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The essential role of multi-point measurements in investigations of turbulence, three-dimensional structure, and dynamics: the solar wind beyond single scale and the Taylor Hypothesis

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Preface.

Space plasmas are three-dimensional dynamic entities. Except under very special circumstances, their structure in space and their behavior in time are not related in any simple way. Therefore, single spacecraft in situ measurements cannot unambiguously unravel the full space-time structure of the heliospheric plasmas of interest – in the inner heliosphere, in the Geospace environment, or the outer heliosphere. This shortcoming leaves numerous central questions incompletely answered. Deficiencies remain in at least two important subjects - Space Weather and fundamental plasma turbulence theory - due to a lack of a more complete understanding of the space-time structure of dynamic plasmas. Only with multispacecraft measurements over suitable spans of spatial separation and temporal duration can these ambiguities be resolved. We note that these characterizations apply to turbulence across a wide range of scales, and also equally well to shocks, flux ropes, magnetic clouds, current sheets, stream interactions, etc. In the following, we will describe the basic requirements for resolving space-time structure in general, using "turbulence" as both an example and a principal target or study. Several types of missions are suggested to resolve space-time structure throughout the Heliosphere.

Introduction.

Turbulence refers to complex dynamics of fluid and plasma systems when nonlinear effects, such as advection, are stronger than dissipative effects. Dimensionless parameters such as a Reynolds number measure the ratio of the strengths of nonlinear and dissipative effects. The usual picture of turbulence begins with a source of large-length scale fluctuations which, through nonlinear processes, transfers energy by cascade processes across the 'inertial range' to the shorter scale lengths of the 'dissipation range' where the energy is dissipated into thermal motions of the fluid or plasma. Small-scale turbulent motions become so disorderly that theory frequently employs statistical descriptions, even if the dynamics is formally deterministic [1]. Revealing the physics of turbulence in the heliosphere (with implications for astrophysical plasmas in general) will require multi-point observations and an array of spacecraft with a range of inter-spacecraft spatial separations [2]. The technology for such missions has been in large part demonstrated by Cluster and MMS, wtth needed additional capabilities under development for Helioswarm. Therefore the multispacreaft missions suggested below, with their diverse scientific goals, are feasible, and their implementation will have an enormous impact on heliospheric applications [3].

Complex dynamical couplings in turbulence lead to small-scale dissipation of the energy supplied at large scales – a process described as a cascade. Experiments, observations, and numerical simulations all show that analogous descriptions apply to hydrodynamic fluids, magnetofluids (MHD), and weakly collisional plasmas. The greatest similarities are found at the larger scales, while plasmas differ at small scales and high frequencies due to the deficit of collisions and the concomitant emergence of complex kinetic physics. Understanding turbulence requires intensive study of statistical properties for the varying parameters found in nature. We argue that multi-point measurements over a range of scales are required to make significant progress in solar wind physics, which remains the only turbulent space plasma for which such a program is feasible. An array of spatially distributed spacecraft making measurements at moderately high time cadence can provide a wealth of information for space physics applications, including space weather [37], and would be of great importance in more distant plasma venues, from the corona to the interstellar medium.

Turbulence effects in the heliosphere.

The effects of turbulence are intrinsically multi-scale, and the solar wind cascade process spans decades of spatial and temporal scales. One might view the turbulence cascade as a primary way in which cross-scale couplings are enabled, connecting macroscopic and microscopic physics in essential ways. Among the most impactful macroscopic influences of heliospheric turbulence [3] is the putative heating of the corona [31] and subsequent solar wind acceleration, which, despite numerous supporting observations, remains to be fully established; a fundamental role is played by missions such as Parker Solar Probe and Solar Orbiter. In the solar wind, extended heating is likely also due to turbulent cascade [4], which operates at different rates in the high cross helicity fast wind, and in the lower cross helicity slow wind [5]. Cross helicity (Alfvénicity) slows the development of turbulence initially, but eventually expansion [6] and shear [7] cause a systematic reduction of this Alfvénic correlation. Similar turbulence effects account for the radial behavior of the Alfvén ratio (or, residual energy), and for spectral steepening in Helios data [8]. Accordingly, turbulence also appears to account well for the radial evolution of the (low-frequency) spectral breakpoint that is closely associated with the systematic increase of the correlation scale of the fluctuations. It is noteworthy that these effects are inconsistent with WKB theory of non-interacting waves [3].

All of the above effects are essentially at the larger collective fluid-like scales. Over a range of scales, extending several decades towards the smaller range, theory suggests that the dynamical development of heliospheric turbulence is responsible for the very important observed features of anisotropy [9] and intermittency [10]. See [3] for details.

Another arena in which turbulence is a major player is the transport, scattering, and acceleration of suprathermal and energetic charged particles. In this case, effects such as pitch-angle scattering also operate in a truly cross-scale manner; with solar wind thermal protons resonantly interacting with turbulent fluctuations at the scale of a few hundred kilometers at 1au, while 10Gev cosmic rays or SEPs resonantly interact with fluctuations at scales of millions of kilometers. Turbulence amplitudes and spectral anisotropy are central in controlling interactions, including resonances, with these energetic particles, e.g. [11].

Given all these demonstrated or anticipated influences, one may reasonably ask at what level do we understand the turbulence that produces these diverse effects in the heliosphere? The answer seems to be that, even with numerous accumulated observational constraints and a reasonable level of progress based on simulation and theory, there are fundamental questions that remain to be addressed experimentally. Simple, idealized steady-state inertial range phenomenologies can provide motivation for observed spectral slopes, but physical understanding of these diverse cross-scale effects, even in the inertial range, requires deeper knowledge and more advanced observations. Beyond inertial range issues, there are questions about dissipation that involve structures and dynamics at sub-proton kinetic scales [12 -14]. Due to the cross-scale couplings and cascade mechanisms involved, the kinetic processes are necessarily driven by the cascade from larger energy-containing scales [15]. This poses further observational challenges for understanding dissipative structures and bulk heating in the corona and solar wind.

Major questions in solar wind turbulence. There are numerous outstanding issues about heliospheric turbulence that have not yet been addressed in observations, *in particular, due to a* *lack of sufficient spatial and temporal resolution.* Without the associated observations, the field cannot advance beyond its current status. A few examples are given here.

Unraveling correlations & structure in space and time. Solar wind researchers are accustomed to employing the Taylor hypothesis, while plasma wave theorists are accustomed to invoking linear dispersion relations. Both of these provide a one-to-one correspondence of variations in space and variations in time. However, in general, spatial and temporal structures are independent entities. For example, the correlation scale is properly defined using single-time multi-point measurements [16]. In general, unraveling the space-time relationship is a necessary goal in quantifying and distinguishing the effects of turbulence, waves, reconnection, and other phenomena in space plasmas. Revealing how turbulent energy in a space plasma is distributed in space & time requires multi-point measurements.

Anisotropy of the spectrum at varying scales. Spectral information relative to preferred directions, e.g., radial and magnetic field directions, is required to validate or controvert available theoretical explanations. Purely phenomenological treatments do not provide strong conclusions. Anisotropic measurements are required, necessitating simultaneous multi-point measurements that span 3D spatial directions.

Direct measurement of scale transfer. What are the cascade rate and the heating rate? Can turbulence explain observed heating and the origin of the solar wind? The Yaglom - Kolmogorov 3rd order laws [17-19] provide a direct evaluation of energy transfer rates at a given scale. The simplest forms require isotropy or some other simple symmetry. Anisotropic forms of the 3rd order law have been applied using Cluster or MMS at single scales, but understanding cross-scale transfer requires anisotropic measurement at several scales. Simultaneous 3D multi-point measurements are needed to reveal how turbulent energy is transferred anisotropically across scales.

Higher order statistics and coherent structures. Intermittency or patchiness is an essential feature of turbulent heating and cascade processes. Indeed, in strong turbulence at high Reynolds numbers, most statistical measures of spectral transfer and dissipation are highly non-uniform. The fourth-order (single-time) statistics provide a baseline measurement of intermittency. The sixth-order statistics are a natural measure of the patchiness of energy transfer. These are fundamental but have not been fully characterized and measured in the solar wind, as they must be measured in anisotropic form, due to the strong influence of the large-scale magnetic field and strong gradients, e.g., in stream interaction regions and shear layers [32], as well as in regions of interaction of turbulence with waves [33, 34]. Measurement of 3D structure and orientation of coherent structures near the kinetic proton scales is needed to reveal the role of higher order moments in dissipation, thus requiring simultaneous 3D multi-point measurements at two or more spatial scales.

Anisotropic scale-dependent relaxation times. Because of the classic "closure problem" (the n-th moment depends on the (n+1)-th), higher order statistics at least up to 4th order contain fundamental information about dynamics. The single-time statistics are important, but so too are the decay rates of higher-order correlations. For example, the decay time of the 3rd order correlations controls spectral evolution. In the context of closures [20], the decay times of the triple correlations are identified with scale-dependent Lagrangian correlation times and are usually treated as the local Kolmogorov time scale, because the dominant sweeping timescale does not induce spectral transfer. In plasma, there are additional available time scales, and understanding 3rd and higher-order correlations become more complex. Observational constraints, including measurement of anisotropic 4th order (and higher) moments, are needed to understand this basic physics. Multi-spacecraft measurements are required over a wide range of scales to assess these crucial dynamical time scales.

Key Turbulence measurements.

A central quantity of interest is the two-point, two-time correlation of a primitive variable (e.g., a magnetic field component b.) This *4D* space-time correlation may be defined as $R_{ij}(r,\tau) = \langle b_i(x,t) b_j(x+r,t+\tau)\rangle$, where the brackets denote an ensemble average, or a suitable space-time average. The (trace) wave vector spectrum is $S(k) = [\frac{1}{2\pi}]^3 \int d^3r R(r,0) e^{ik\cdot r}$, in which the time lag is zero, as well as the Eulerian frequency spectrum $E(\omega) = \frac{1}{2\pi}\int d\,\tau\, R(0,\tau) e^{i\omega\tau}$, in which the spatial lag is zero. The full space-time (trace) $S(k,\omega)$ spectrum is analogously defined as the fourier transform of the 4D space-time R. This is an analog of a dispersion relation, but without the expectation of a definite relationship between frequency and wavevector. If a nonzero time lag is retained when the spatial transform is carried out, one arrives at the important quantity $S(k,\tau) = S(k)\Gamma(k,\tau)$. This defines the scale-dependent time correlation (in the Eulerian frame) $\Gamma(k,\tau)$, alluded to in the prior section. The space-time correlation also permits direct tests of the Taylor hypothesis. Observational determination of the 2nd order, two-time, two-point correlation contains much information that we require, but this is not all that is needed to describe interplanetary turbulence.

Relation of central importance is the third order law, which directly measures energy transfer across scales. The contribution of incompressive transfer is given by the Politano-Pouquet law [17]. Hall effect contributions and compressive contributions [21] can be treated additively. With suitable conditions on time stationarity, a pristine inertial range, etc., the relevant incompressive divergence form is $\nabla_s \cdot \langle \delta z_s^{\mp} |\delta z_s^{\pm}|^2\rangle = -4\epsilon^{\pm}$ for the ± Elsasser field increments and lag *s*. Integrating over a volume in lag space and employing Gauss' law yields a surface integral that determines the total incompressive transfer of the ± Elsasser fields across that surface. A suitable multi-spacecraft configuration (say, a regular tetrahedron) enables an approximate evaluation of this transfer [18, 22]. It is also clear that estimates based on the third order law will improve when a sufficient number of lag directions are available which enables direction averaging carried out in an appropriate way [40, 41, 42]. *Carrying out this multi-spacraft measurement provides a direct evaluation of scale transfer with no approximations on rotational symmetry.* In various combinations, this approach can be supplemented with, or compared to, single spacecraft results using the frozen-in flow (Taylor) hypothesis, including methods that assume isotropy and other symmetries, e.g. [23]. This approach can also reveal potential cascades to both large and small scales [33, 34, 35].

Fourth-order correlations are also crucial, as they quantify intermittency, and drive all important third-order correlations. In MHD the effect of a mean magnetic field appears in the moment hierarchy at the same order as the 4th order correlations [36]. Together with the mean field, the 3rd and 4th order correlations influence the production of spectral anisotropy [9, 24], a major issue in plasma cascade and dissipation [14]. Also at 4th order is the anisotropic *scale-*

dependent kurtosis, a quantity that reveals scale-varying intermittency, anisotropy of coherent structures, and incoherent wave activity, as seen in the reference [25] using MMS data.

Much theoretical attention is paid to the inertial and kinetic cascade ranges in plasma turbulence. However, ***the last stages in which collective flow and field energies are converted into microscopic motions or "heat" are crucial for understanding dissipation***. Two quantities of great importance in this regard are the work done on particles of species 〈, that is, $J_\alpha \cdot E$ by the e/m fields, where E is the total electric field, and the pressure strain interaction $P^{\alpha}_{ij}\ S^{\alpha}_{ij}$, where P^{α} is the pressure tensor and $S^{\alpha}_{ij} = \partial_i u^{\alpha}_j + \partial_j u^{\alpha}_i$ is the symmetric rate of strain tensor, each of species 〈 [26, 27]. Even though these quantities are not sign-definite, as viscous dissipation would be, their net (averaged) values are interpreted as the conversion of e/m energy into flow energy, and the conversion of energy in the flow into internal energy. These channels of energy conversion are agnostic regarding specific mechanisms (e.g., reconnection) that may be producing heating, and are therefore crucial diagnostics for understanding the termination of the cascade and the degeneration of collective motions into internal energy. Multi-spacecraft techniques again enter prominently, as the total J can be evaluated by curlometer techniques, while the rate of strain tensor can be similarly evaluated by differencing the velocities across spacecraft pairs.

Summary. Statistical quantities such as those above are essential to fully understand turbulence, energy cascade and dissipation in magnetized plasmas. Correlations are expected to be anisotropic and proper analysis requires measurement at several lag scales. Reasonable choices are, first, near the ion kinetic scales, and then at two or more larger scales, where the fluid MHD-like cascade remains operative. Methods have been developed to extract space-time information from multi-spacecraft datasets, including wave telescope (or k-filtering) [28] and direct methods that rely on ensemble statistics [29]. Some such methods have been successfully applied in plasma laboratory experiments [e.g., 38].

Mission Concepts

Near 1 au: Turbulence and Helioswarm. The recently selected *HelioSwarm* mission consists of nine spacecraft in orbit near 1 au, providing 36 baselines for two point measurements. (See Decadal white paper by Klein et al.) This is a breakthrough mission with regard to resolving spacetime ambiguity and for the study of fundamental turbulence properties, covering almost two decades of scale from less than 100 km to more than 1000 km. This is the first Heliophysics mission devoted principally to multiscale turbulence physics and will act as a pathfinder for demonstration of a variety of turbulence analysis techniques. It is expected to reveal fundamental physics of turbulence in the solar wind [30], with immediate implications for the corona [31] and other space and astrophysical plasmas. Helioswarm targets fundamental science done mainly with second and third order statistics, as well as higher order multifractal scalings. A significant step beyond this would be to employ a larger number of spacecraft so that the topology of the turbulence and even images of the turbulence can be obtained. A recently discussed mission of this type is MagnetToRE [39], which deploys "many magnetometers over 100's of km" to enable reconstruction of images of the magnetic field using AI techniques. Analysis of scintillations of the interspacecraft radio communications signals can also provide images of the plasma density. Preliminary analysis shows that a constellation of approximately 30 nanosats each carrying radio and magnetometer, would be able to accomplish such image reconstruction. This new type of helioscience would contribute to Space Weather as well as a general understanding of the morphology of the interplanetary magnetic field and density. MagneToRE is just one example of the next steps in understanding the 3D structure of the Heliospheric plasma and a mission of this type is a natural significant step beyond Helioswarm.

L1 Cluster/"MMS in Solar Wind". The numerous breakthrough discoveries in space plasma physics due to MMS mission are of enormous significance. However, the scientific goals of the mission are mainly magnetospheric and not interplanetary, so the instrument design is less effective in the solar wind. MMS has achieved enormous progress in revealing details of reconnection and turbulence in the magnetosphere, as well as other fundamental areas of magnetospheric kinetic plasma physics. Consequently there is significant motivation to pursue analogous progress in solar wind kinetic physics by developing and deploying a four spacecraft interplanetary mission having high resolution instruments based on adaptations of MMS technology. Such a mission can be expected to revolutionize understanding of solar wind kinetic physics as much as MMS has advanced magnetospheric kinetic physics.

If such a solar wind-adapted MMS-like cluster is placed at L-1 in a halo orbit similar to ACE, the mission can also contribute significantly more to heliospheric physics. Even without continuous transmission of the highest resolution measurements, the L1 cluster would provide considerable support for contemporaneous L1 spacecraft serving the role of space weather monitoring (such as ACE or IMAP) . The L1 cluster would provide a second point for estimation of structure sizes and large scale gradients that can refine boundary conditions for use in global magnetospheric modeling of response to solar wind conditions. Determination of local gradients and rates-of-strain can also provide inputs to more advanced modeling. Curvature of field lines, shocks, surfaces of discontinuity, and measurement of internal structure of the heliospheric current sheet are other major enhancements to L-1 monitoring that would be enabled.

Space Weather Missions in the Inner heliosphere It has long been recognized that multispacecraft missions situated well inside of 1AU can make major contributions to the science underlying space weather as well as contributions to prediction or operations. In this regard, over the past two decades, there have been concept studies such as Solar Wind Sentinels, Inner Heliospheric Mappers, Solar Flotilla, Heliospheric Constellation and Multispacecraft Heliospheric Mission. These have been discussed as contributions to Space Weather and NASA Living with a Star science and as such have emphasized detection of the size, shape and trajectories of relatively large scale structures such as CMEs, CIRs, shock surfaces, etc. A major goal usually to improve predictions of solar wind conditions later at Earth. Various formations have been suggested. For example, *Sentinels* employed solar sails (10 m x 100 m) to reduce the solar-directed acceleration on the spacecraft so that they can orbit the Sun with a one-year period well inside the L1 point at approximately 0.98 AU or climb out of the ecliptic plane to provide information on vertical gradients. These spacecraft would be spaced up to 0.1 AU apart longitudinally bracketing the Sun-Earth line and providing additional warning time for corotating solar wind structures above that available from monitors on the Sun-Earth line. Measurements of the gradients of the encountered structures enable more accurate models to be developed, further allowing better predictions of their size and duration at Earth to be produced. Other ideas would place multiple microsatellites in solar elliptic orbits to enable a greater understanding of how particles and fields are transported from the solar corona. Such a mission might study the physics of shocks and shock particle acceleration; utilize spatial and temporal scales for energy dissipation and transfer in the solar wind, study the magnetic structure of the inner heliosphere, and visualize the dynamics of the inner heliosphere. This kind of *Solar Flotilla or Heliospheric Constellation* could consist of multiple autonomous microsatellites in several (three) principal solar elliptic orbits at 0.2 to 0.4 AU, with two to six microsatellites per orbit. Significant variation in inter-spacecraft separation would be possible depending on the orbital details. This class of mission, while primary goals would be space weather-related, would also make significant contributions to very large scale nonlinear plasma dynamics (e.g., origins of turbulence) and the understanding of magnetic structure and SEP propagation.

Cluster of Solar Probes. The Parker Solar Probe Mission has made and will continue to make breakthrough discoveries in the regions of the inner heliosphere that have never been explored before. The unprecedented close approach to the Sun allows the pioneering discovery of new features in the young turbulence and structures. However, ambiguities remain. In the future, a cluster of Parker Solar Probes will be needed to try and answer the questions that the single spacecraft alone cannot resolve. Among these, disclose the (local) 3d structure of the Alfvén zone [Rohit], investigate the substructure of the heliospheric current sheet, revolve the topology of newborn magnetic structures, and more accurately measure the transfer of energy in the lower corona.

Conclusion

Future interplanetary missions with a sufficient number of spacecraft will employ multi-point methods to answer many basic questions about three dimensional structure and dynamics, related to turbulence and Space Weather, that can be experimentally addressed in no other way [2]. Such missions may span various ranges of scales, varying inter-spacecraft separations depending on their particular emphasis, and may make significant contributions by placing them in various positions in the heliosphere, including near Earth, close to the Sun, and at intermediate distances. These missions are technologically feasible and can be appropriately prioritized to maximize scientific and societal benefit.

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