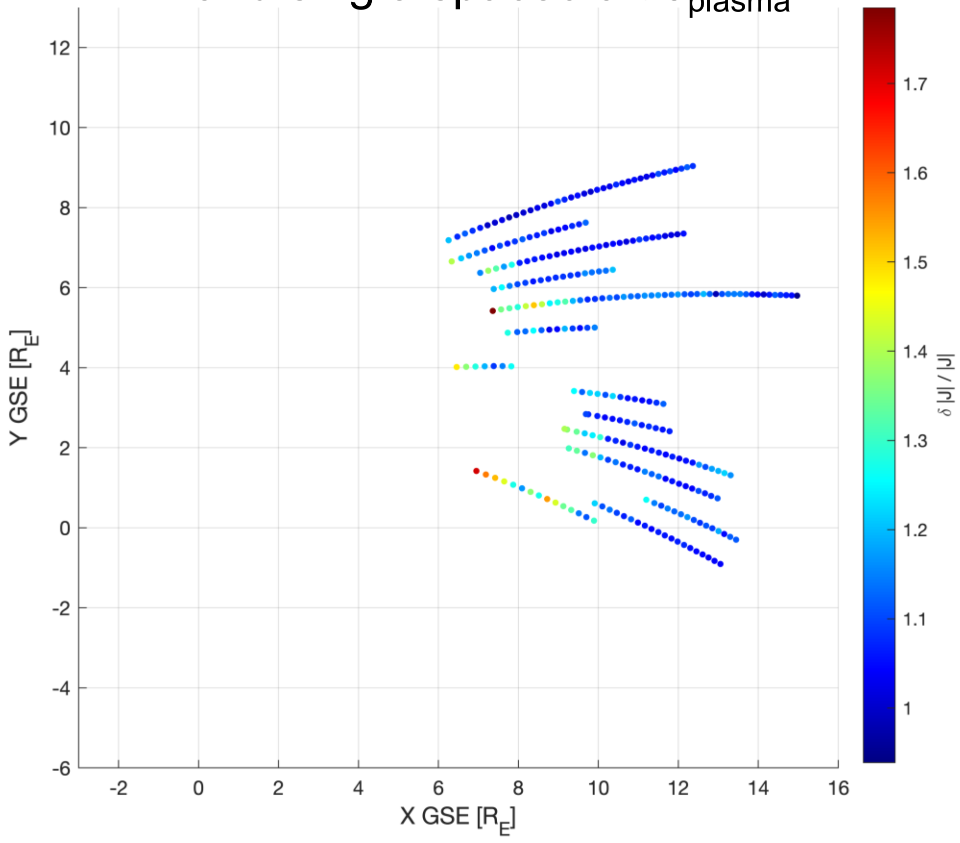


Evaluating with current data

Evolution of divB/curlB

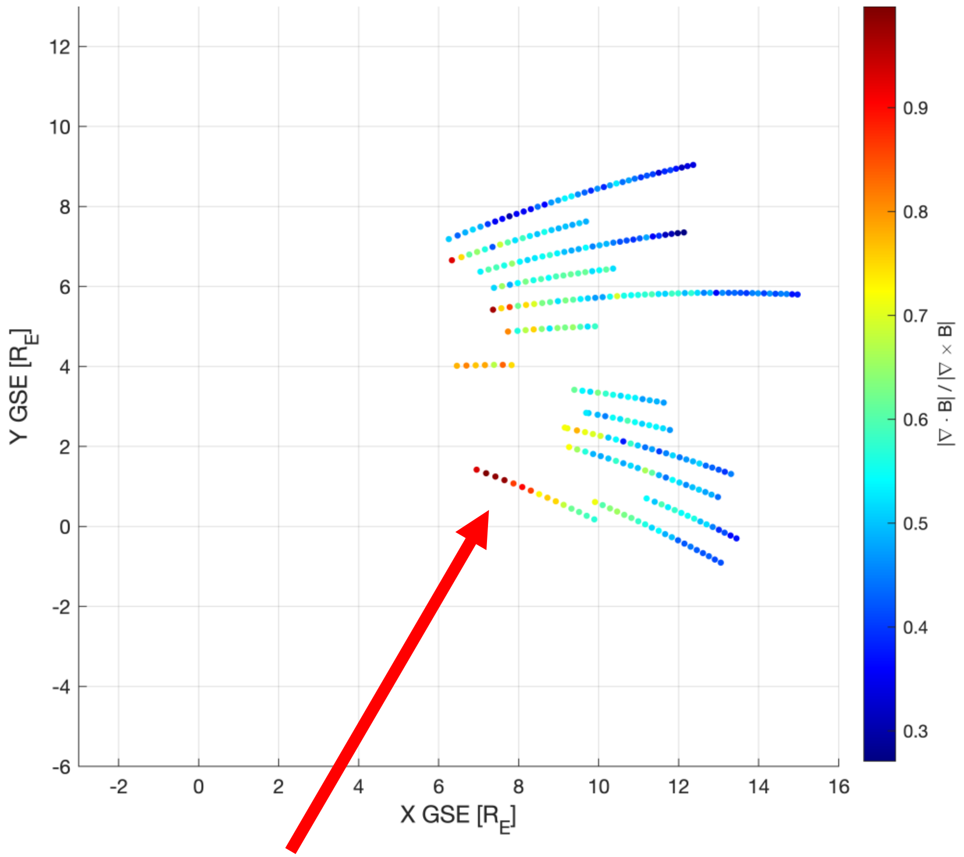


Evolution of deviation between 4-spacecraft $J_{curlometer}$ and single-spacecraft J_{plasma}

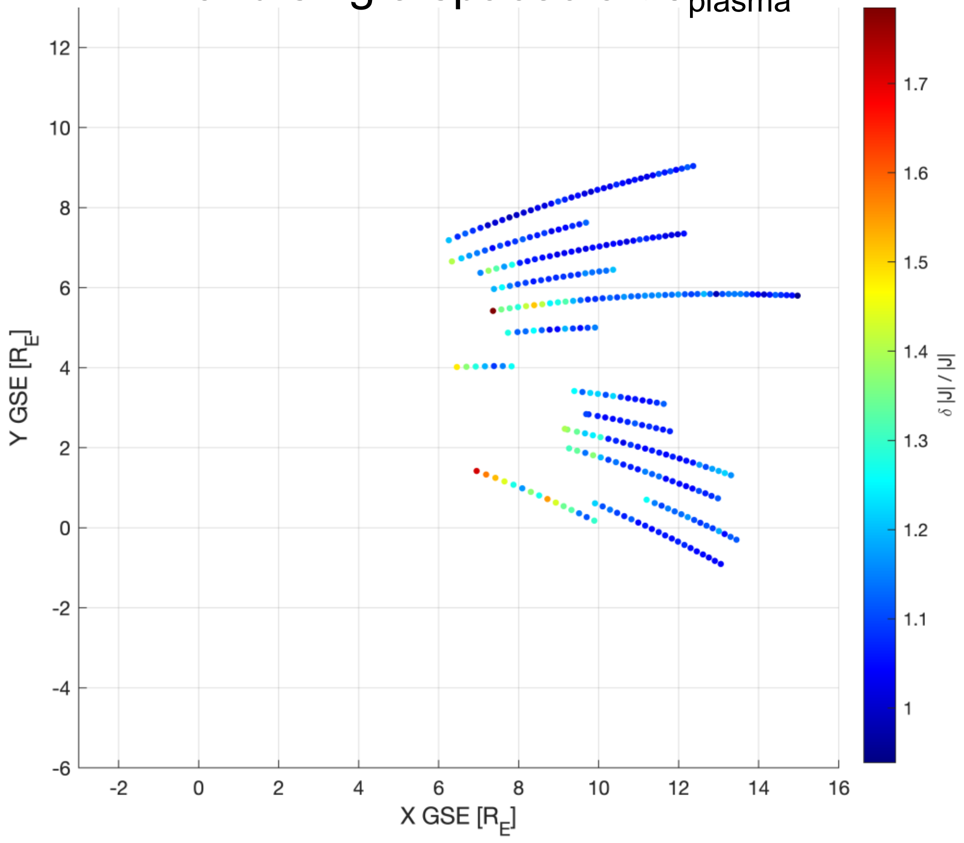


Evaluating with current data

Evolution of divB/curlB



Evolution of deviation between 4-spacecraft $J_{curlometer}$ and single-spacecraft J_{plasma}



We observe the expected decrease of accuracy as the spacecraft formation evolves away from a regular tetrahedron

Estimating Accuracy

Magnetic field:
Curl of B: Current density

$$\nabla \times \vec{B} = \vec{J}$$

- Independent measurement derived from plasma instruments
  (less accurate, single point, lower time resolution)

- Divergence of B

$$\nabla \cdot \vec{B} = 0$$

Test of the linearity assumption

Velocity: More complicated
No independent measurement to compare against

- Roberts et al. JGR 2023 provides uncertainty estimates

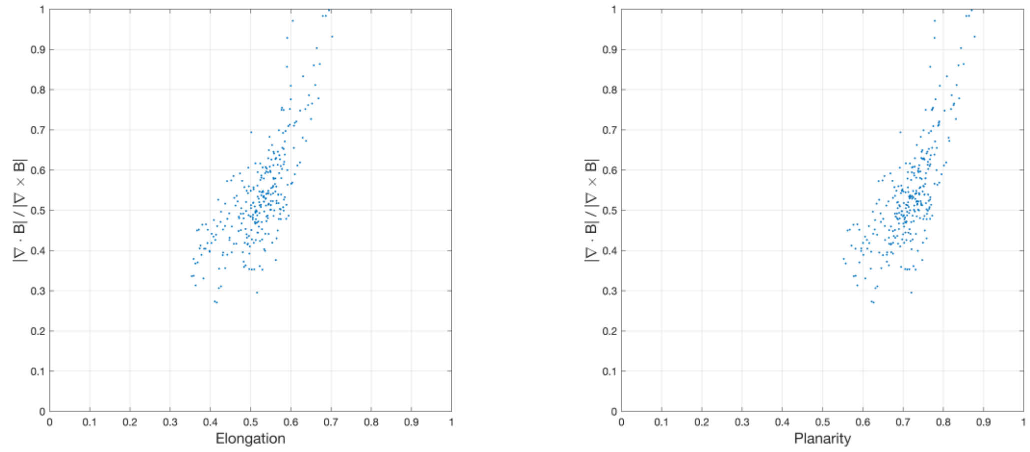
Evaluating with current data

We can directly measure the evolution of the accuracy of the curlometer estimates as the spacecraft formation quality factors change

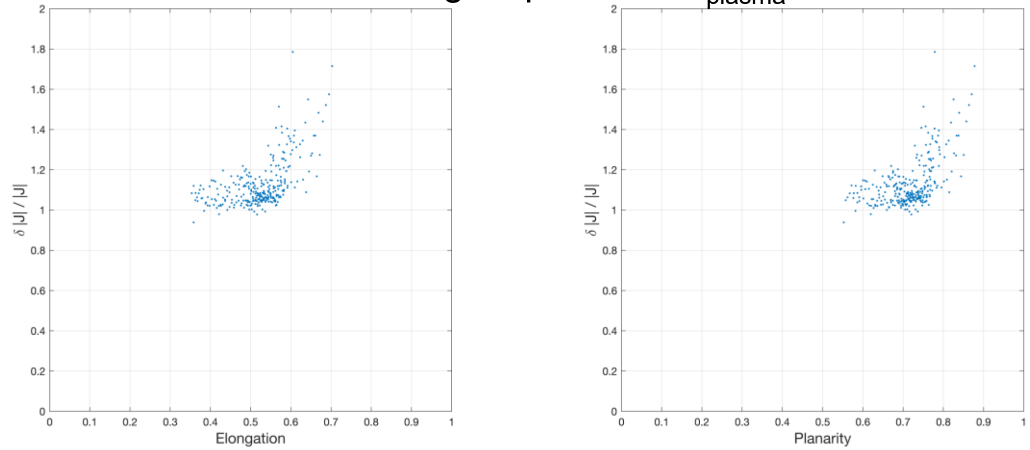
- We observe the expected decrease of accuracy as the spacecraft formation evolves away from a regular tetrahedron

- These observations can be used to support the thresholds set for current MMS studies and future mission design

Evolution of divB/curlB



Evolution of deviation between 4-spacecraft $J_{curlometer}$ and single-spacecraft J_{plasma}

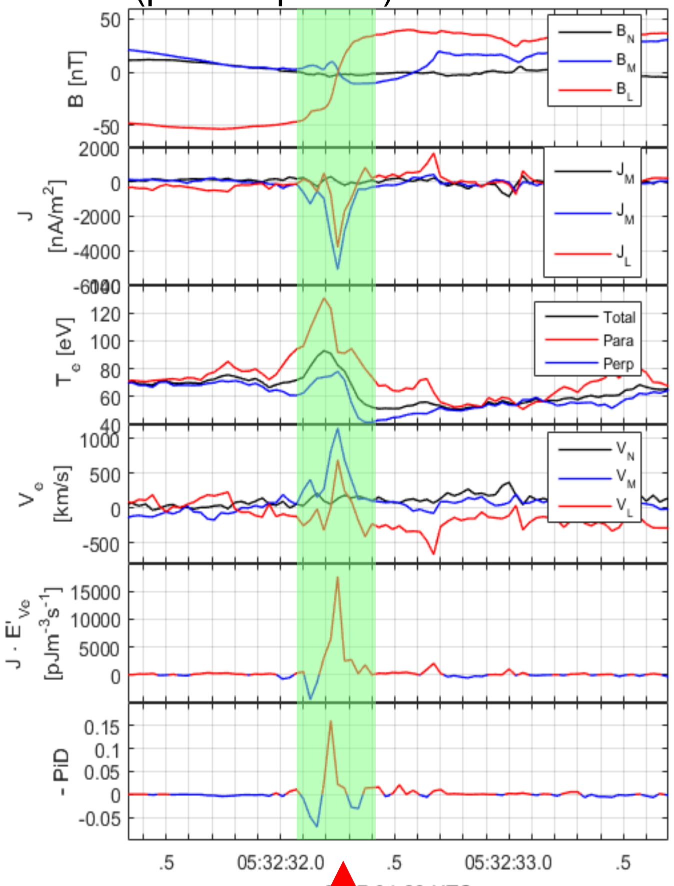


Measuring dissipation

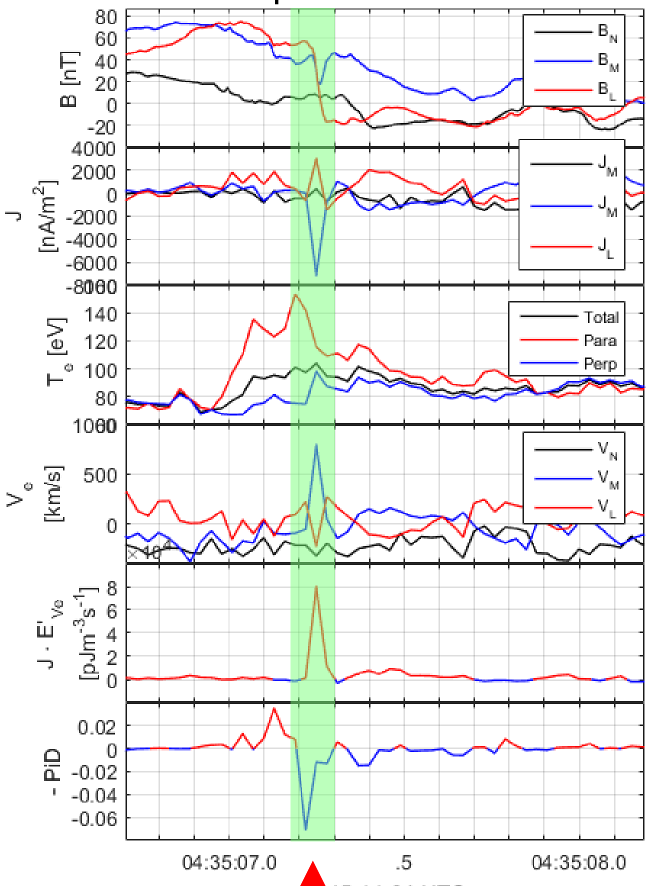
Events from Wilder et al. JGR 2018

- MMS provides multi-point high-resolution plasma and fields measurements
- Compute quantities relevant to kinetic physics:
  e.g. VDFs, 3D gradients of fields
- **We can resolve kinetic dissipation**
- **Directly observe energy conversion pathways in turbulence**
- **Track particle heating and energization**

Anti parallel Reconnection
(plasma β ~1.3)



Stronger Guide-field (~1.3)
Plasma β~10



Pressure-strain measurements in Chasapis et al. 2018

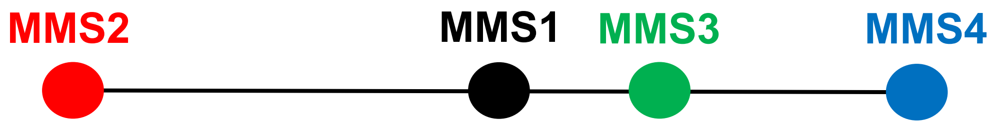
Electrons are being heated

Electrons are locally cooling

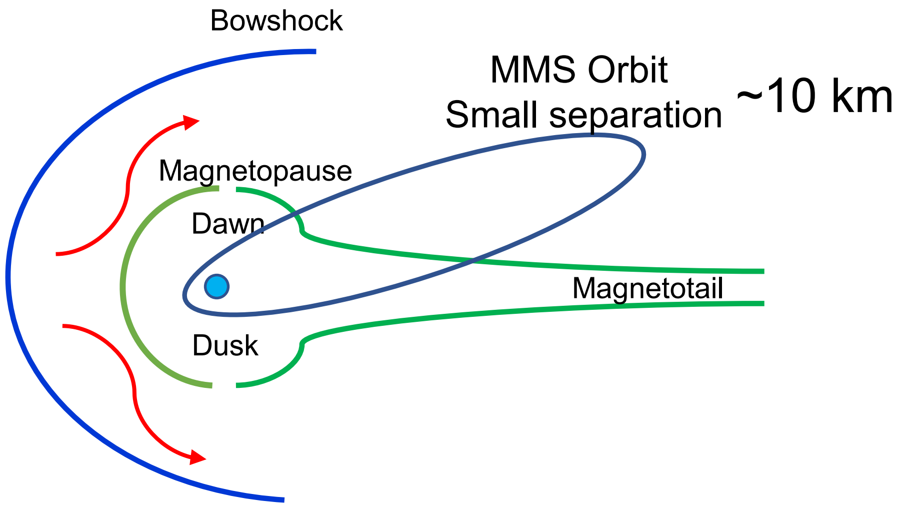
Variable separation - Tail

The MMS spacecraft can be arranged in a colinear configuration

The spacecraft slowly drift apart, increasing the separation on each orbit



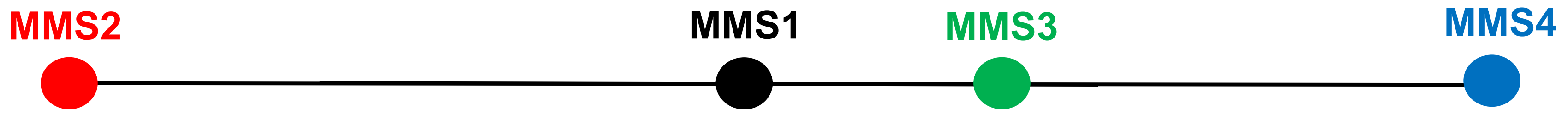
- Small to large separation during the tail season



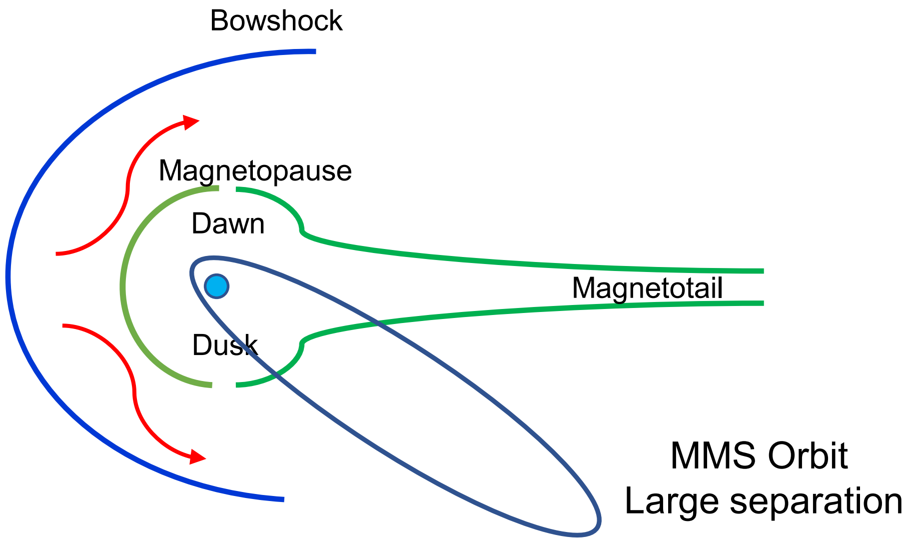
Variable separation - Tail

The MMS spacecraft can be arranged in a colinear configuration

The spacecraft slowly drift apart, increasing the separation on each orbit



- Small to large separation during the tail season

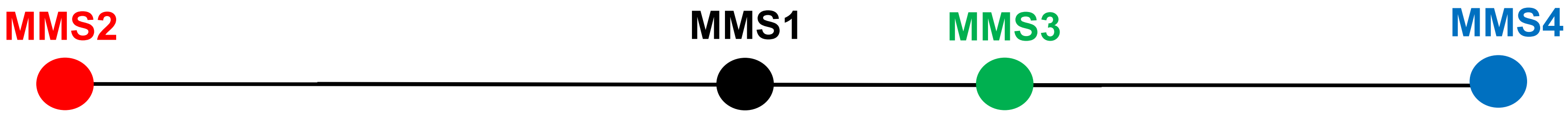


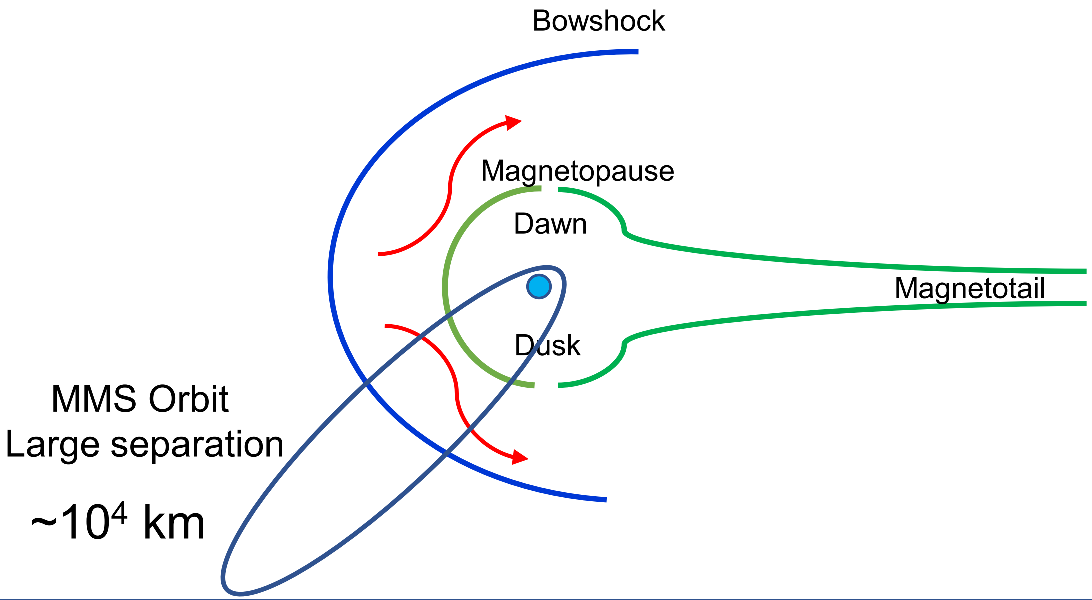
~10^4 km

Variable separation - Dayside

The MMS spacecraft can be arranged in a colinear configuration

The spacecraft slowly drift apart, increasing the separation on each orbit



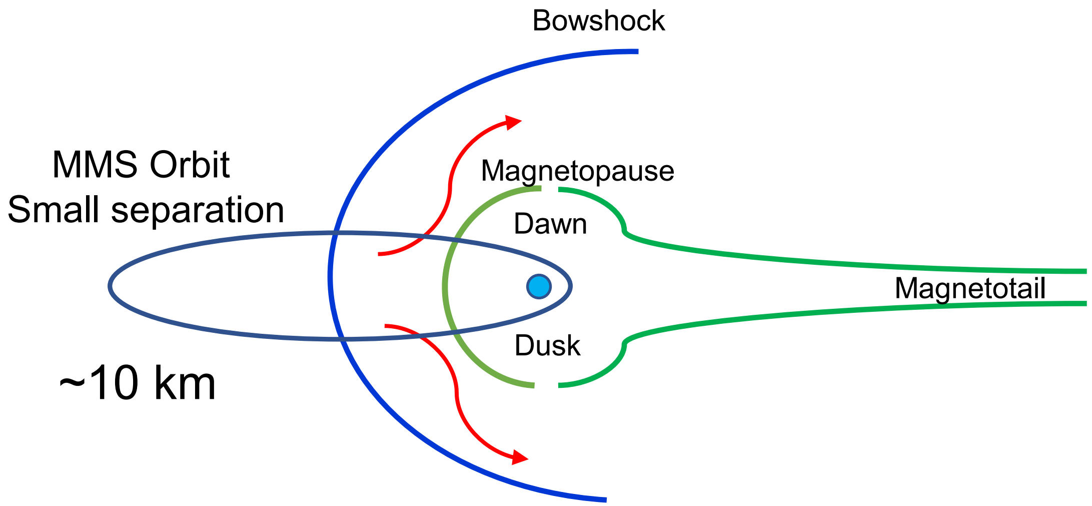


- Small to large separation during the tail season
- Large to small separation during the following dayside season

Variable separation - Dayside

The MMS spacecraft can be arranged in a colinear configuration

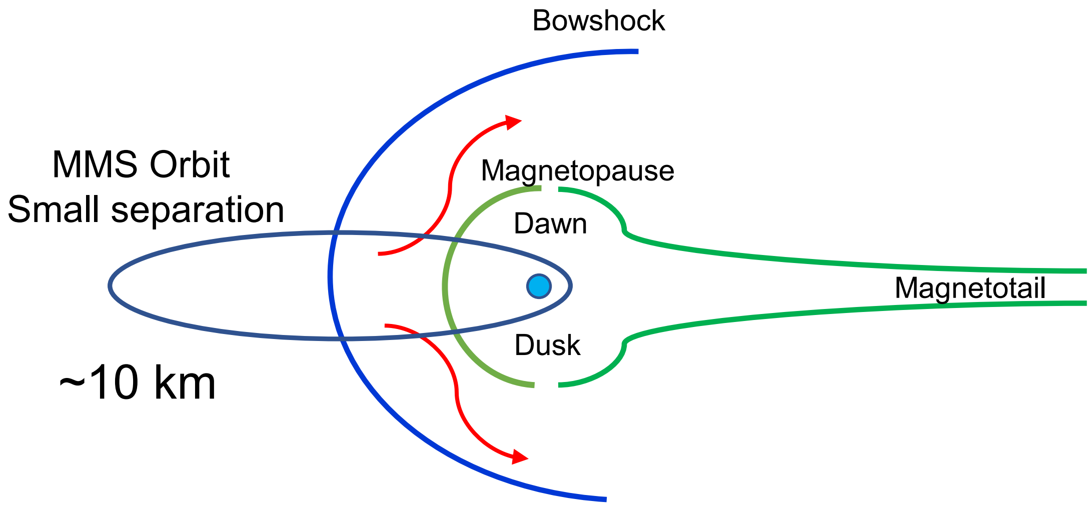
The spacecraft slowly drift apart, increasing the separation on each orbit



- Small to large separation during the tail season
- Large to small separation during the following dayside season

The MMS spacecraft can be arranged in a colinear configuration

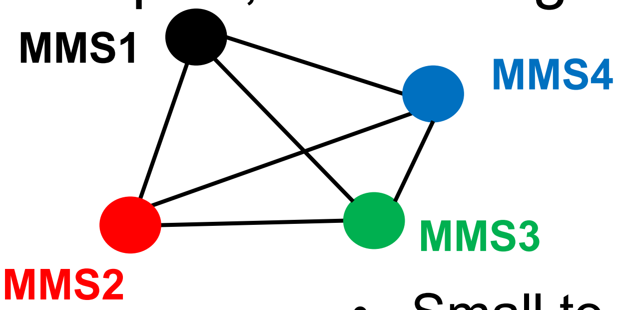
The spacecraft slowly drift apart, increasing the separation on each orbit

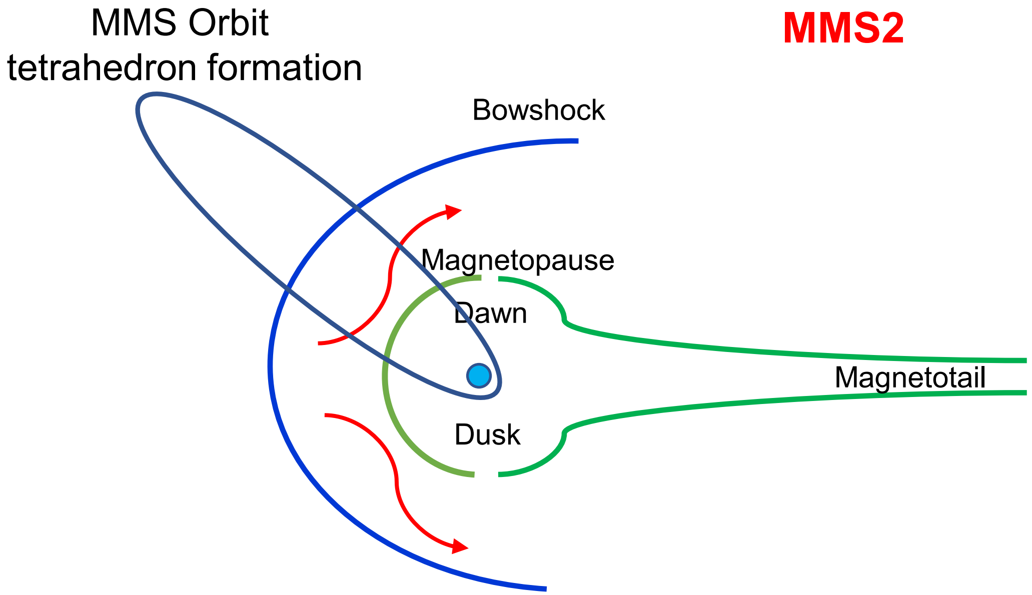


- Small to large separation during the tail season
- Large to small separation during the following dayside season *with one trailing spacecraft

The MMS spacecraft can be arranged in a colinear configuration

The spacecraft slowly drift apart, increasing the separation on each orbit





- Small to large separation during the tail season
- Large to small separation during the following dayside season *with one trailing spacecraft
- Back to tetrahedron for the next phase