

The heliospheric space plasma
physics in the era of multipoint
space missions

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New Multipoint Techniques for the Comprehension of Turbulence in the Heliosphere

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Motivation

The exploration of space plasmas is entering a new era of multi-satellite constellations, enabling the determination of the properties of the interplanetary medium with unprecedented precision. This progress requires moving beyond imprecise approximations to advanced analytical approaches

Multipoint measurements are crucial for investigating central phenomena in heliospheric dynamics, such as magnetic reconnection, plasma turbulence, shock waves, and particle acceleration. These lectures will focus on the role, methods, and relevance of multipoint observations, with a key emphasis on advanced techniques for analyzing the turbulent cascade in the collisionless, interplanetary medium



Overview on the Lectures

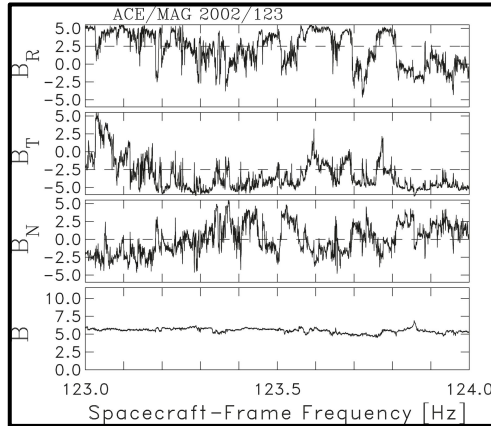
- (I) Turbulence in the heliosphere: an overview**
- (II) Simulations to support observations**
- (III) Virtual spacecraft with constellations: second-order statistics**
- (IV) An exact law of turbulence**
- (V) A novel method for measuring the turbulent dissipation rate**
- (VI) From virtual to real: MMS**
- (VII) Beyond 3D: space-time statistics and time decorrelation**
- (VIII) Space weather applications and conclusions**

(I) Turbulence in the Heliosphere: an Overview

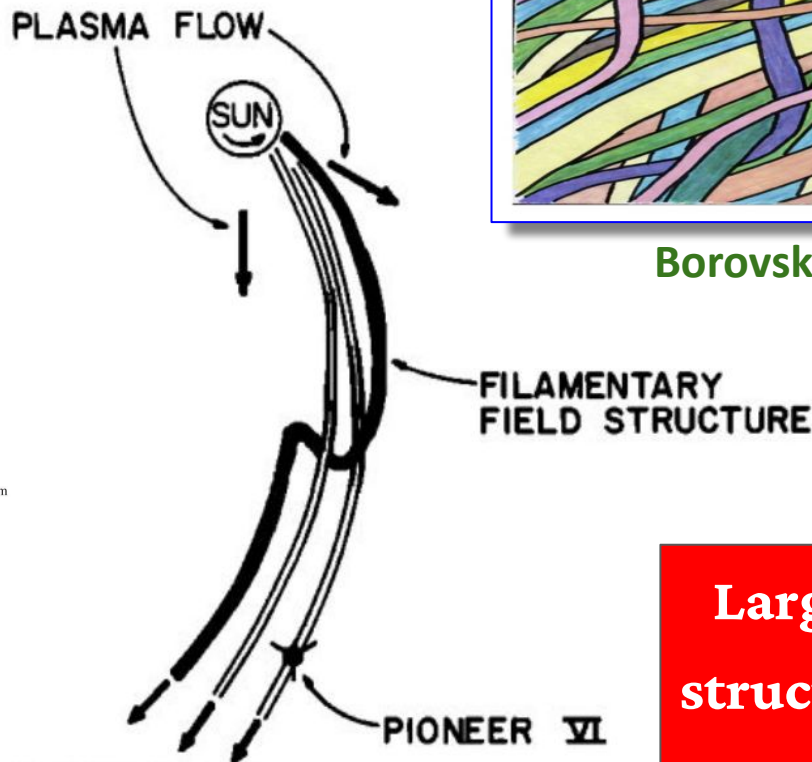
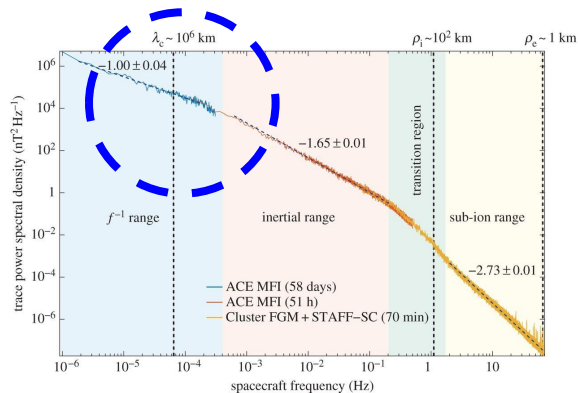




Turbulence (and more) in the Heliosphere

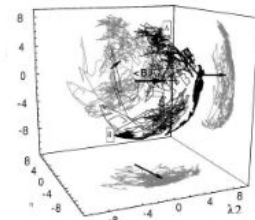


Kiyani, PTRSA (2008)



Borovsky+, JGR (2008)

Bruno+

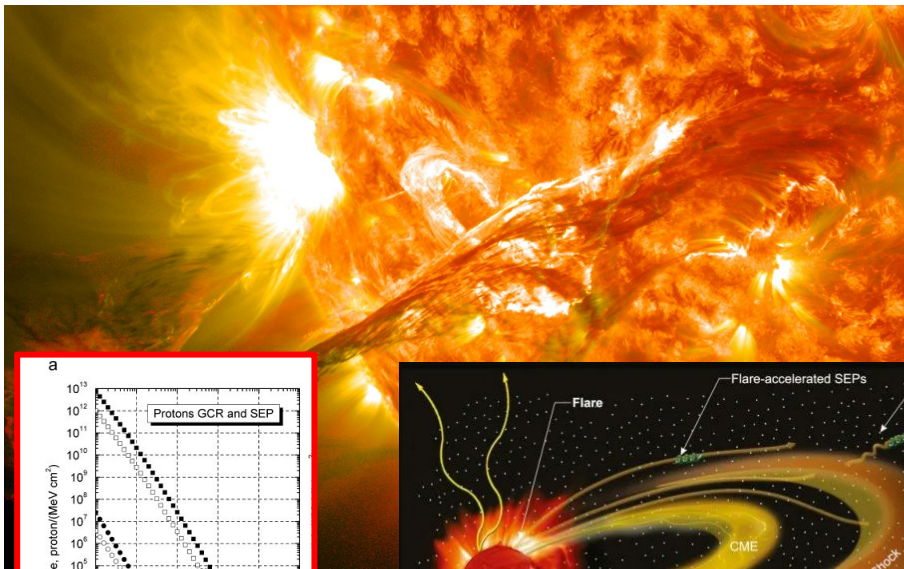


Large scale magnetic structures populate the wind



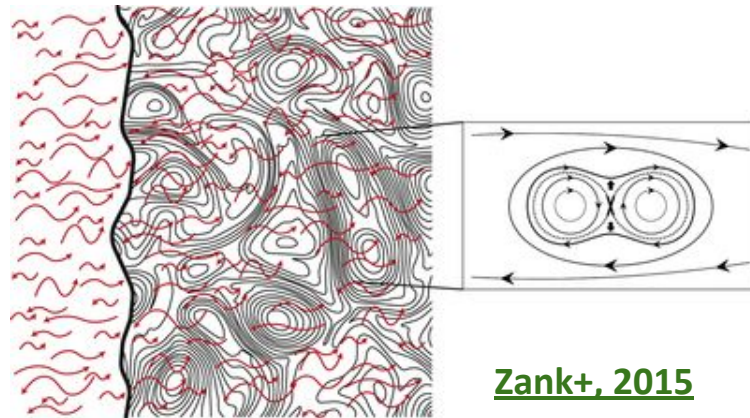
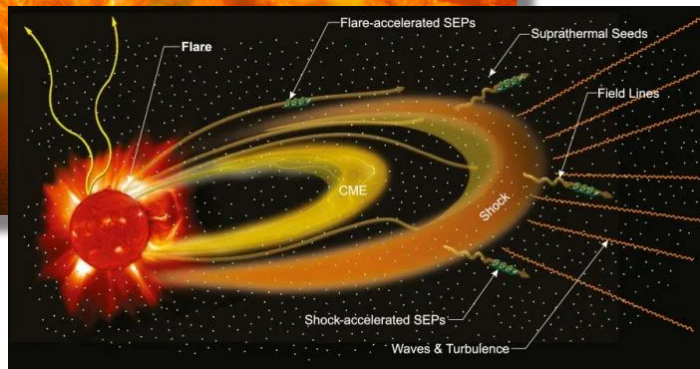
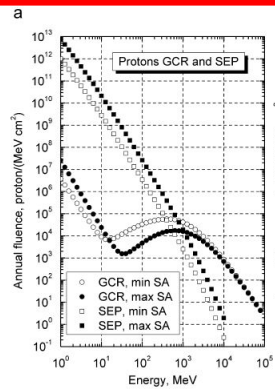
Explosive Events over Turbulence: Complexity²

SEPs originate from the Sun, primarily during events like solar flares and coronal mass ejections (CMEs)



Turbulence can spread out particles and local reconnection events might cause an extra-acceleration.

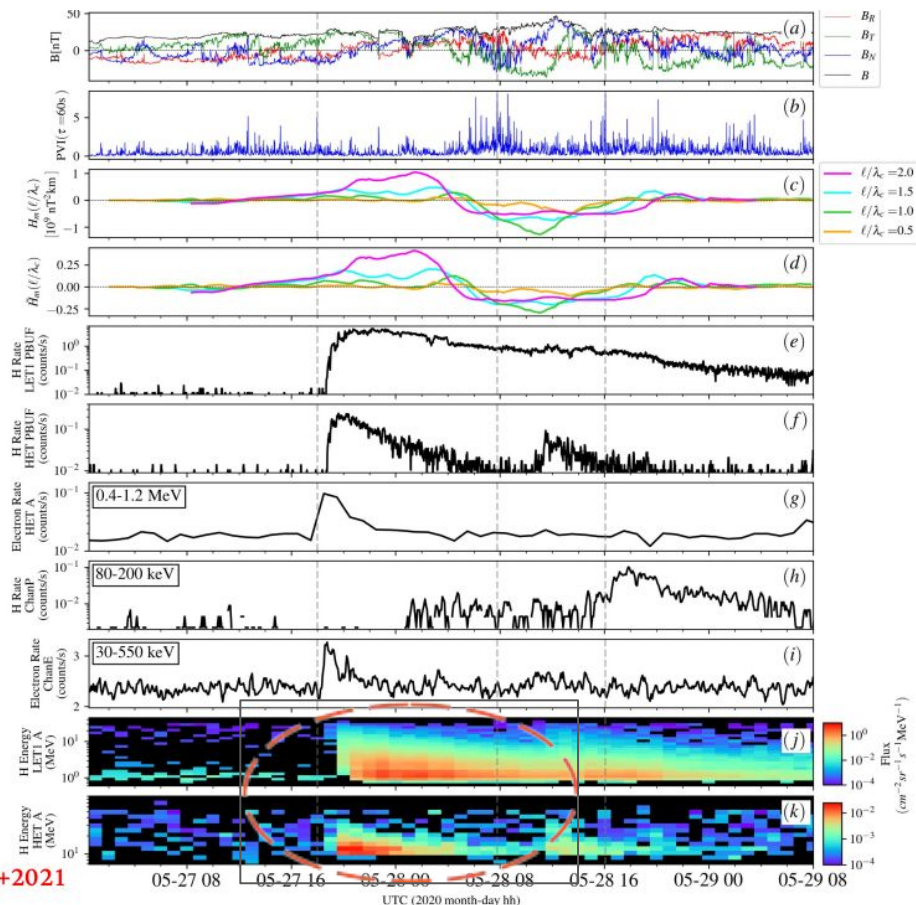
It can slow down the path. but also accelerate. It can defocus the beam!



Zank+, 2015



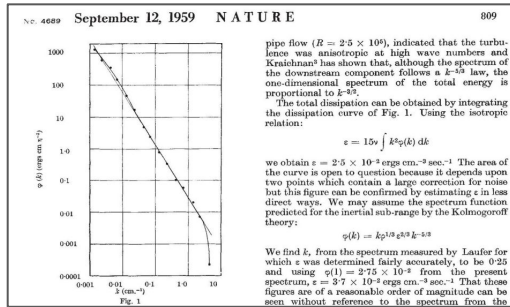
Details from in Situ Measurements



Parker Solar Probe reveals
“structured” energetic
particles events

The Starting Point: Hydrodynamics

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v} \cdot \nabla \mathbf{v} - \nabla P + \nu \nabla^2 \mathbf{v}$$

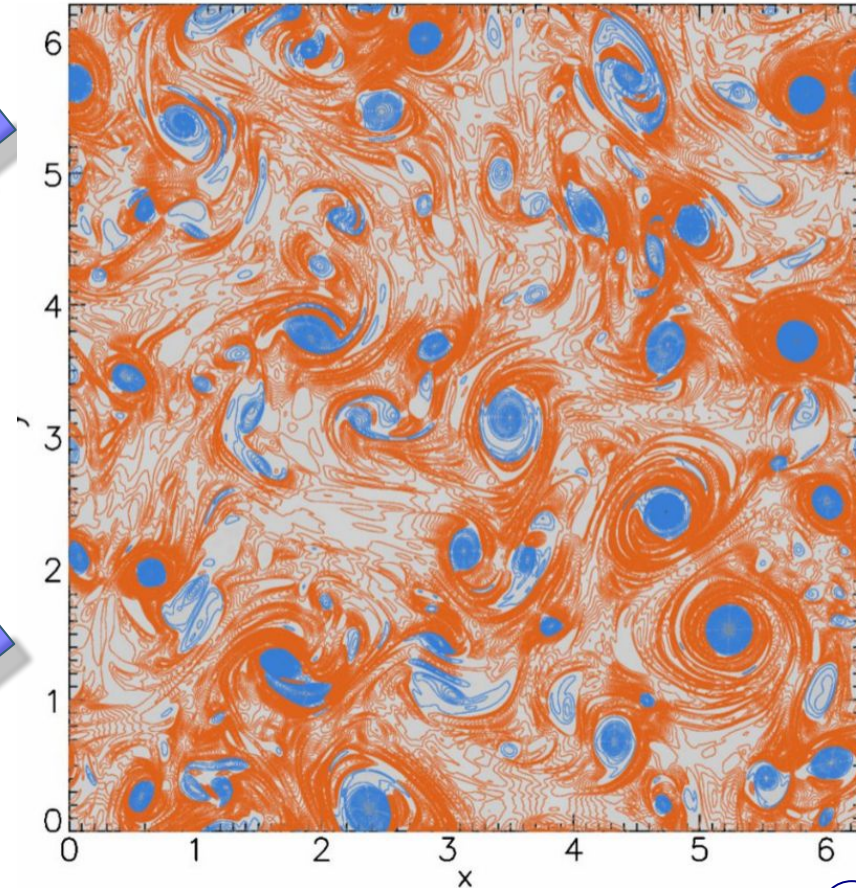


H. L. Grant, A. Moilliet, R. W. Stewart, A spectrum of turbulence at very high Reynolds number, Nature volume 184, 808–810 (1959)

- Energy is distributed across the scales with a power law distribution

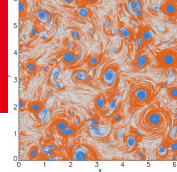
- Coherent, long-lived structures appear as a result of a local relaxation process

Pelz+ (1985)
SS+, POP (2010)





The Kolmogorov Theory in one Slide

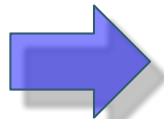


$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v} \cdot \nabla \mathbf{v} - \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{f} \longrightarrow \frac{\partial \mathbf{v}_k}{\partial t} = -i \sum_{p+q=k} (\mathbf{v}_q \cdot \mathbf{p}) \mathbf{v}_p - i \mathbf{k} P_k - \nu k^2 \mathbf{v}_k + \mathbf{f}_k$$

$$(I) \quad \varepsilon_k = \frac{|\mathbf{v}_k|^2}{\tau_k} = \nu \langle \omega^2 \rangle = \langle \mathbf{v} \cdot \mathbf{f} \rangle = const \equiv \varepsilon$$

K41

$$(II) \quad \tau_k \sim \frac{1}{k v_k}$$



$$P(k) = C \varepsilon^{2/3} k^{-5/3}$$

but also:

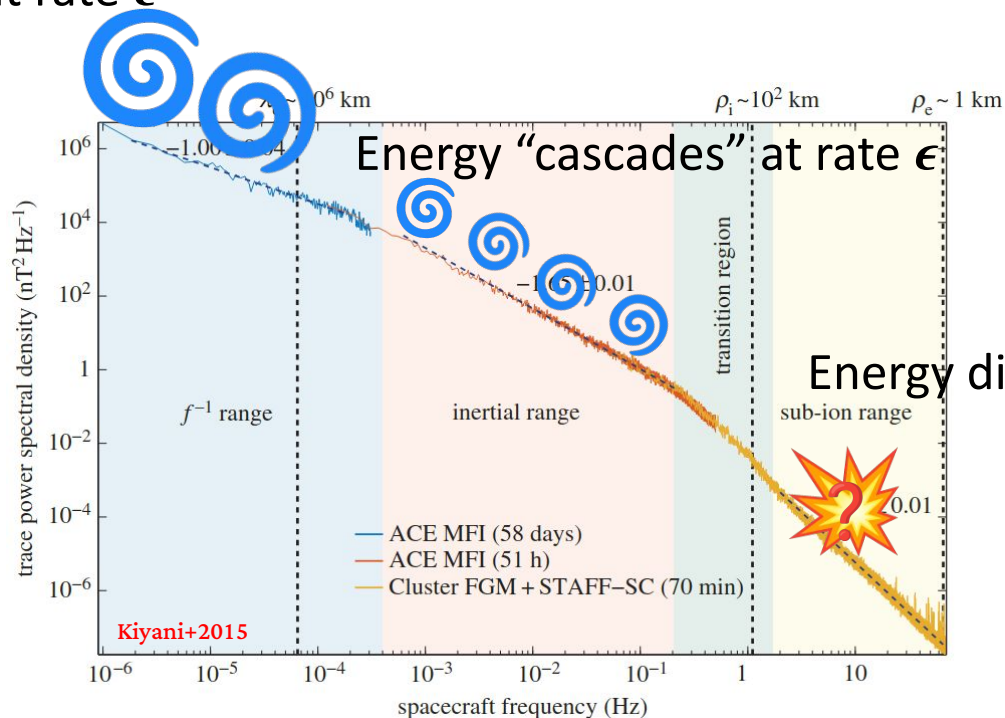
$$S_2(\ell) = \langle |\mathbf{v}(\mathbf{x} + \ell) - \mathbf{v}(\mathbf{x})|^2 \rangle \sim \ell^{2/3}$$

$$S_m(\ell) = \langle |\mathbf{v}(\mathbf{x} + \ell) - \mathbf{v}(\mathbf{x})|^m \rangle \sim \ell^{m/3}$$

$$(III) \quad E = \langle v^2 \rangle = \sum_k v_k^2 = \int P(k) dk$$

A Basic Picture of Turbulence

Energy injected at rate ϵ



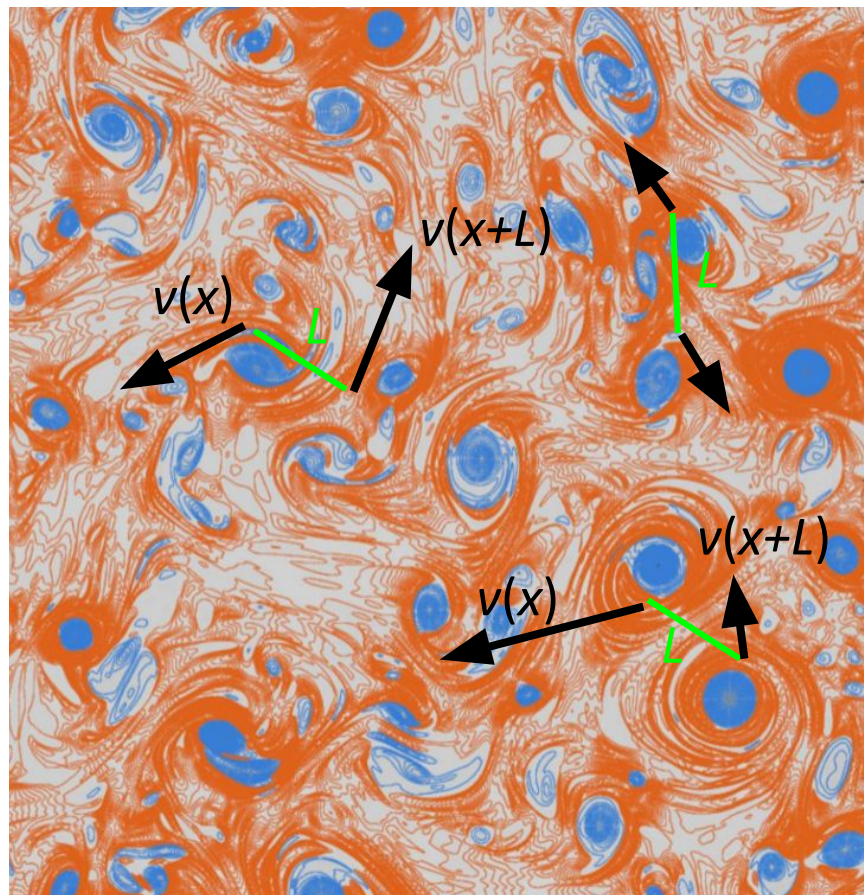
Injection

Cascade

Dissipation



Structure Functions and Spectra



$$S_2(\ell) = \langle |\mathbf{v}(\mathbf{x} + \ell) - \mathbf{v}(\mathbf{x})|^2 \rangle \sim \ell^{2/3}$$



Blackman–Tukey

$$C(\ell) = E - \frac{S_2(\ell)}{2}$$

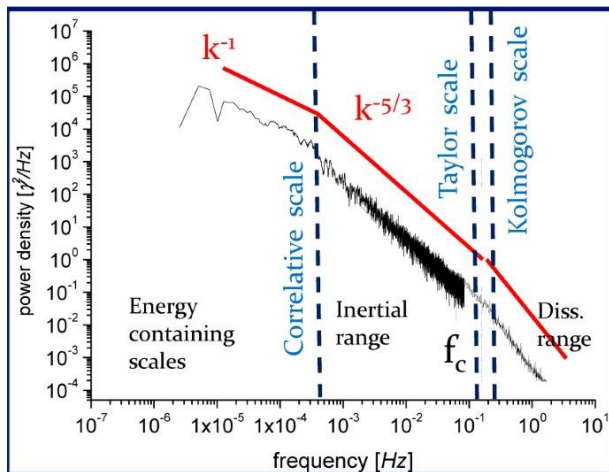


$$P(k) = \int C(\ell) W_\ell e^{-ik\ell} d\ell$$



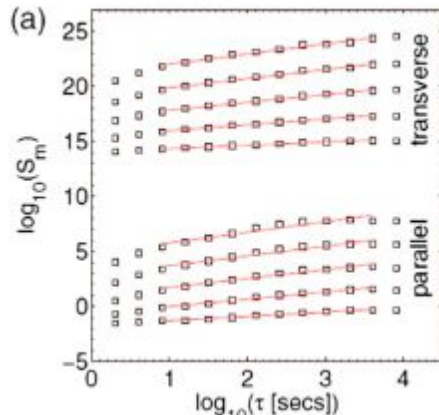
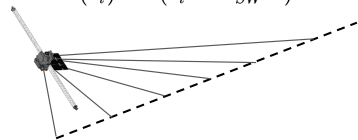
Power Spectra and Intermittency in the Solar Wind

Bruno & Carbone, LRSP (2013)



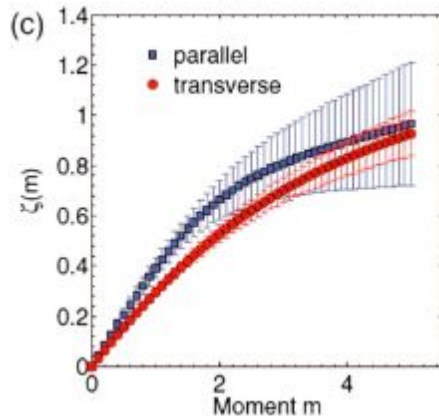
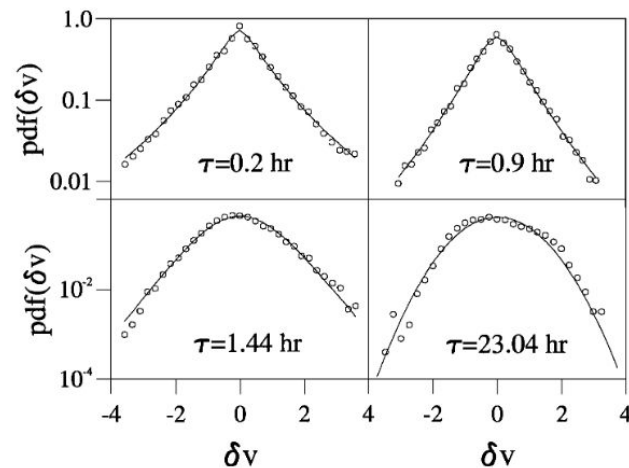
Taylor

$$\delta \mathbf{b} = \mathbf{b}(\mathbf{r}_i) - \mathbf{b}(\mathbf{r}_i - \mathbf{V}_{SW} \delta t)$$



Distributions of field gradients

SORRISO-VALVO ET AL.: PDF OF MHD FLUCTUATIONS

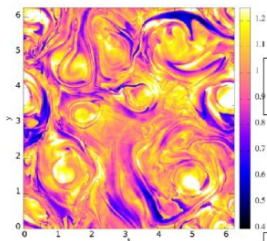
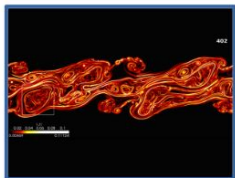
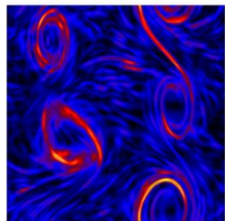


(II) Simulations to Support Observations



A “Menu” of Models

information
loss



Liouville



Vlasov-Maxwell

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha} \left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right] \cdot \nabla_v f_\alpha = 0 \quad +$$

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \end{aligned}$$



integrando

Equazioni a 2 fluidi



approssimando

Magnetoidrodinamica (MHD)



trascurando $\mathbf{B}$

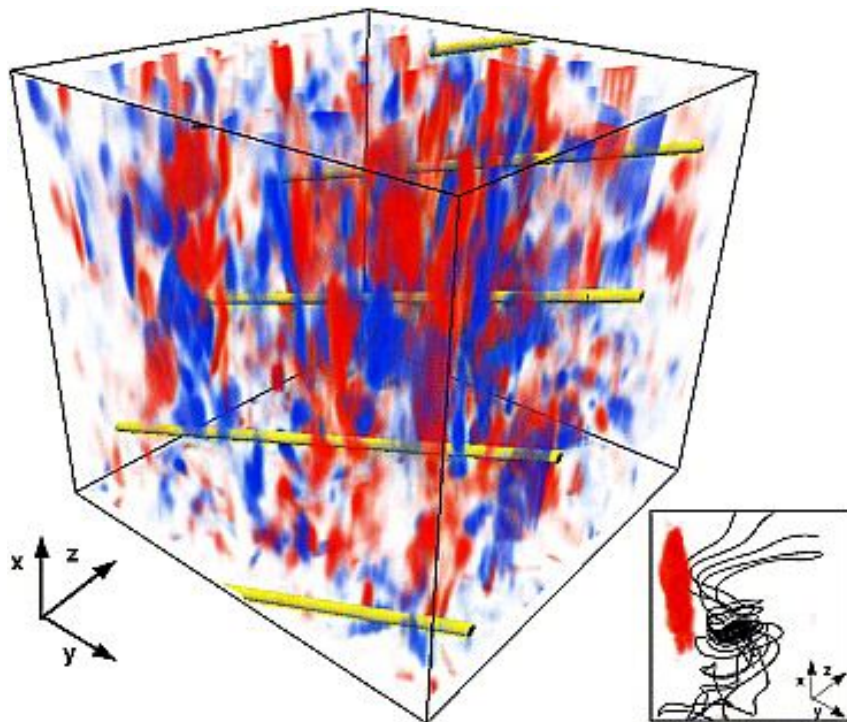
Idrodinamica (Navier-Stokes)



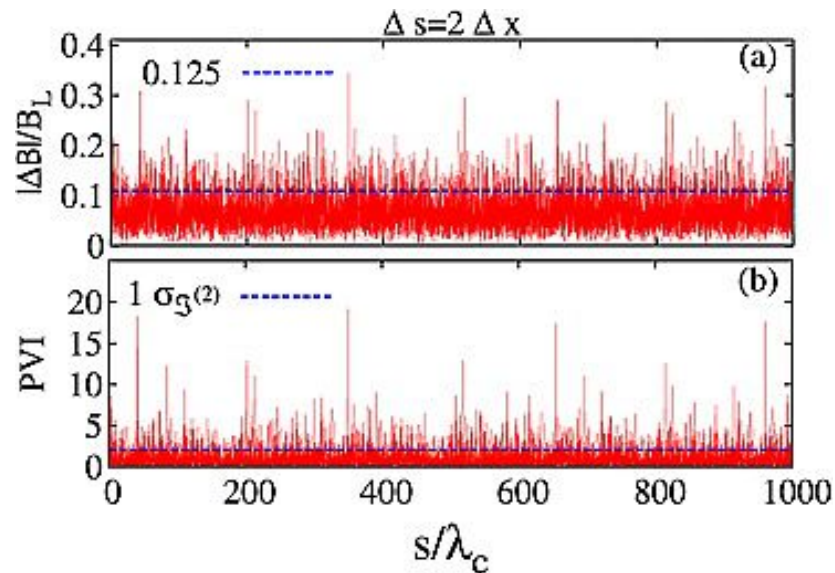
complexity

Virtual Spacecraft: the Way to Go

$$\mathfrak{J}^{(2)} = \frac{|\Delta \mathbf{B}|^2}{\Sigma^2},$$



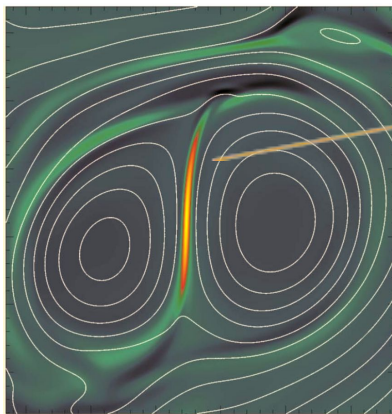
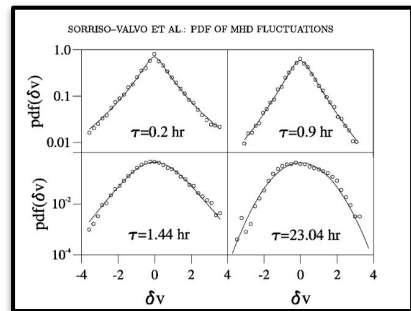
Greco+, GRL (2008), APJ (2009)



...from 19 years ago



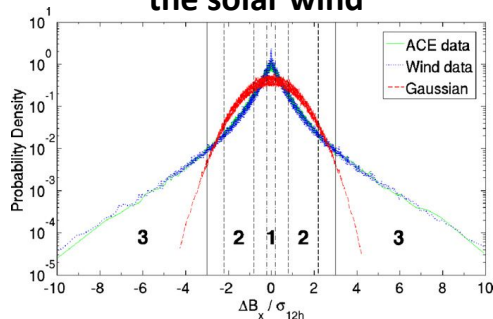
Anomalies: Reconnection & Dissipation



**In between flux tubes,
magnetic gradients are
present**

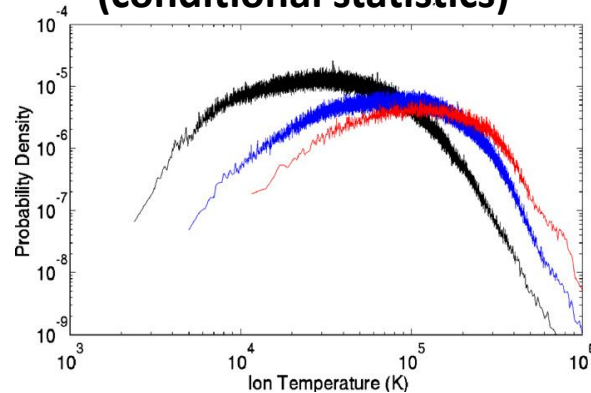
Retino+, Nature, 2007
SS, PRL, 2009

**PDF of magnetic increments in
the solar wind**



**Intermittent magnetic
gradients are sites of
enhanced heating**

**PDF of ion heating
(conditional statistics)**





Combined Large- and Small-Scale Analysis?

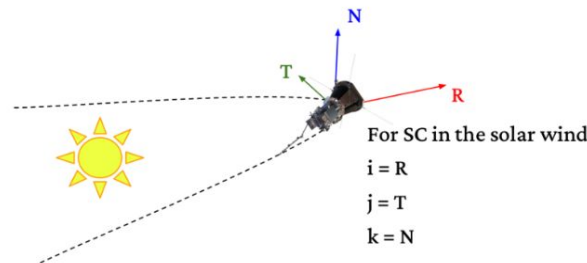
- Small scales: we identify strong (reconnecting) discontinuities via the Partial Variance of Increments (PVI)

$$\text{PVI} = \frac{|\Delta \mathbf{B}|}{\sqrt{\langle |\Delta \mathbf{B}|^2 \rangle}} \quad \text{where } |\Delta \mathbf{B}| = |\mathbf{B}(t + \tau) - \mathbf{B}(t)|. \quad \text{Greco+, APJ (2008)}$$

- Large scales: we use a novel technique to identify flux tubes, based on the local analysis of magnetic helicity (in real space!)

$$H_m(x, \ell) = \int_0^\ell dl_i h(l_i) \int_{x-\frac{w_0}{2}}^{x+\frac{w_0}{2}} d\xi \epsilon_{ijk} b_j(\xi) b_k(\xi + l_i),$$

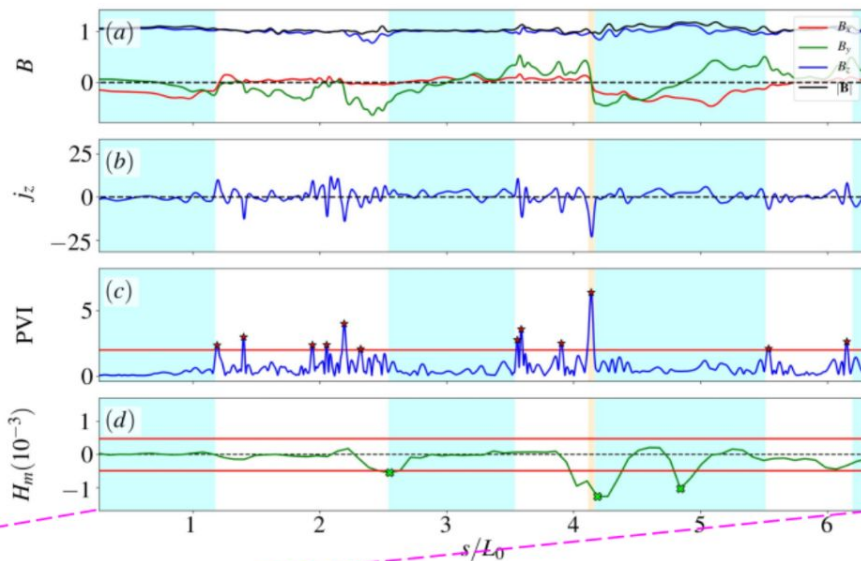
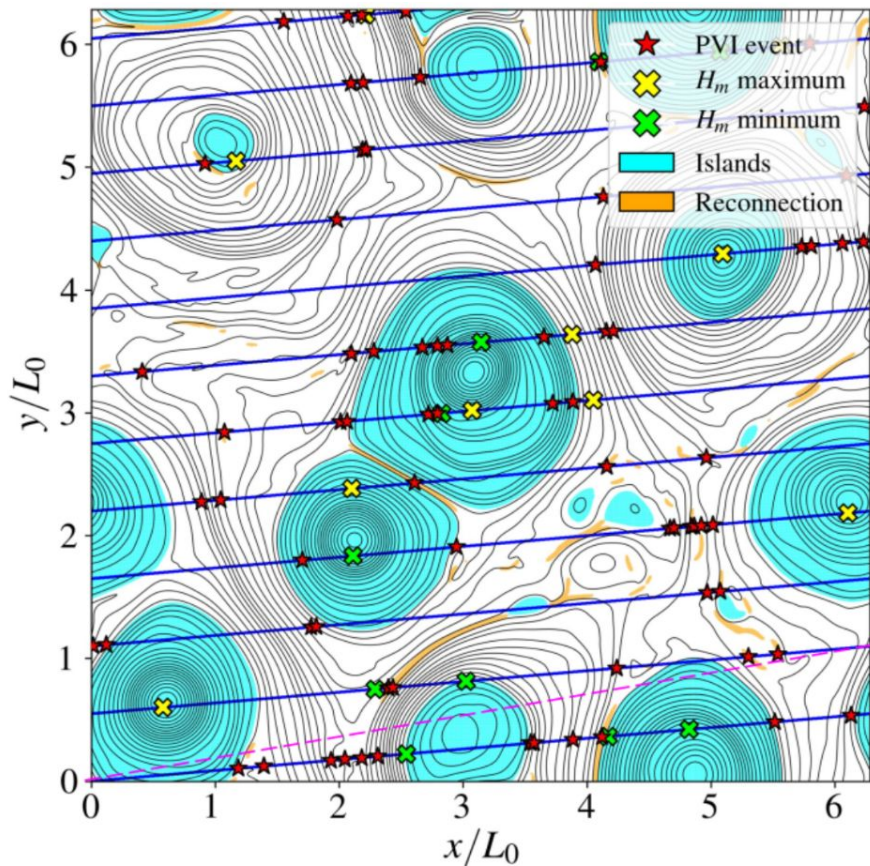
Pecora+, A&A (2021)



for other methods, see Zhao+, A&A (2021); Telloni+, APJ 2020



Validation via Virtual Spacecraft

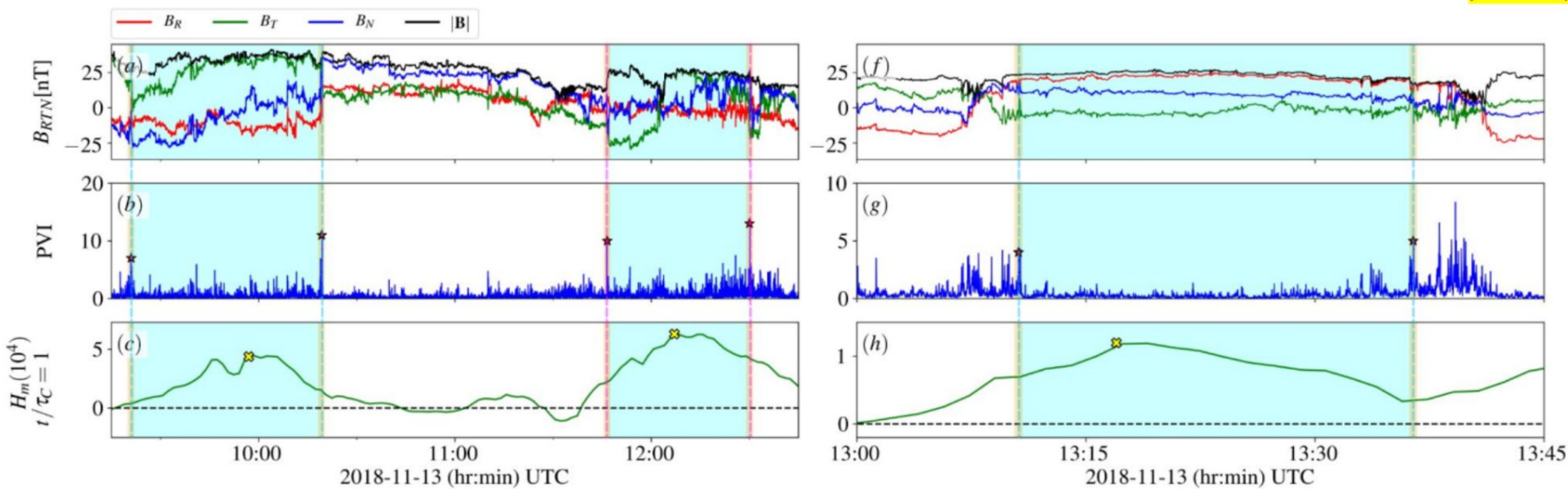


The technique can identify tubes and sheets



From Simulations to PSP

(observations)

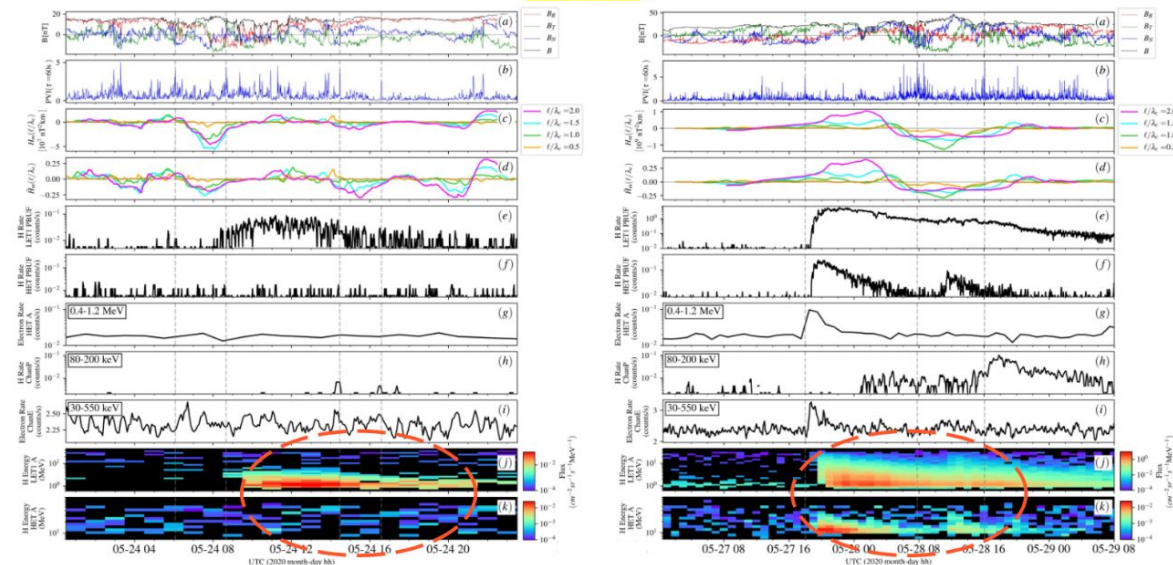


Data from PSP suggest that large scale helical tubes are bounded by sharp (reconnecting) current layers



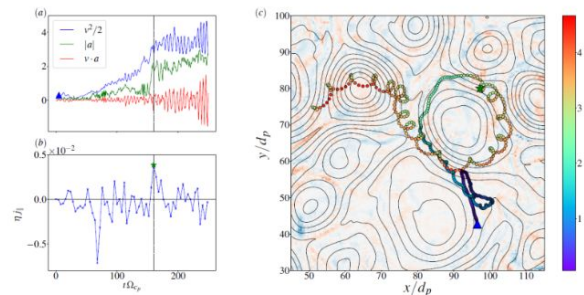
Reconnecting Flux Tubes and Energetic Particles

(observations)



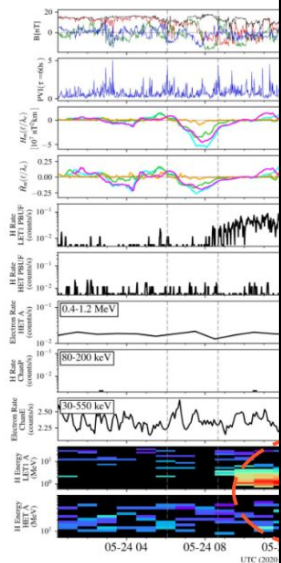
Pecora +, JPP (2018)
Pecora+, AA (2021)
Pecora+, MNRAS (2021)

Large and small scale coherent structures influence particle energization and energy conversion mechanisms in astrophysical plasmas

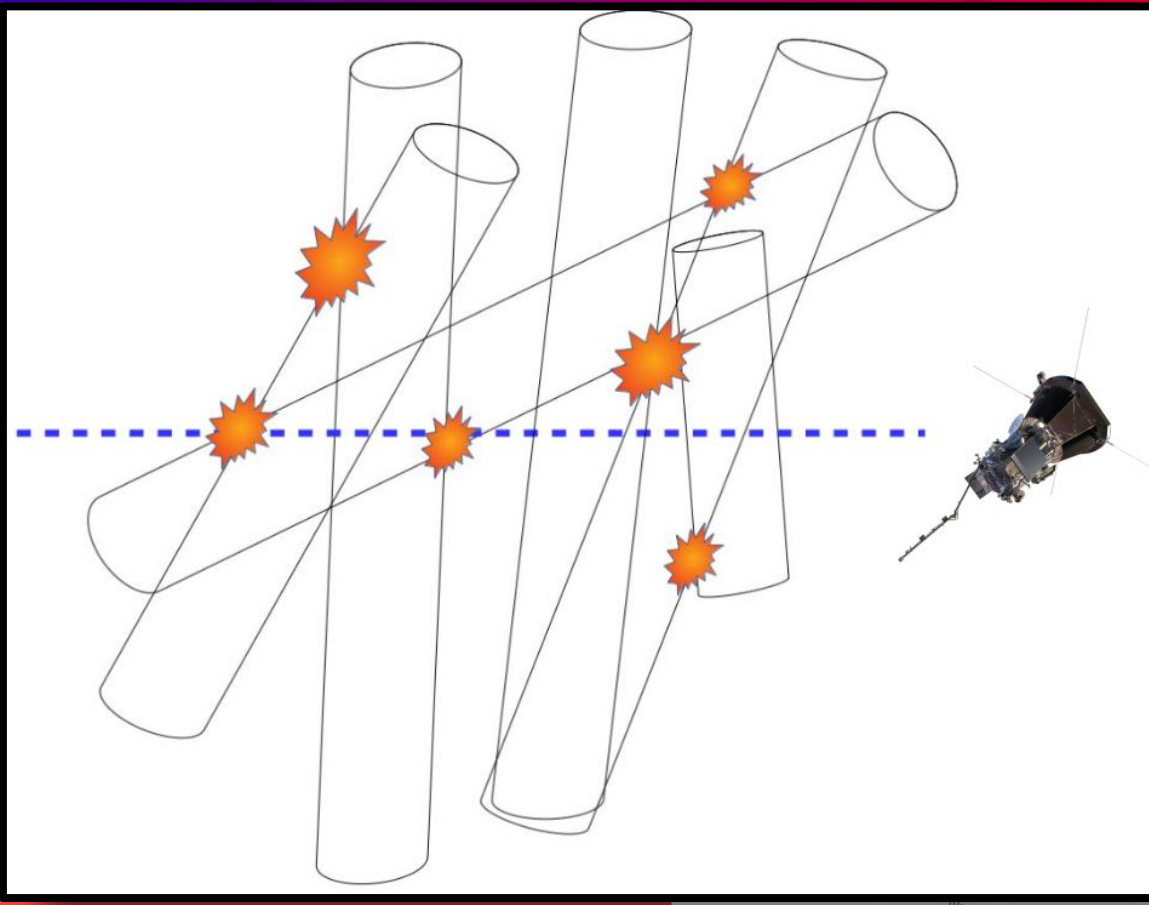




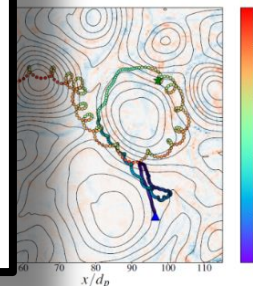
Reconnecting Flux Tubes and Energetic Particles



Large a
struct
energizat
mechanism

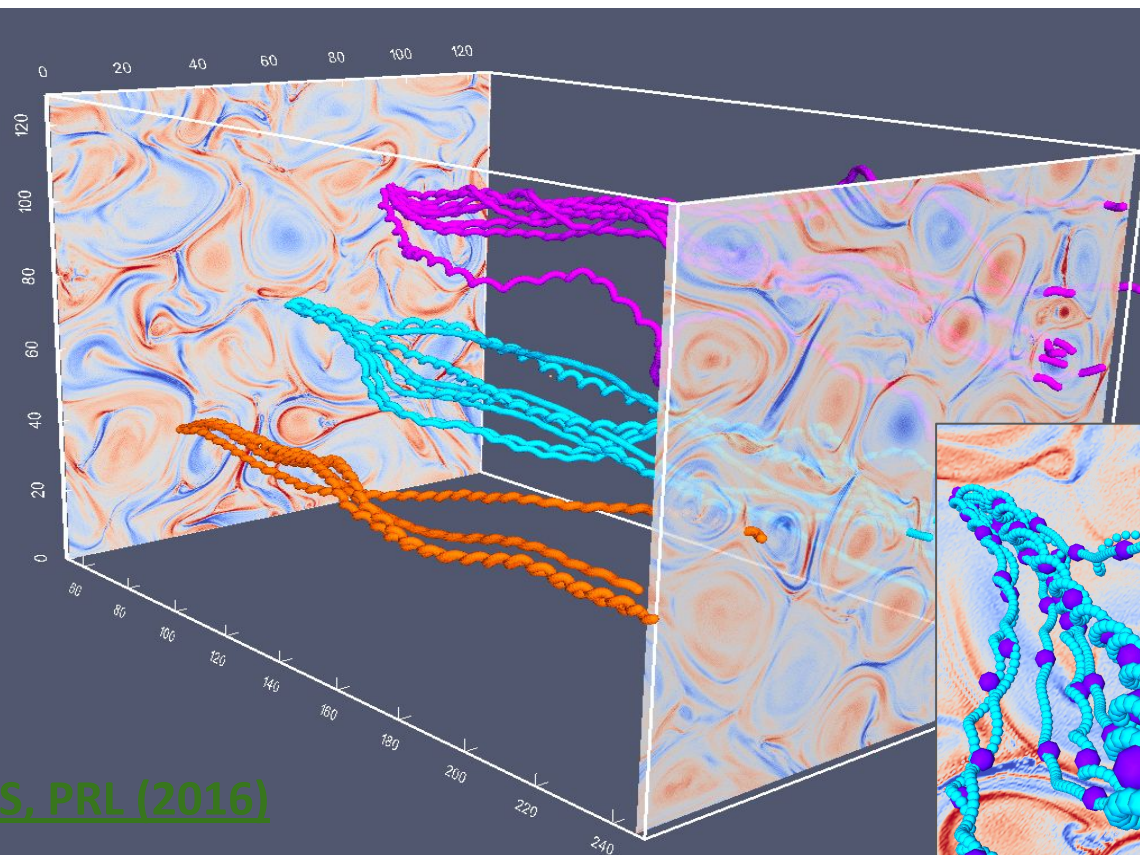


, JPP (2018)
, AA (2021)
, MNRAS (2021)

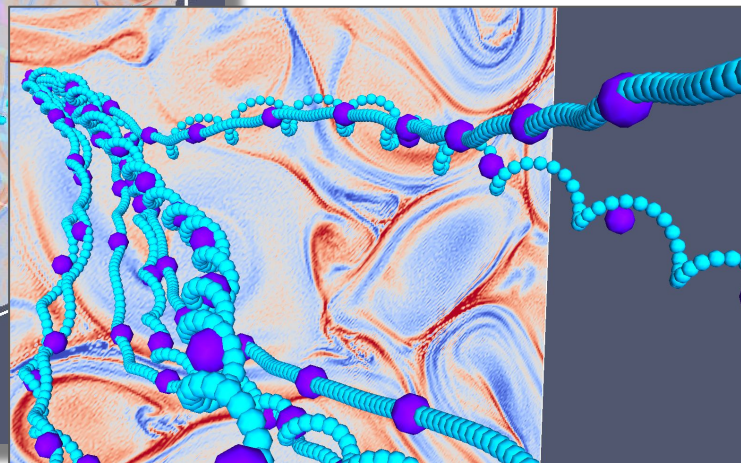




Acceleration due to Turbulence



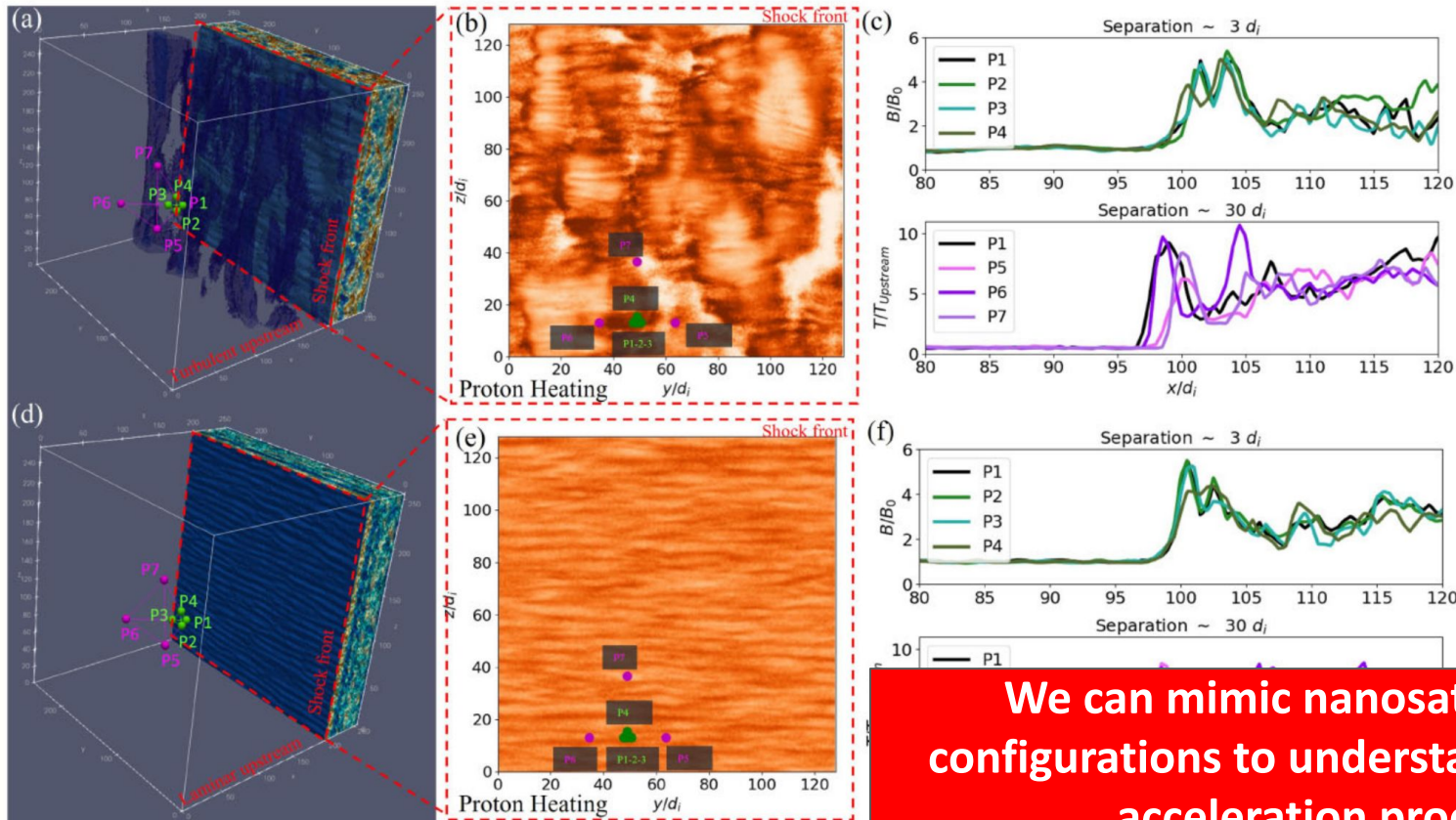
Energetic particles spread out (and accelerate) more because of turbulence



SS, PRL (2016)



From Simulations to New Spacecraft Design

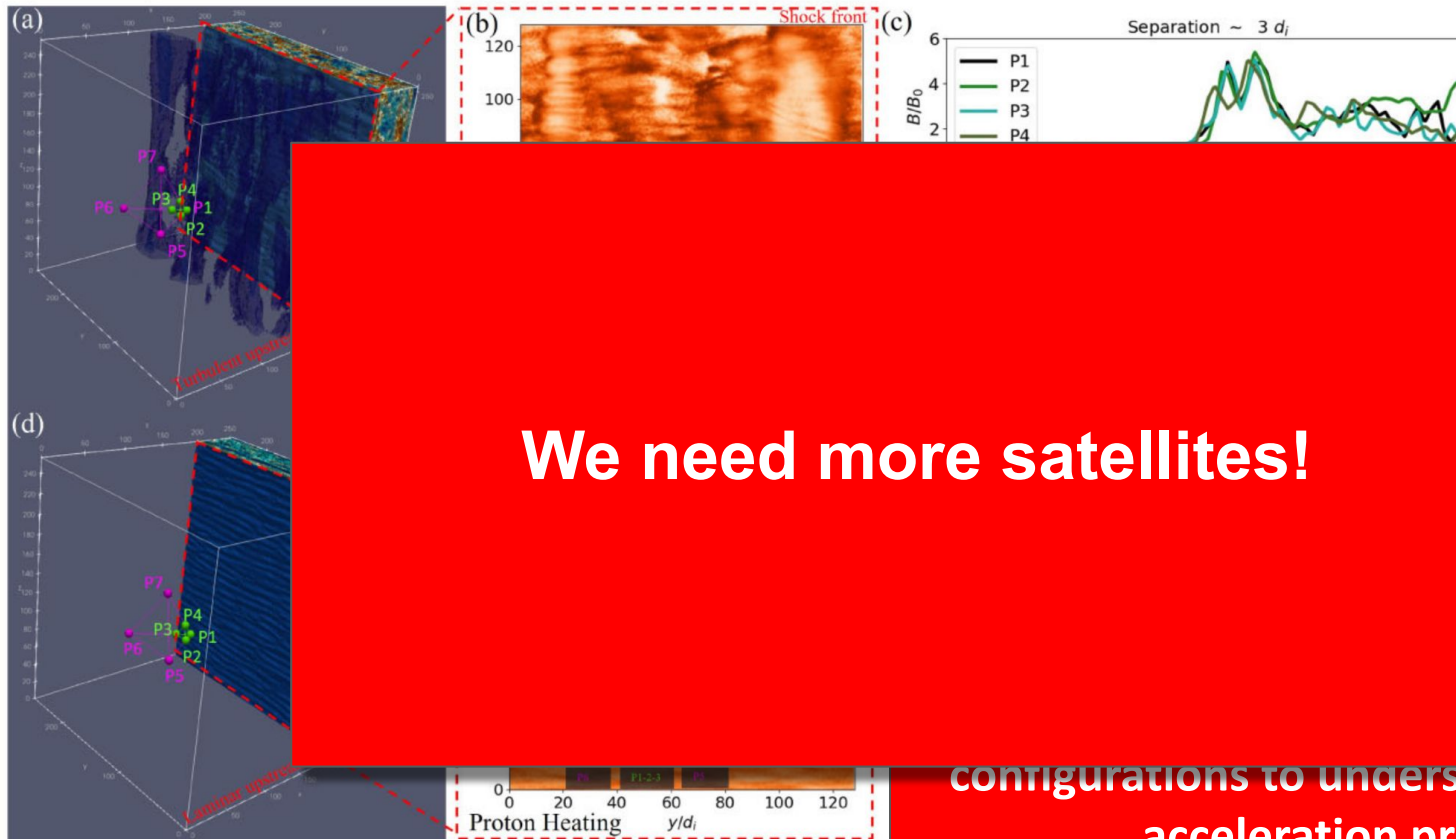


Trotta+, PNAS (2021),
MNRAS (2023)

We can mimic nanosats or swarm configurations to understand and predict acceleration processes



From Simulations to New Spacecraft Design

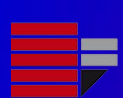


We need more satellites!

Potter+, PNAS (2021),
MNRAS (2023)

for swarm
configurations to understand and predict
acceleration processes

(III) Virtual spacecraft with constellations: second-order statistics



Space Missions

Near Sun



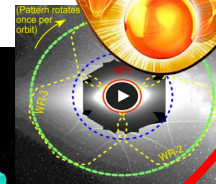
- Trace the flow of energy that heats and accelerates the solar corona and solar wind



- Determine the structure and dynamics of the solar wind

Near Earth

Constellations of satellites to understand the the wind and the magnetosheath





Space Missions

Near Sun



- Trace the flow of energy that heats and accelerates the solar corona and solar wind

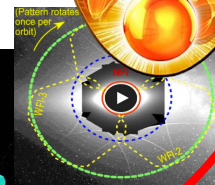


- Determine the structure and dynamics of the solar wind

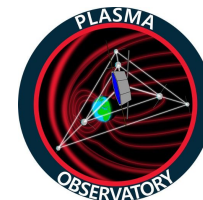
To improve our understanding of the Sun-Earth connection

Near Earth

Constellations of satellites to understand the the wind and the magnetosheath

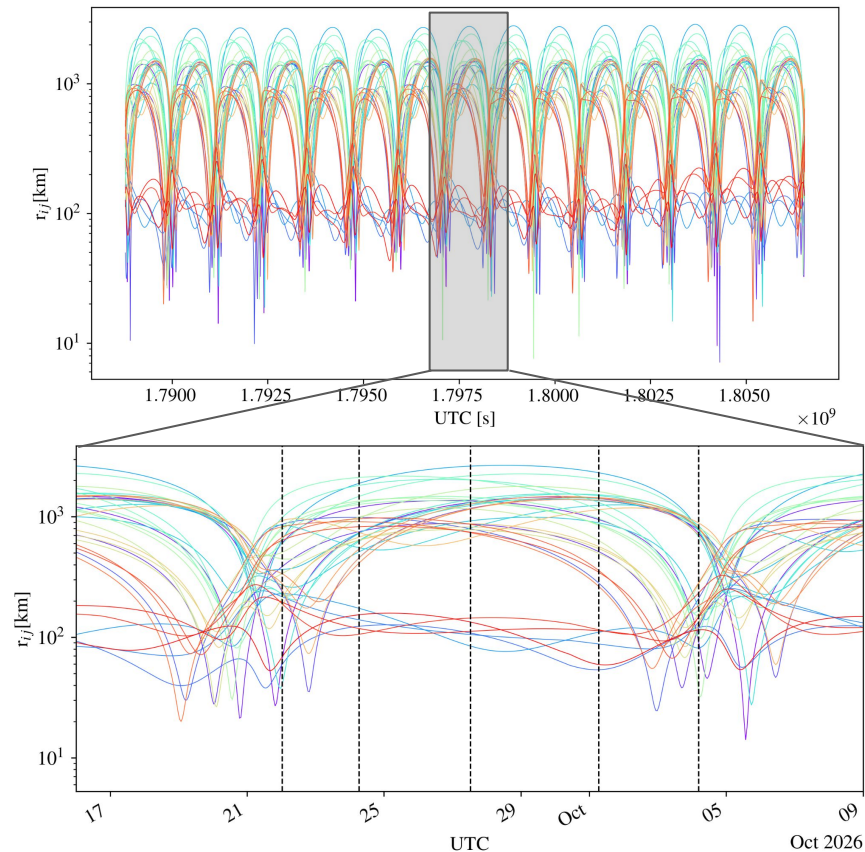
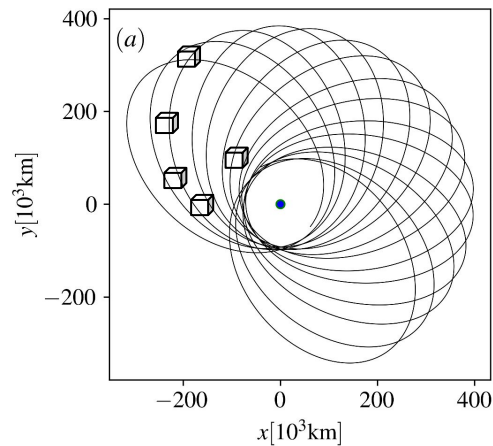


hopefully soon:

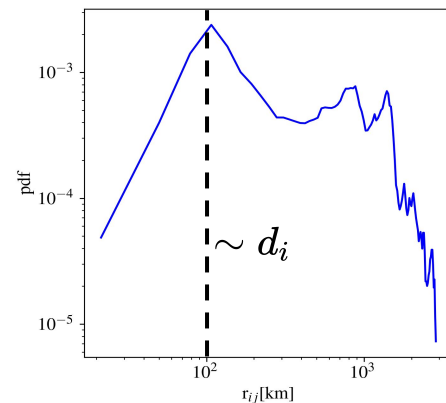




A Virtual Flight with a Swarm



$$r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$$
$$i \neq j = 1, \dots, 9$$





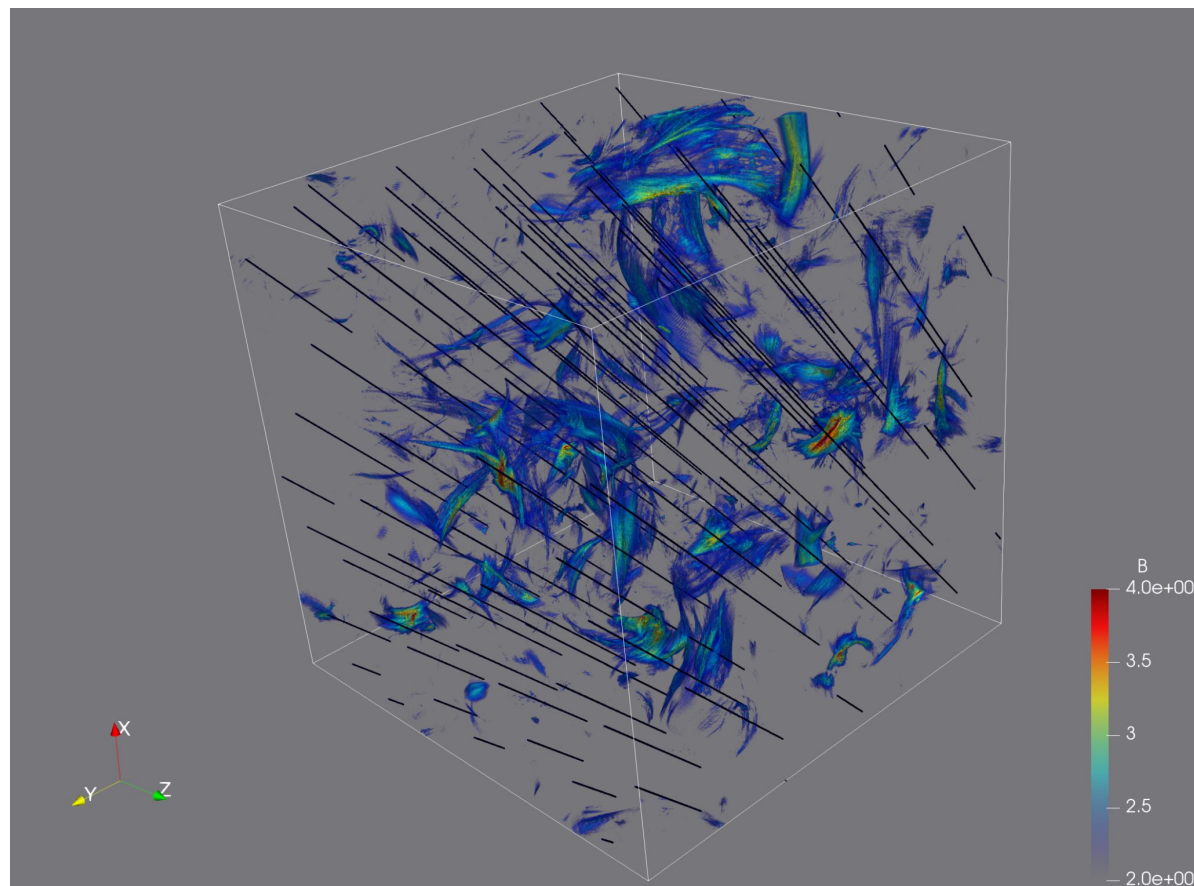
Testing with Simulations

- ◆ 3D MHD
- ◆ 1024^3
- ◆ With and without mean field

- ◆ 3D Hybrid-PIC
- ◆ 512^3

- ◆ 3D Full PIC
- ◆ 512^3
- ◆ With and without mean field

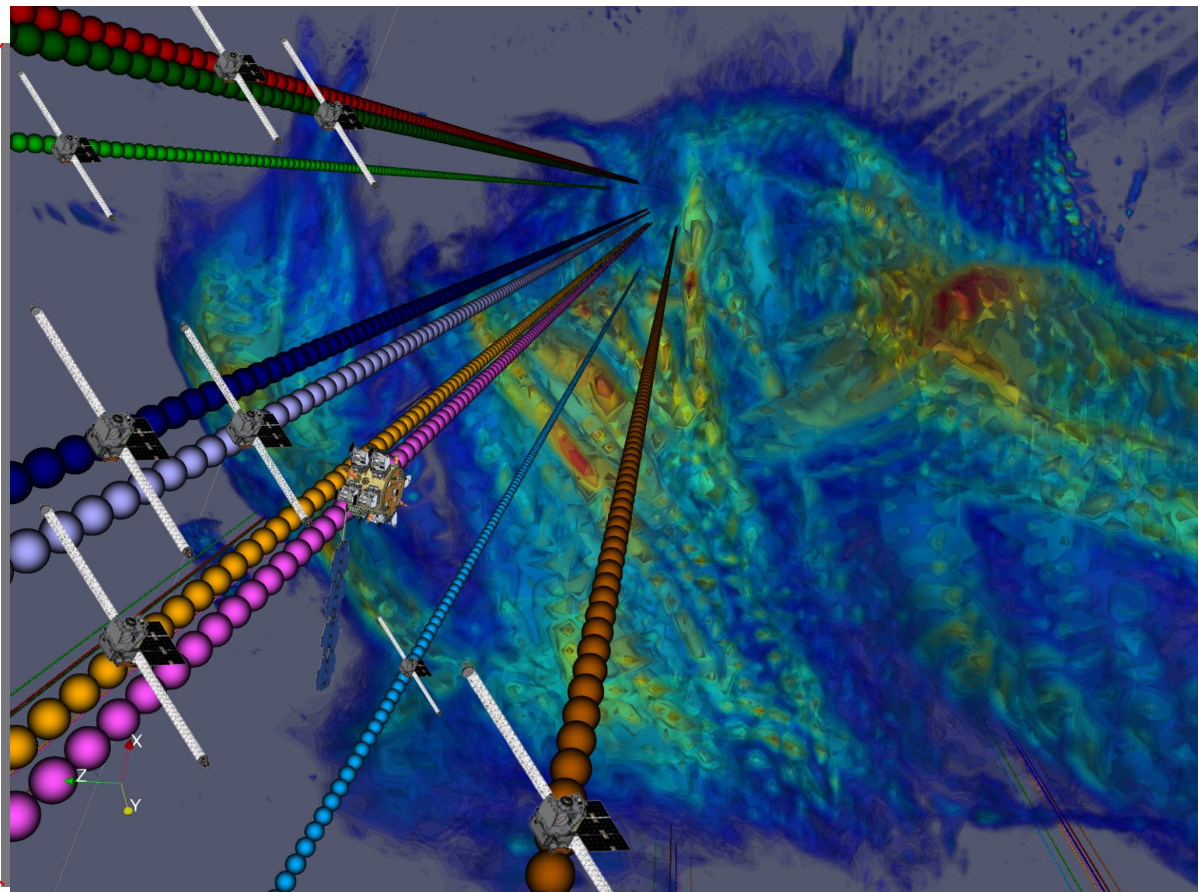
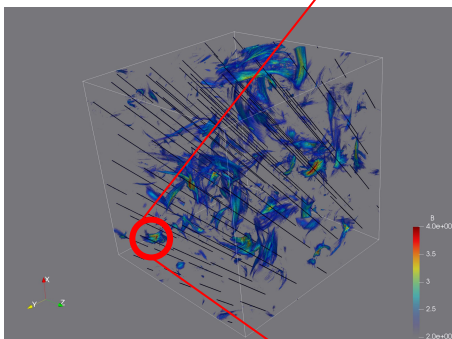
- ◆ 3D Hall-MHD
- ◆ 2048^3 – on their way





A First Flight through MHD

- ◆ 3D MHD simulations
- ◆ 1024^3
- ◆ With and without mean field



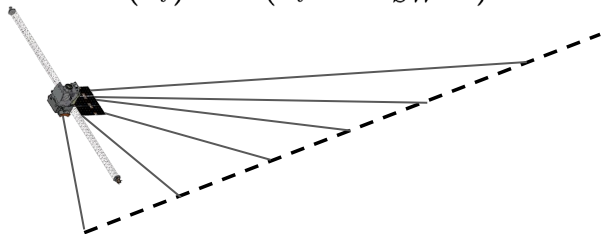


Increments in More Dimensions

method I

Taylor

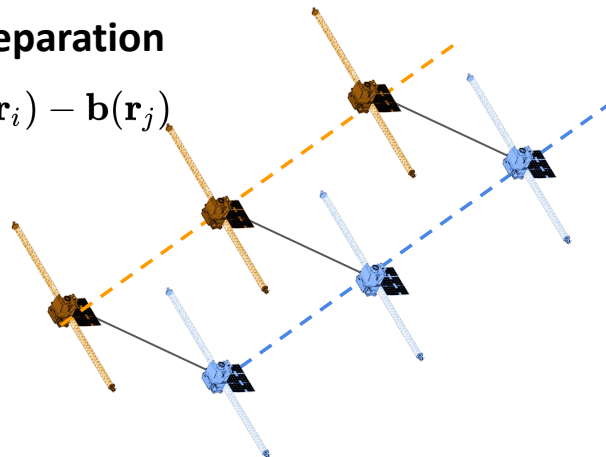
$$\delta \mathbf{b} = \mathbf{b}(\mathbf{r}_i) - \mathbf{b}(\mathbf{r}_i - \mathbf{V}_{SW} \delta t)$$



method II

Fixed separation

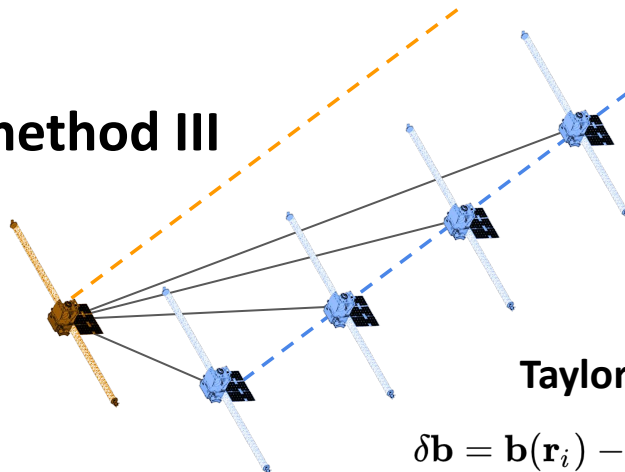
$$\delta \mathbf{b} = \mathbf{b}(\mathbf{r}_i) - \mathbf{b}(\mathbf{r}_j)$$



method III

Taylor-like

$$\delta \mathbf{b} = \mathbf{b}(\mathbf{r}_i) - \mathbf{b}(\mathbf{r}_j - \mathbf{V}_{SW} \delta t)$$



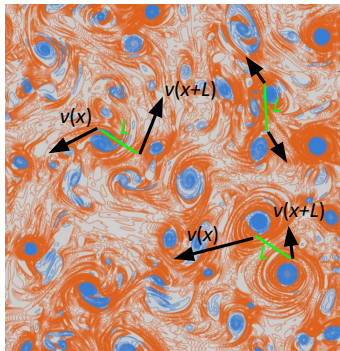
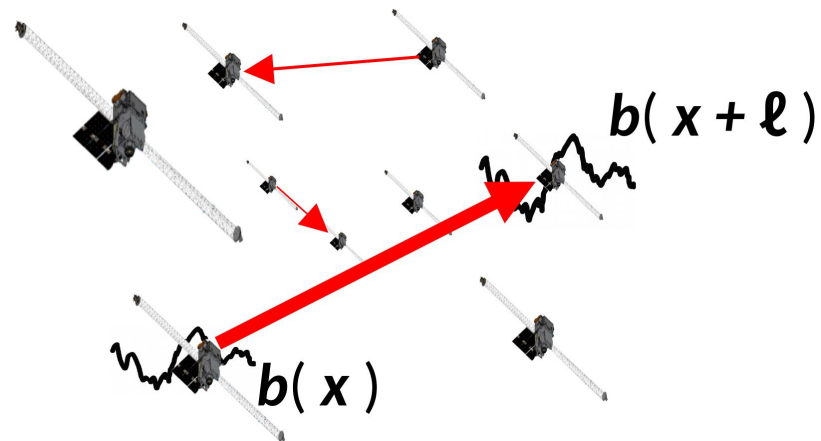
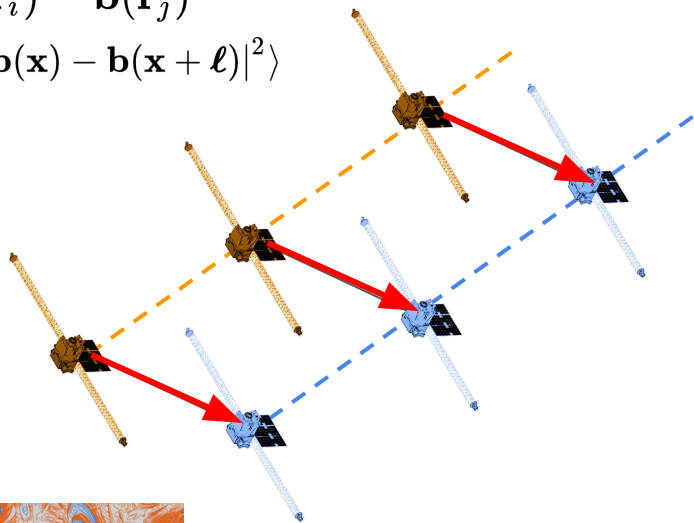


Multi-Spacecraft Strategies

For each satellite pair:

$$\delta \mathbf{b} = \mathbf{b}(\mathbf{r}_i) - \mathbf{b}(\mathbf{r}_j)$$

$$S_b^2(\ell) = \langle |\mathbf{b}(\mathbf{x}) - \mathbf{b}(\mathbf{x} + \ell)|^2 \rangle$$



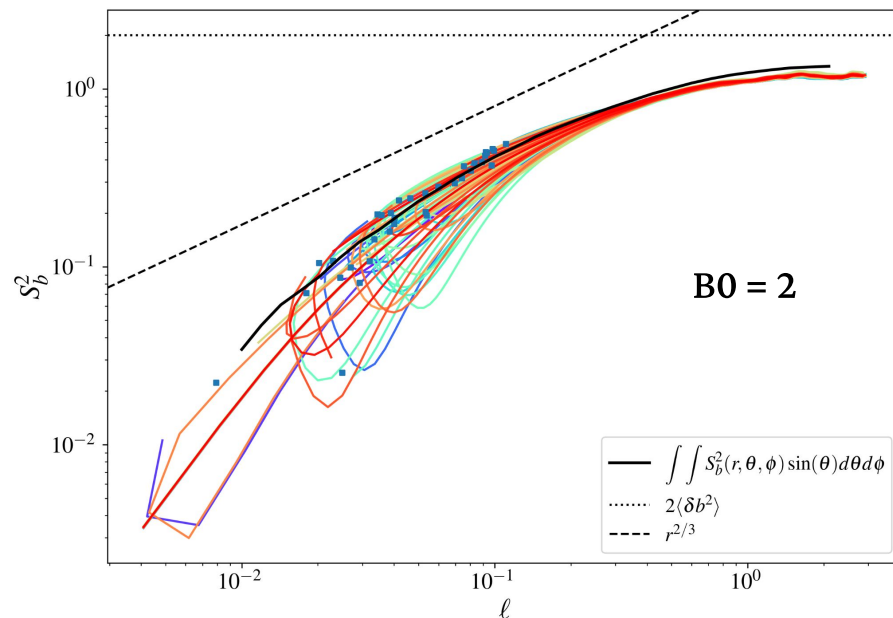
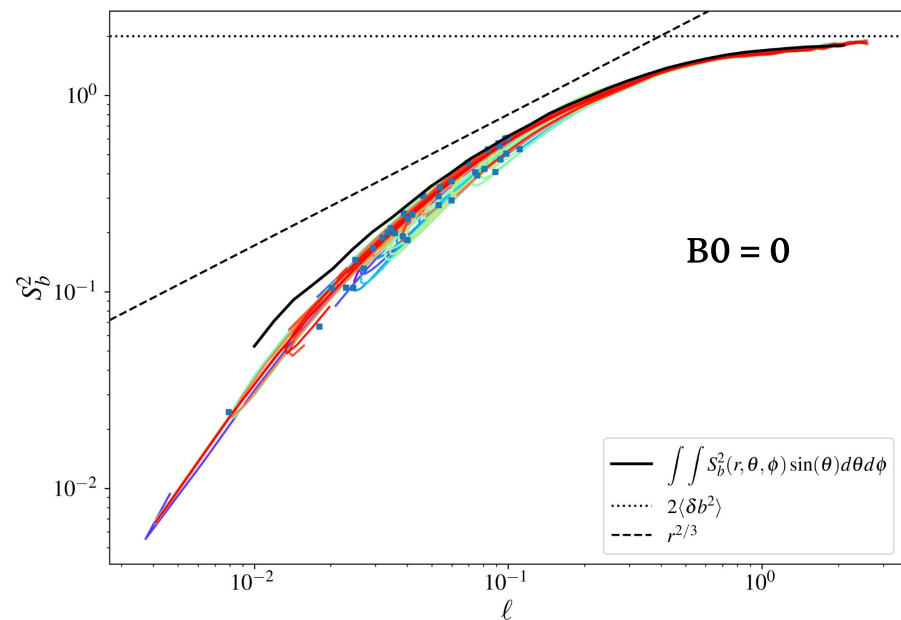
One value of $\mathbf{Y}$ for each pair of spacecraft $\rightarrow$
One value at the corresponding point in lag space



1D, Second Order Structure Function

- Exact 1D S^2
- 9+36 Taylor-like S^2
- 36 S^2 values at fixed separation

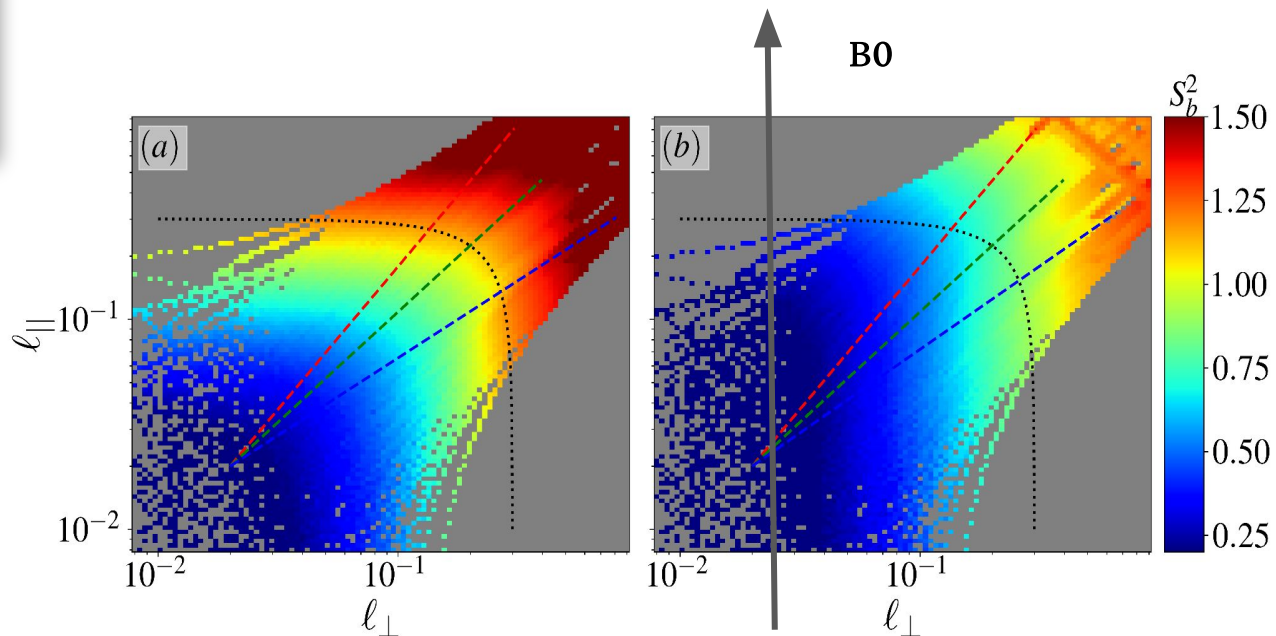
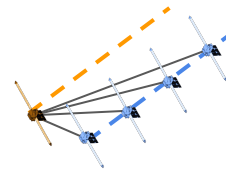
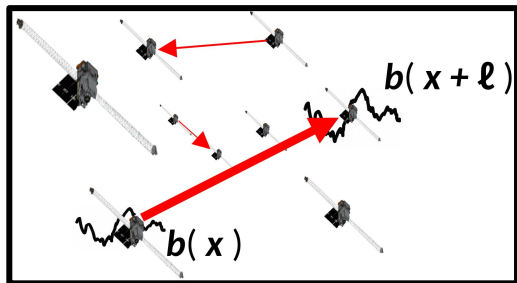
$$S_b^2(\ell) = \langle |\mathbf{b}(\mathbf{x}) - \mathbf{b}(\mathbf{x} + \ell)|^2 \rangle$$





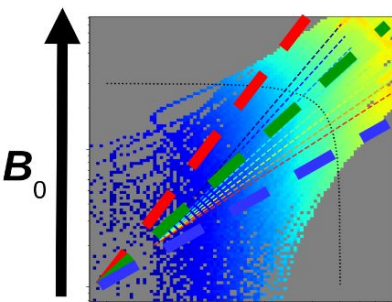
1D, Second Order Structure Function

$$S_b^2(\ell) = \langle |\mathbf{b}(\mathbf{x}) - \mathbf{b}(\mathbf{x} + \ell)|^2 \rangle$$





Finally some Spectra from Multi-Spacecraft



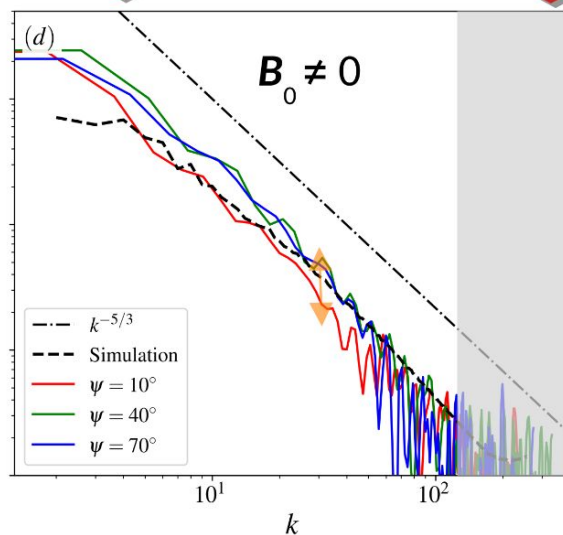
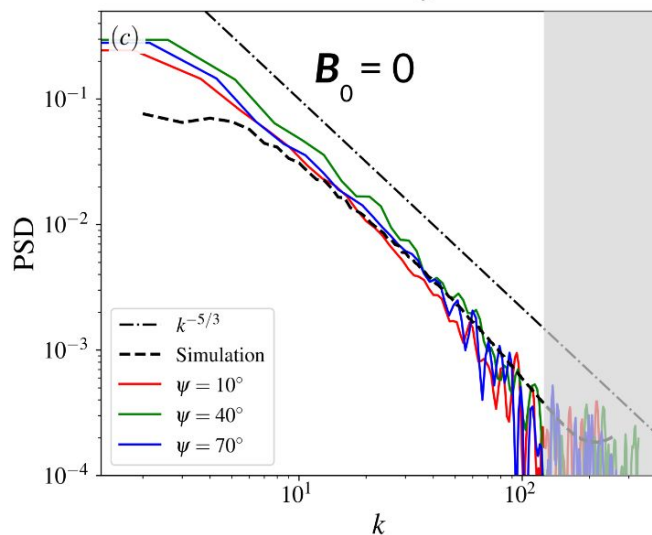
Directional
autocorrelation function

$$C(\ell) = E - \frac{S_2(\ell)}{2}$$

Blackman-Tukey

$$P(k) = \int C(\ell) W_\ell e^{-ik\ell} d\ell$$

Power spectra

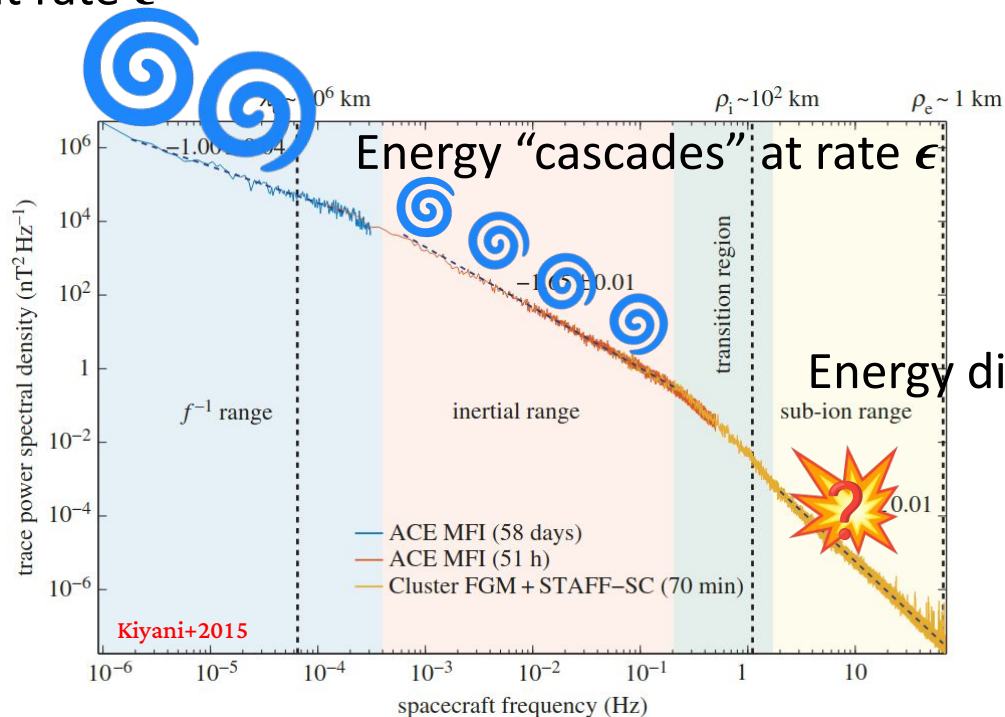


- Contrary to S_2 , a clear scaling law is recovered
- Spectral anisotropy is correctly evaluated



Can just a Spectrum Tell you the Entire Story?

Energy injected at rate ϵ



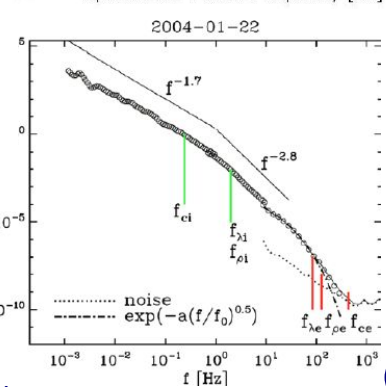
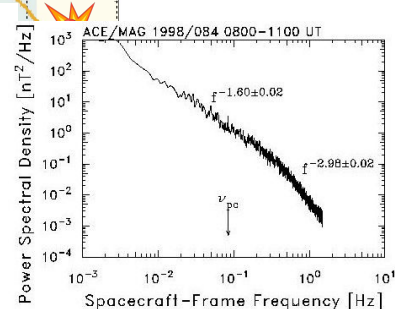
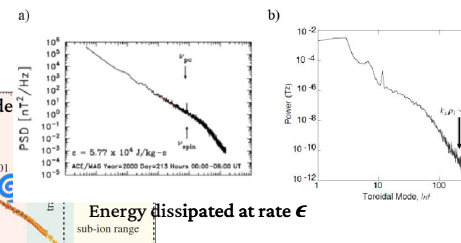
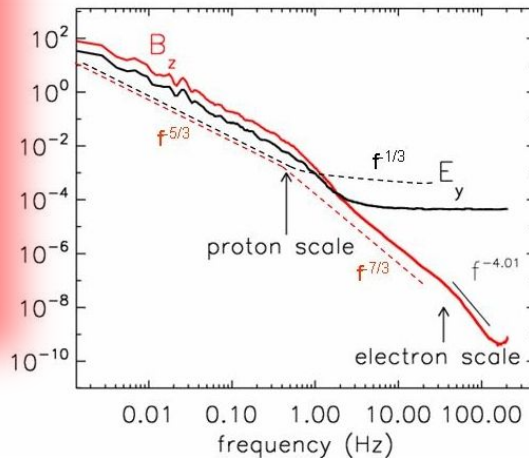
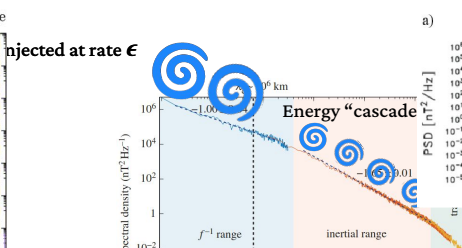
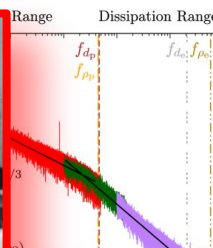
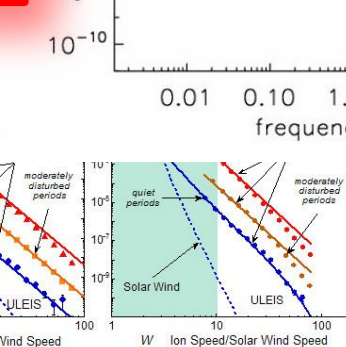
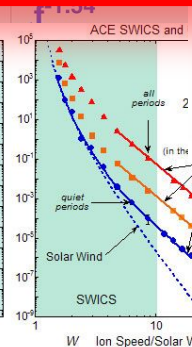
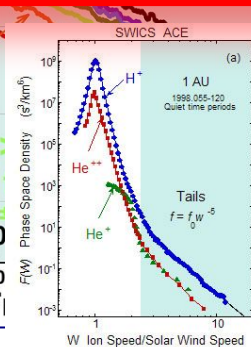
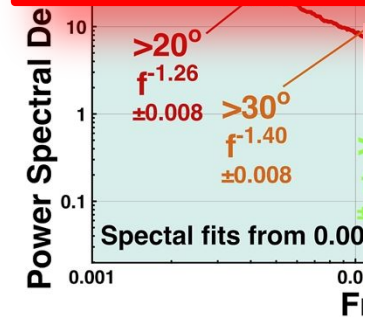
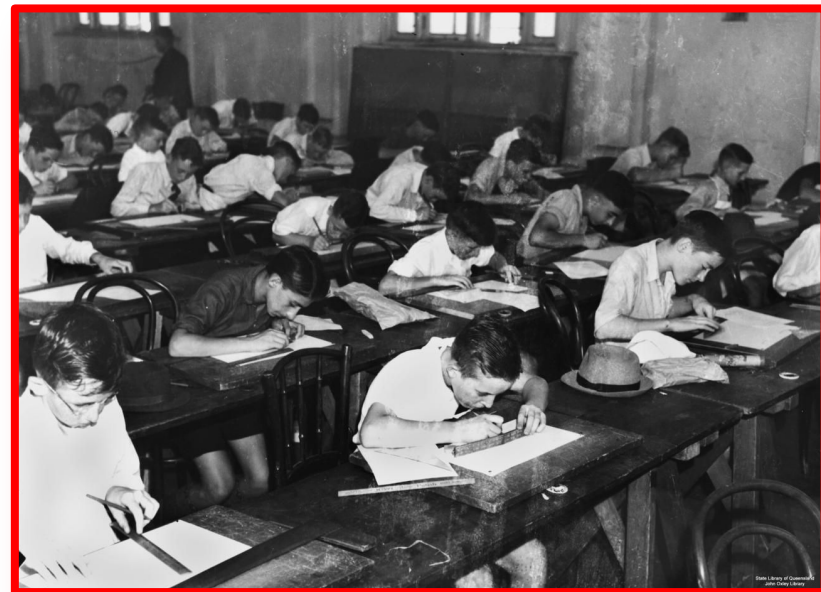
Injection

Cascade

Dissipation

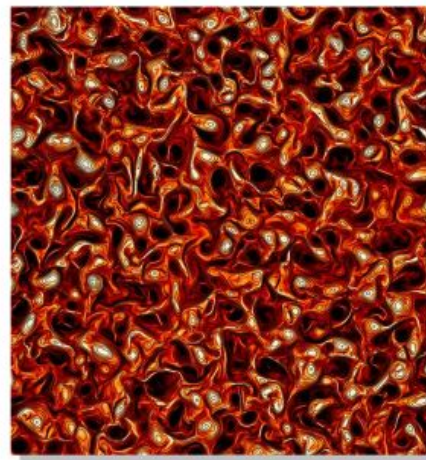
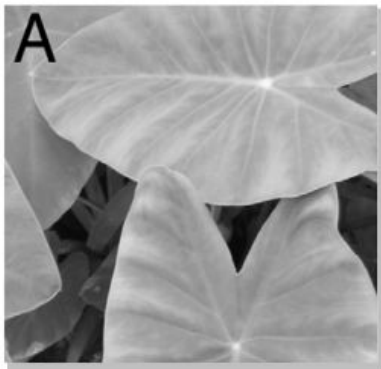


Spectra, spectra, spectra...

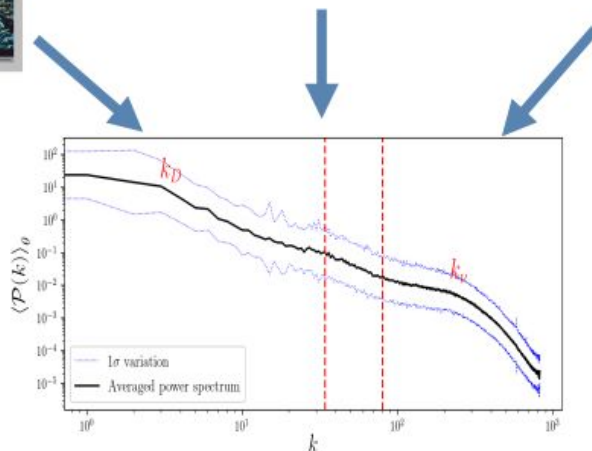




Spectra, spectra, spectra...



**Power spectra are not
enough to characterize a
turbulent cascade**



we need refined analyses



(IV) An Exact Law of Turbulence



An Exact Law of Turbulence

von Kármán–Howarth equation

Incompressible, homogeneous MHD

Time dependence and dissipation terms could (ideally) be neglected in the inertial range →

Yaglom's law

$$\mathbf{z}^{\pm}(\mathbf{x}) = \mathbf{u}(\mathbf{x}) \pm \mathbf{b}(\mathbf{x})$$

$$\delta \mathbf{z}^{\pm}(\mathbf{x}, \ell) = \mathbf{z}^{\pm}(\mathbf{x} + \ell) - \mathbf{z}^{\pm}(\mathbf{x})$$

$$\frac{\partial}{\partial t} \langle |\delta \mathbf{z}^{\pm}|^2 \rangle = -\nabla_{\ell} \cdot \langle \delta \mathbf{z}^{\mp} |\delta \mathbf{z}^{\pm}|^2 \rangle + 2\nu \nabla_{\ell}^2 \langle |\delta \mathbf{z}^{\pm}|^2 \rangle - 4\epsilon^{\pm}$$

Politano & Pouquet 1998

$$\nabla_{\ell} \cdot \mathbf{Y}^{\pm} = -4\epsilon^{\pm}$$

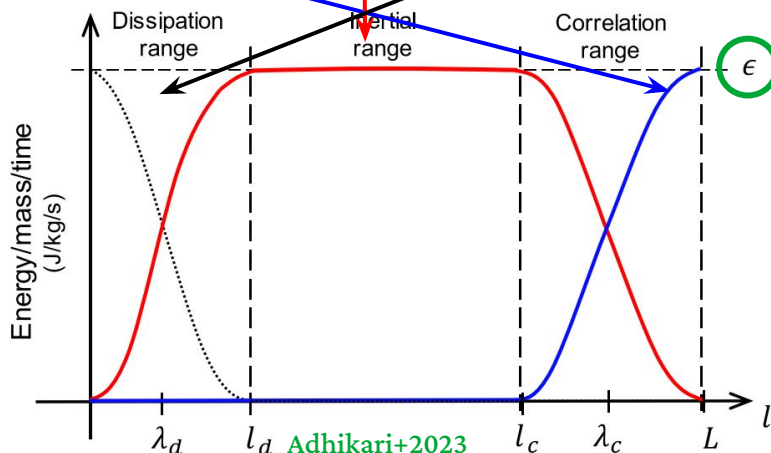


The Ranges of Turbulence

von Kármán–Howarth equation
Incompressible, homogeneous MHD

$$\frac{\partial}{\partial t} \langle |\delta \mathbf{z}^\pm|^2 \rangle = -\nabla_\ell \cdot \langle \delta \mathbf{z}^\mp |\delta \mathbf{z}^\pm|^2 \rangle + 2\nu \nabla_\ell^2 \langle |\delta \mathbf{z}^\pm|^2 \rangle - 4\epsilon^\pm$$

Politano & Pouquet 1998



Adhikari+2023

Additional corrections
for compressibility and
Hall effects can be
added

[Hadid+PRL, 2018](#)

[Hellinger+APJL, 2018](#)

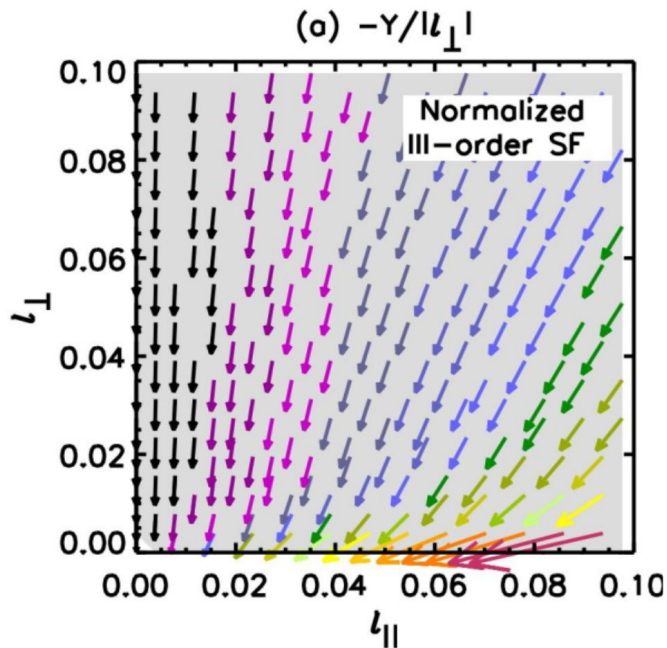
[Andrés+JPP, 2018](#)

[Andrés+PRL, 2019](#)



The Flux

Idea: The energy transfer rate can be computed also in the increments space

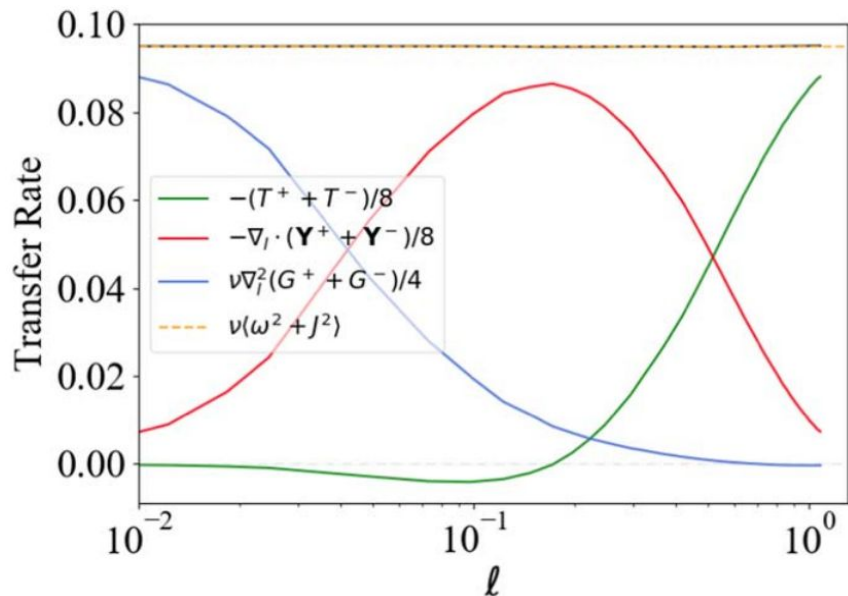


Verdini+, APJ (2015)

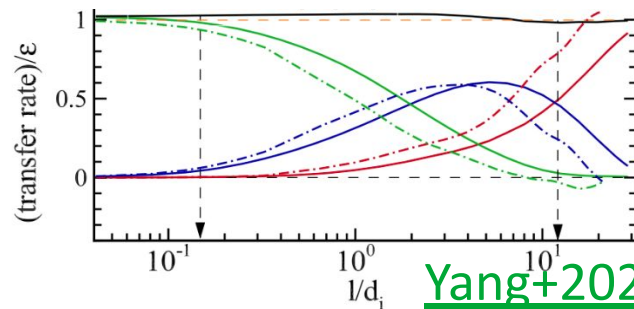


Third-Order Statistics in Simulations

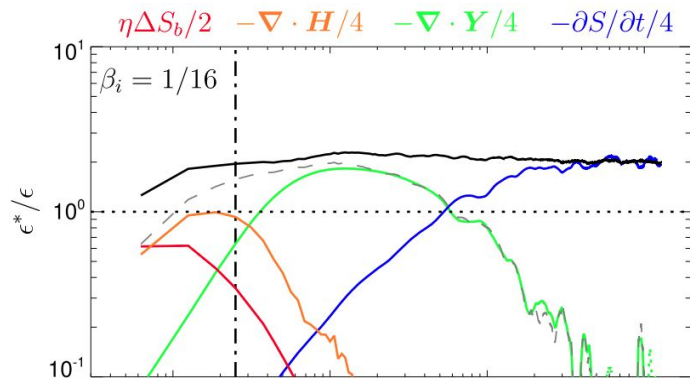
$$\nabla_{\ell} \cdot \mathbf{Y}^{\pm} = -4\epsilon^{\pm}$$



Wang+2022



Yang+2022



Hellinger+2018

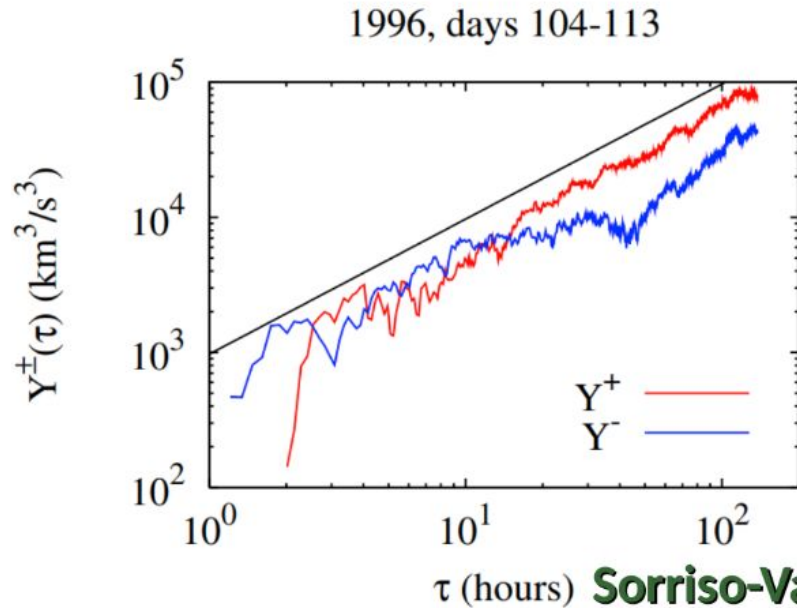


A Standard Simplification

$$\nabla_{\ell} \cdot \mathbf{Y} = -4\varepsilon \quad \Rightarrow \quad \text{Integrating in the increments space on a sphere, assuming isotropy}$$

$$Y_{\ell} = -\frac{4}{3} \varepsilon \ell$$

Politano & Pouquet, GRL (1998)
Marino et al, APJ (2012)



Observation of inertial energy cascade in interplanetary space plasma

For a review:

Marino & Sorriso-Valvo, Phys. Rep (2023)

Sorriso-Valvo et al, PRL (2007)

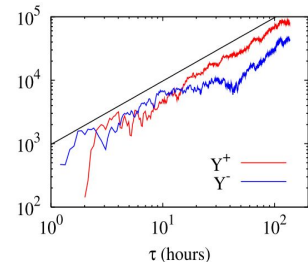
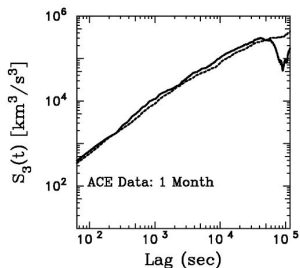


Limitations

Spacecraft limitations

Isotropy & Taylor hypothesis assumption

$$\epsilon = -\frac{3Y}{4\ell}$$



[MacBride+2005](#)

[Stawarz+2009](#)

[Carbone+2009](#)

[Hadid+2017](#)

[Bandyopadhyay+2020](#)

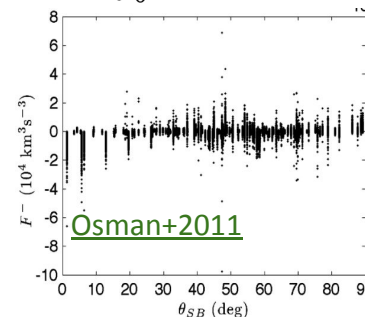
[Sorriso-Valvo+2007](#)

$$\nabla_{\ell} \cdot \mathbf{Y}^{\pm} = -4\epsilon^{\pm}$$

Yaglom's law

Directional average & Taylor hypothesis

$$\epsilon = -\frac{3}{4\ell} \int_0^{\pi/2} Y \sin(\theta_{\ell B}) d\theta_{\ell B}$$



[Nie&Tanveer 1999](#)

[Bandyopadhyay+2018](#)



One Step Back

$$\nabla_\ell \cdot \mathbf{Y} = -4\varepsilon$$



Integrating in the
increments space on a
sphere, assuming
isotropy

$$Y_\ell = -\frac{4}{3} \varepsilon \ell$$

Politano & Pouquet, GRL (1998)

Marino et al, APJ (2012)



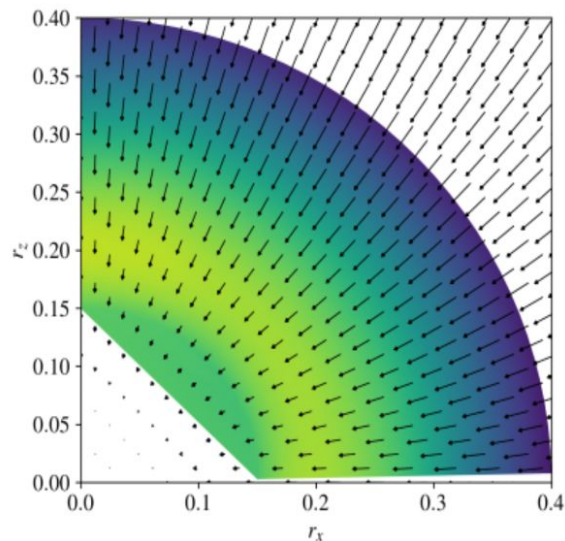
One Step Back

$$\nabla_{\ell} \cdot \mathbf{Y} = -4\varepsilon$$



as in Verdini+15

$$\mathbf{Y}^{\pm} = \langle \delta \mathbf{z}^{\mp} | \delta \mathbf{z}^{\pm} |^2 \rangle$$

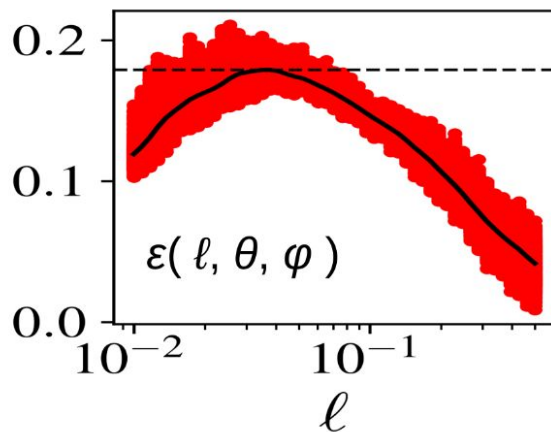


Integrating in the
increments space on a
sphere, assuming
isotropy

$$Y_{\ell} = -\frac{4}{3} \varepsilon \ell$$

Politano & Pouquet, GRL (1998)

Mandel et al, APJ (2012)



**Flux of a smooth
field, with a constant
divergence**

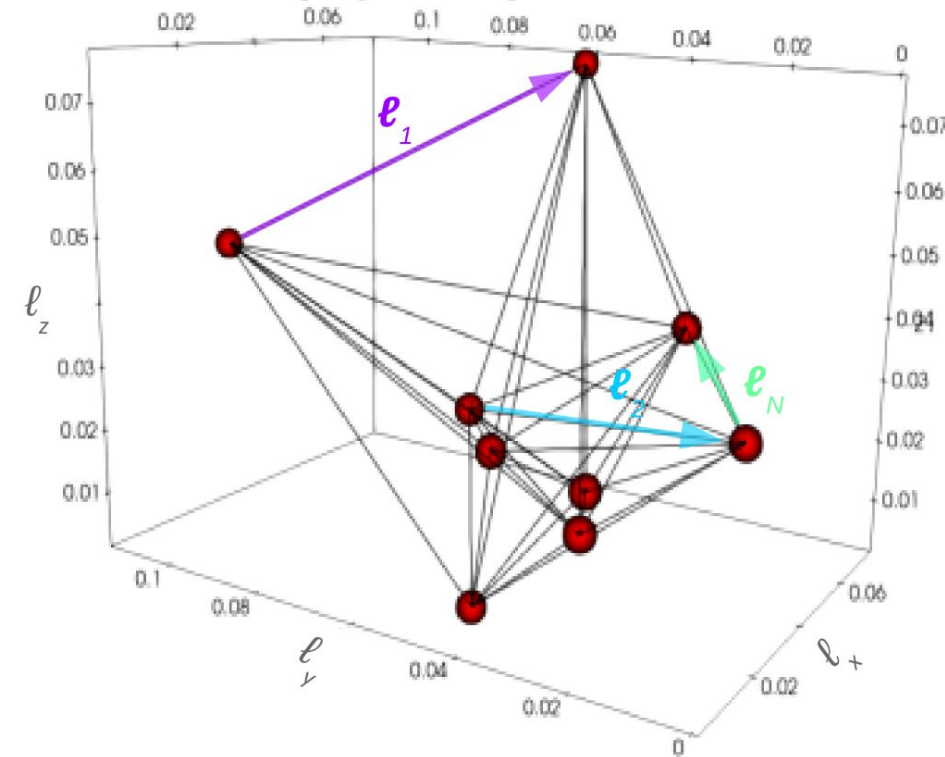


**We measure it directly, in
the increments space,
thanks to the
constellation!**

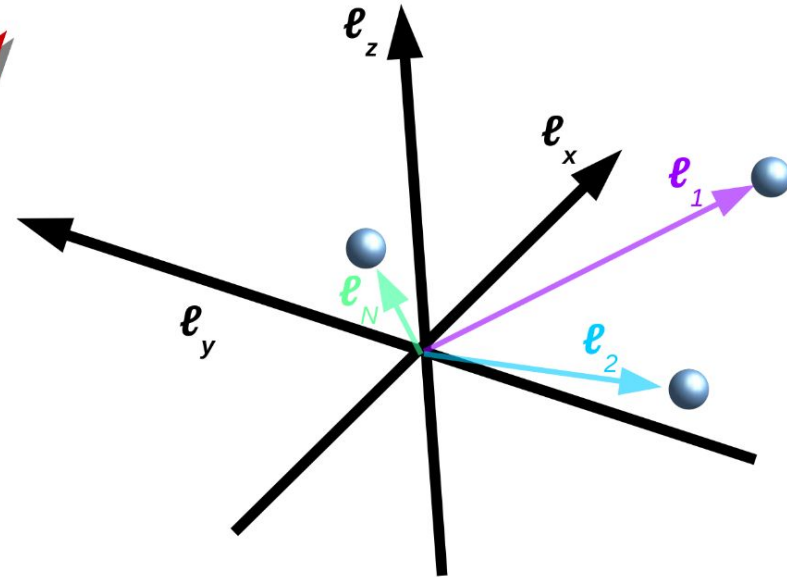
(V) A Novel Method for Measuring the Turbulent Dissipation Rate

From Physical to Increments Space

physical space



increments space



One value for each pair of spacecraft → One value at each point in lag space

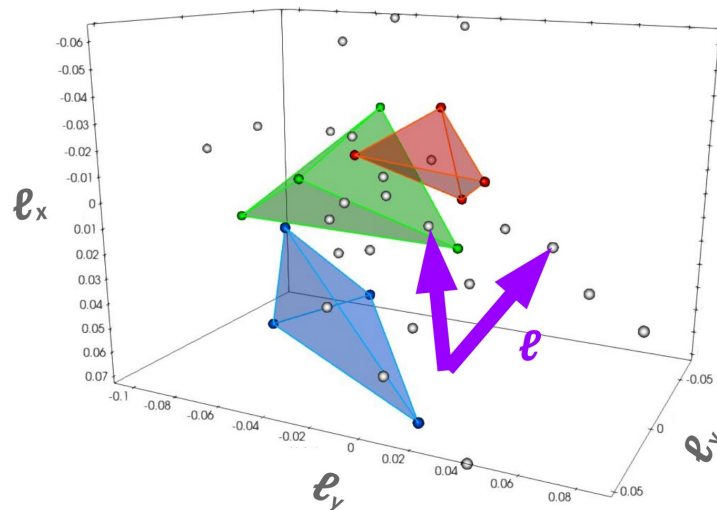
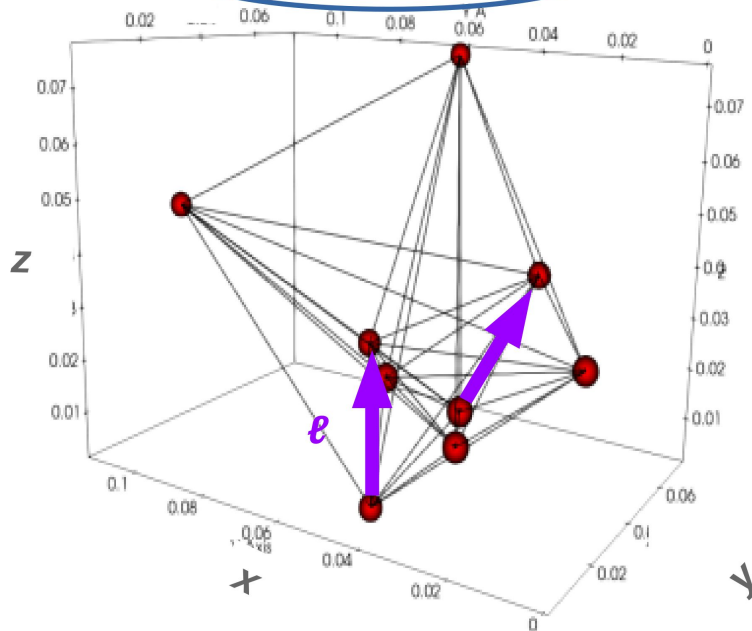


The “Lag-Polyhedra Derivative Ensemble” (LPDE)

We present the novel multi-spacecraft technique
LPDE (Lag-Polyhedra Derivative Ensemble) for
evaluating third-order statistics, using simultaneous
measurements at many points.

58905 tetrahedra!

physical space



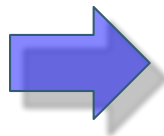
Increment space



The “Elements” in the Increments Space

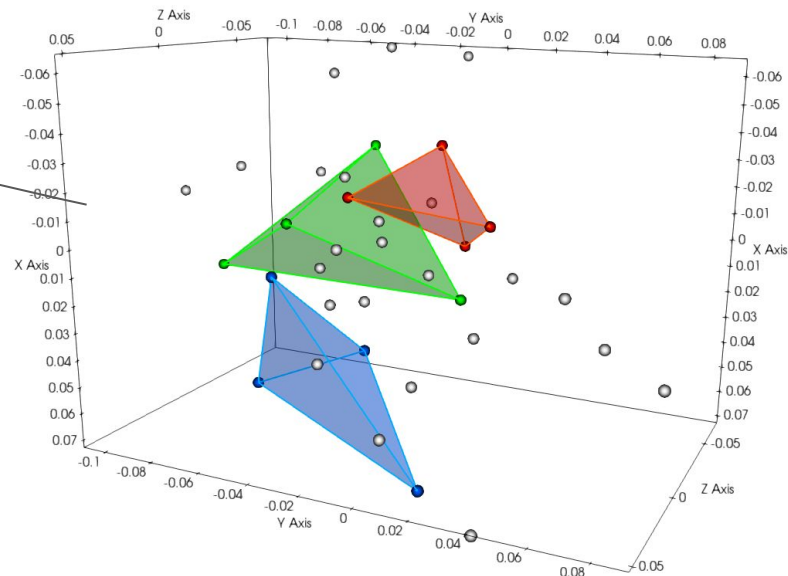
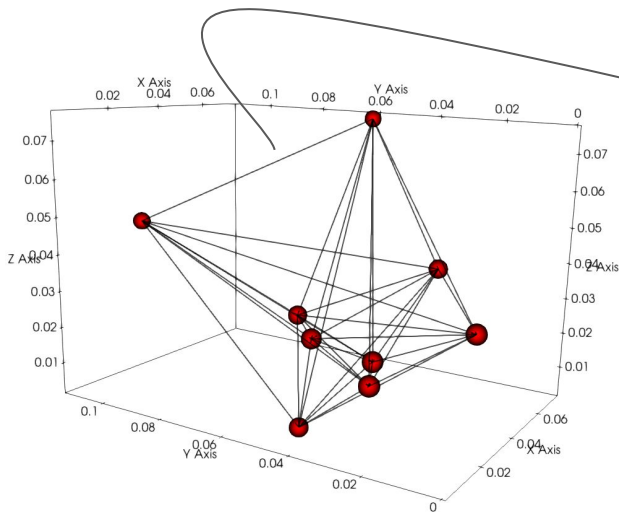
Real space:

- ◆ 9 spacecraft
- ◆ 36 baselines
- ◆ 126 tetrahedra



Lag space:

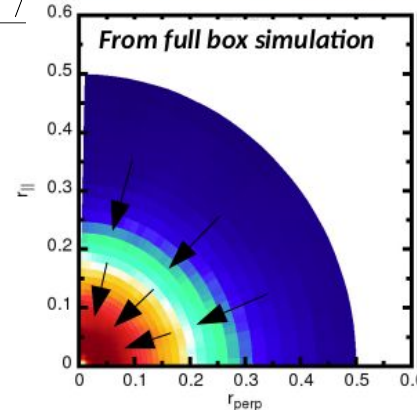
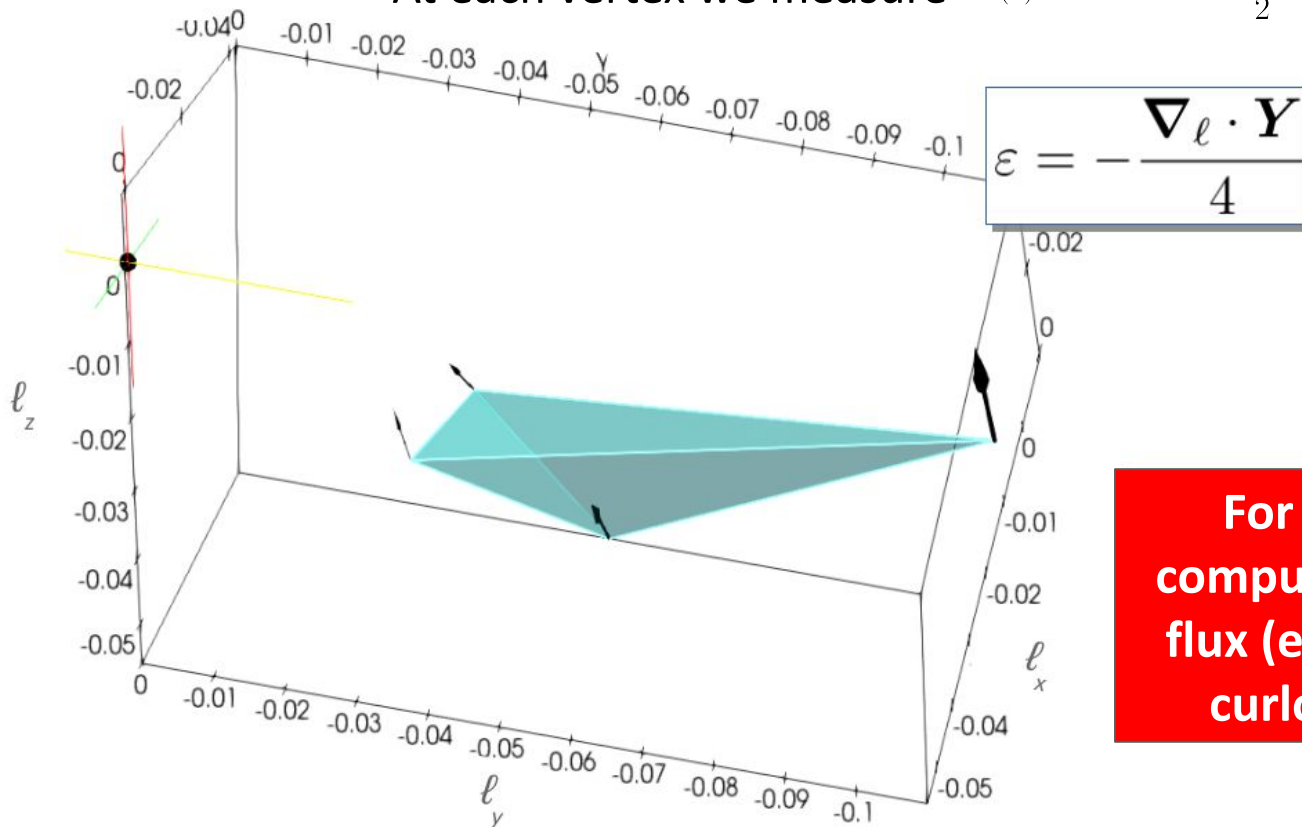
- ◆ 36 “spacecraft”
- ◆ 630 baselines
- ◆ 58905 (!) tetrahedra





The Cascade Rate

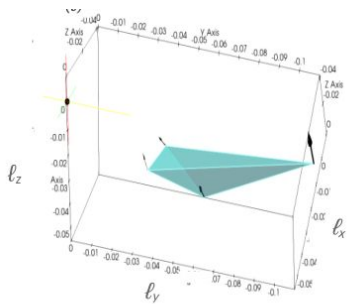
At each vertex we measure $Y(\ell) = \frac{\langle \delta \mathbf{z}^- | \delta \mathbf{z}^+ |^2 \rangle + \langle \delta \mathbf{z}^+ | \delta \mathbf{z}^- |^2 \rangle}{2}$



For each tetrahedron we compute the divergence of the flux (epsilon) via the standard curlometer-like technique



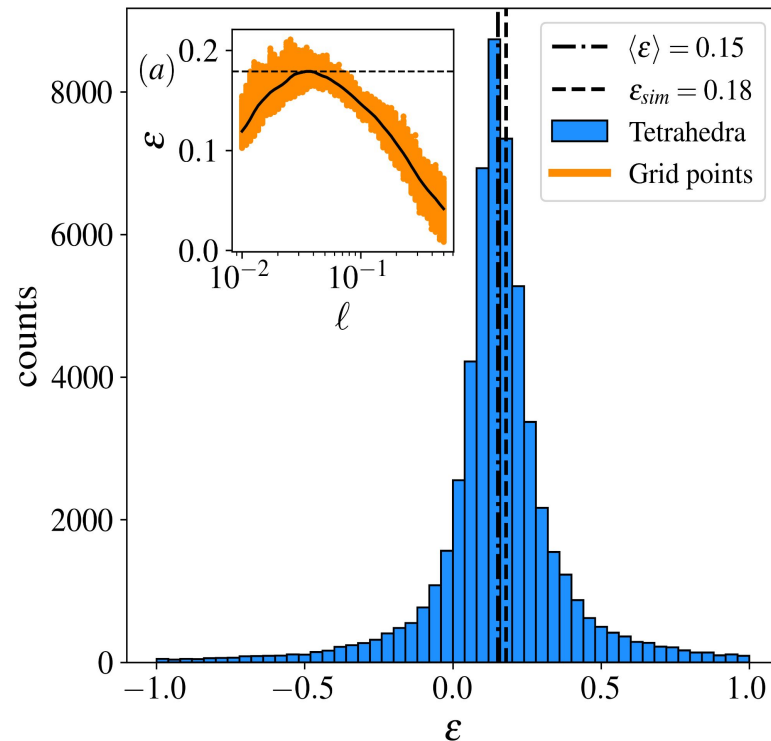
The Average Dissipation Rate for Virtual Swarms



$$\varepsilon = -\frac{\nabla_{\ell} \cdot \mathbf{Y}}{4}$$



The method can
measure very well the
average rate

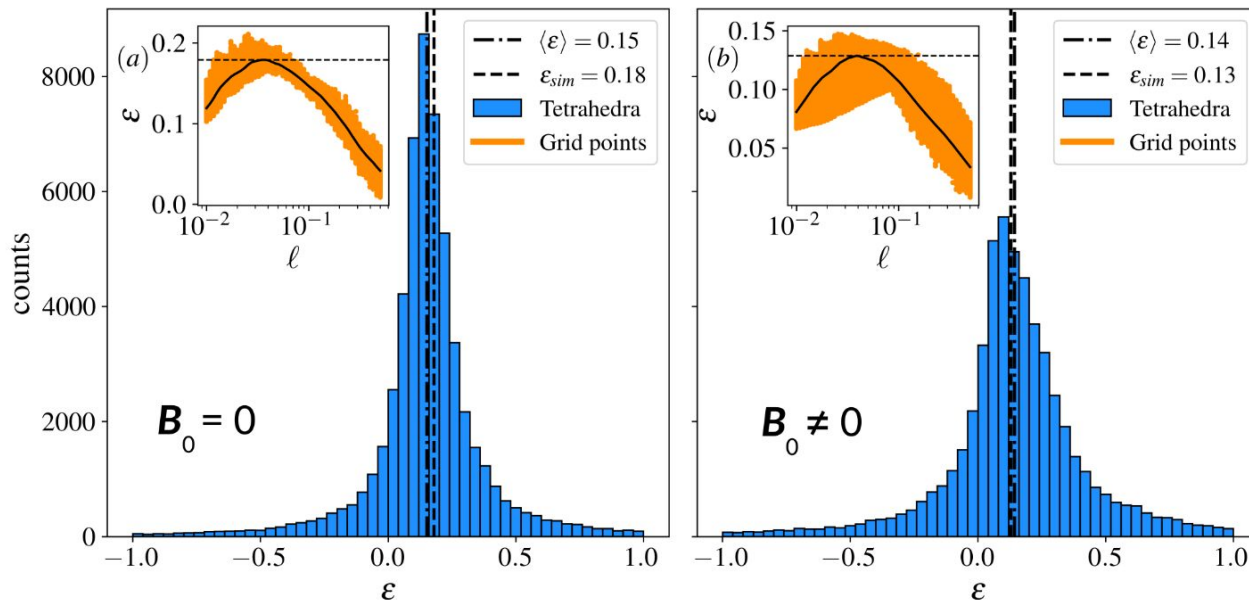


Pecora+, Multipoint turbulence analysis with Helioswarm, APJL (2023)



Anisotropy of the Flux (Cascade)

For each tetrahedron we compute the divergence of the flux (epsilon) via the standard curlometer-like technique [Dunlop *et al* 2002]



Very good measurement of the average cascade rate

(VI) From Virtual to Real: MMS

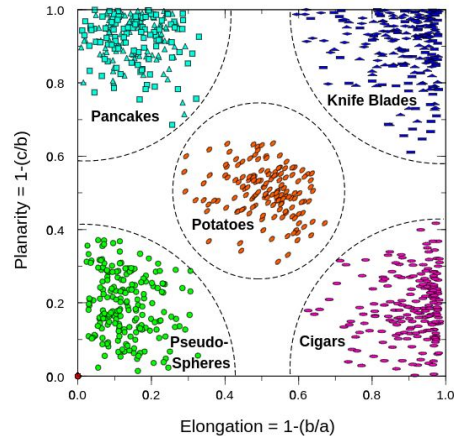


MMS Excellent Data

Magnetosheath intervals

Date (UTC)	$\delta b/B$	$\delta \rho/\rho$	β	$\tau_c(s)$	$d_i(km)$
2017 Sep 28 06:31:33 – 07:01:43	0.51	0.19	5.9	81.0	46
2017 Nov 10 22:35:43 – 22:52:03	3.17	0.43	8.3	2.9	74
2017 Dec 21 07:21:54 – 07:48:01	1.92	0.31	4.7	6.1	50
2017 Dec 26 06:12:43 – 06:52:23	0.82	0.21	4.5	16.3	48
2018 Apr 19 05:10:23 – 05:41:53	2.99	0.29	15.0	5.1	36

Real-space sc configuration does not have control on the quality of lag-space tetrahedra.

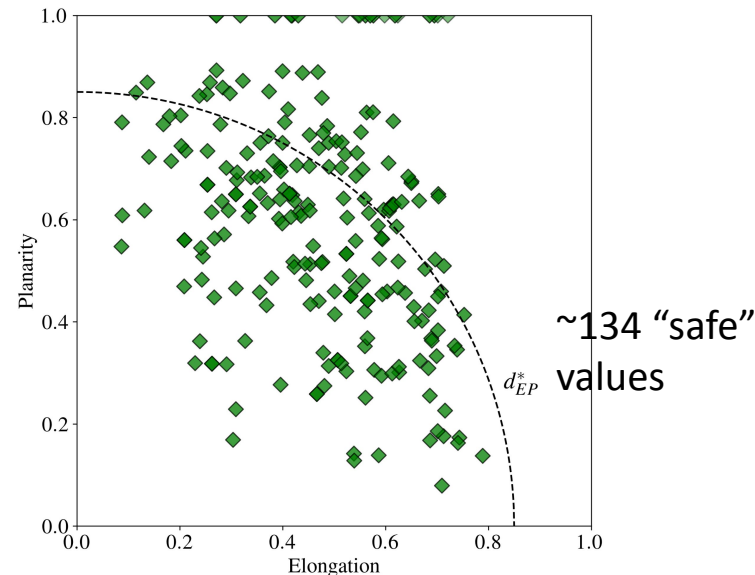


Paschmann&Daly 1998



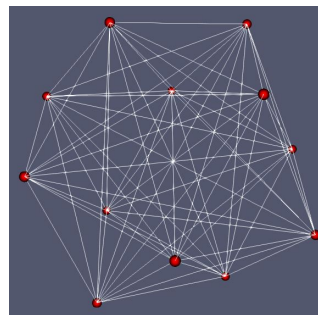
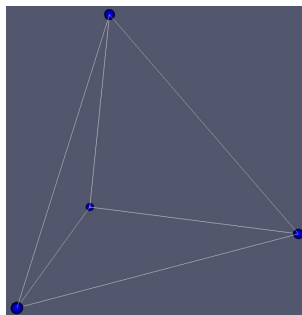
vertices gives the j th component of the mesocenter, $\ell_{bj} = \langle \ell_{aj} \rangle_a$. We use $N = 4$. Calculating the three eigenvalues of the volumetric tensor, $\lambda_1 \geq \lambda_2 \geq \lambda_3$, we can define elongation (E) and planarity (P) parameters as $E = 1 - \sqrt{\lambda_2/\lambda_1}$ and $P = 1 - \sqrt{\lambda_3/\lambda_2}$, respectively. We will consider suitable for our analysis all tetrahedra constrained in a certain region of the EP plane defined by the distance parameter $d_{EP} = \sqrt{E^2 + P^2}$. We initially use only results obtained from lag tetrahedra such that $d_{EP} \leq 0.6$. For these, the errors should be smaller than 20% (see p. 408 of Paschmann and Daly [38]). Additionally, this threshold automatically rejects tetrahedra that are nearly planar or collinear.

Lag-space tetrahedra quality check





MMS - Vector Yaglom flux



Real space:

- ◆ 4 spacecraft
- ◆ 6 baselines
- ◆ 1 tetrahedron

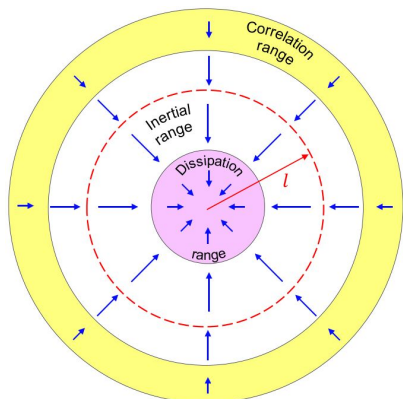
Lag space (2):

- ◆ 12 “spacecraft”
- ◆ 66 baselines
- ◆ 240 tetrahedra

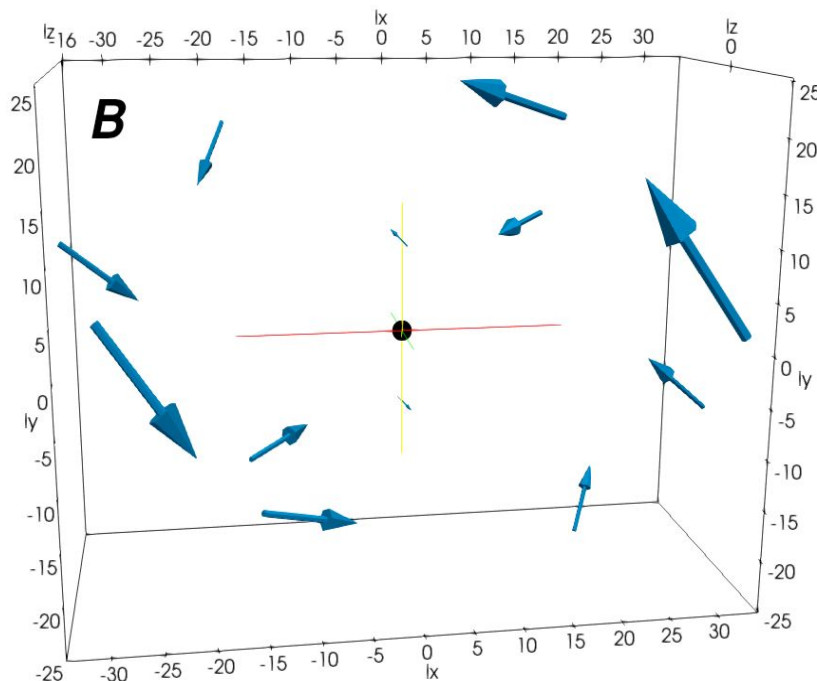


MMS - Vector Yaglom flux

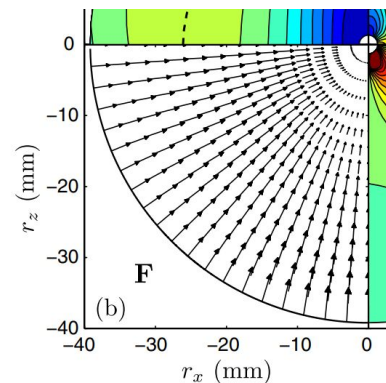
Yaglom flux dives into the origin not radially. Tilting due to anisotropy?



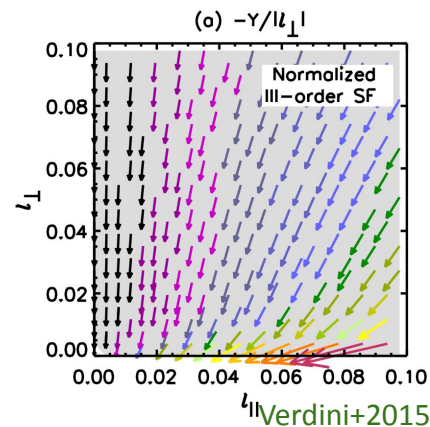
Adhikari+2023



Pecora+2023 PRL



Lamriben+2011 PRL HD experiment



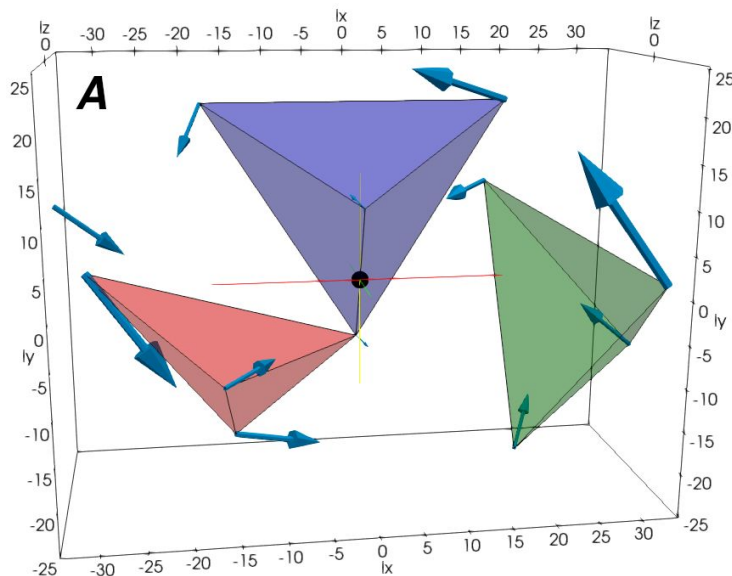
Verdini+2015



The Elements with MMS

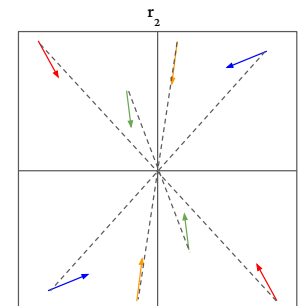
Real space:

- ◆ 4 spacecraft
- ◆ 6 baselines
- ◆ 1 tetrahedron



Lag space (2):

- ◆ 12 “spacecraft”
- ◆ 66 baselines
- ◆ 240 tetrahedra





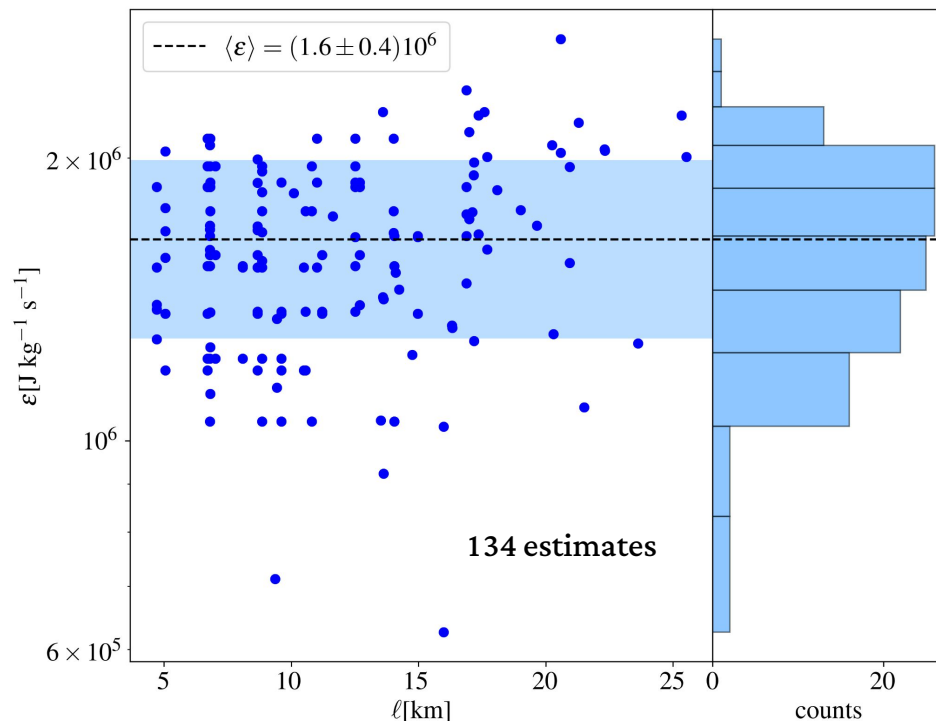
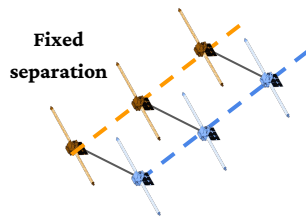
MMS Estimation of the Cascade Rate

Magnetosheath intervals

Date (UTC)	$\delta b/B$	$\delta \rho/\rho$	β	$\tau_c(s)$	$d_i(km)$
2017 Sep 28 06:31:33 – 07:01:43	0.51	0.19	5.9	81.0	46
2017 Nov 10 22:35:43 – 22:52:03	3.17	0.43	8.3	2.9	74
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2018 Apr 19 05:10:23 – 05:41:53	2.99	0.29	15.0	5.1	36

$$\mathbf{Y}^{\pm} = \langle \delta \mathbf{z}^{\mp} | \delta \mathbf{z}^{\pm} |^2 \rangle$$

$$\nabla_{\ell} \cdot \mathbf{Y}^{\pm} = -4\epsilon^{\pm}$$





Improvements from MMS to HelioSwarm

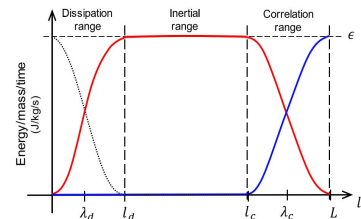
- ◆ Number of cascade rate evaluations
- ◆ Scale-dependent estimates
- ◆ More real space analyses

<i>Real space</i>	MMS	HS
spacecraft	4	9
baselines	6	36
tetrahedra	1	126

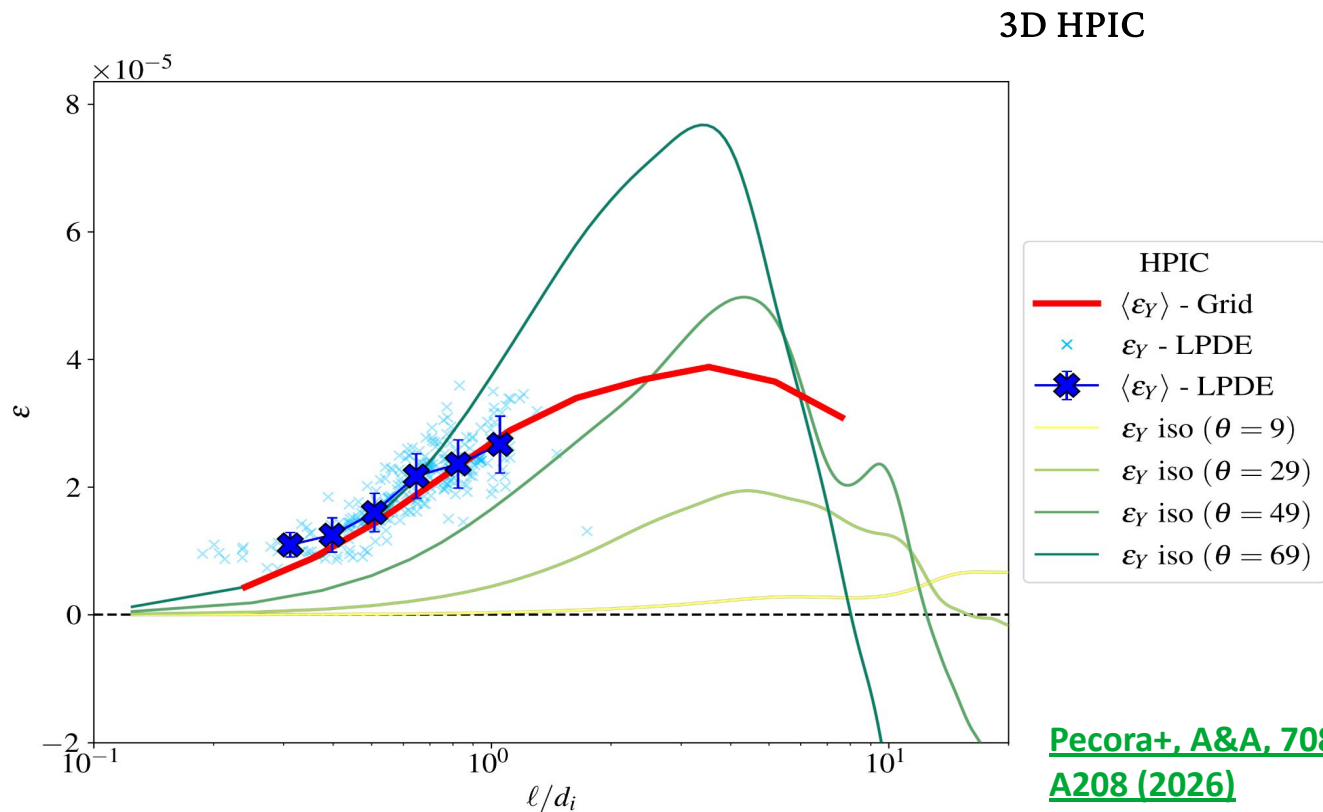
<i>Lag space</i>	MMS	HS
“spacecraft”	12	72
baselines	66	2556
independent tetrahedra	250	514395



Kinetic Simulations



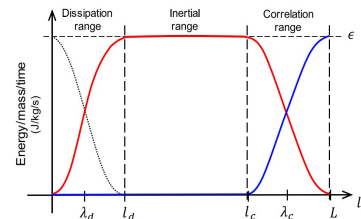
Adhikari+2023



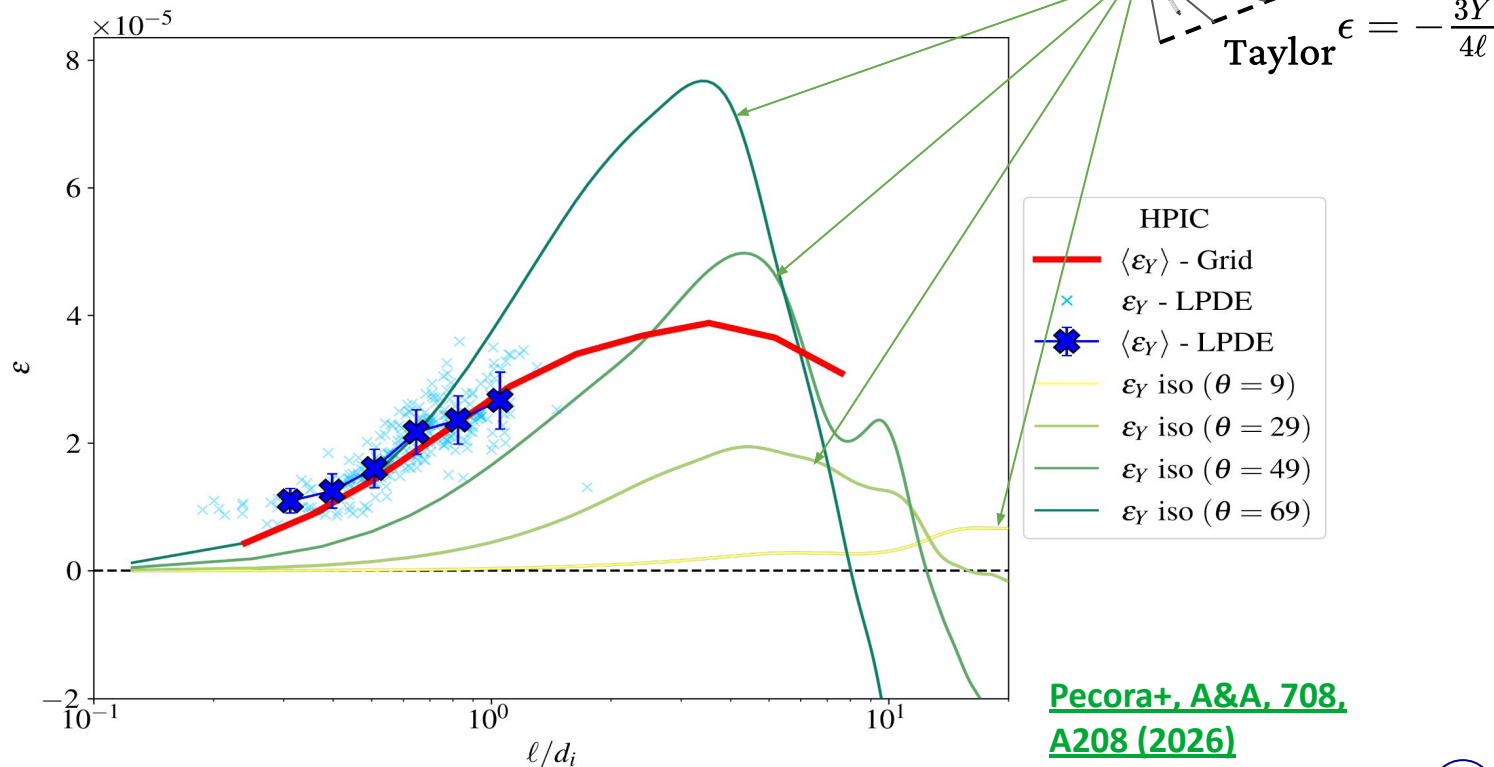
Pecora+, A&A, 708,
A208 (2026)



Kinetic Simulations



Adhikari+2023

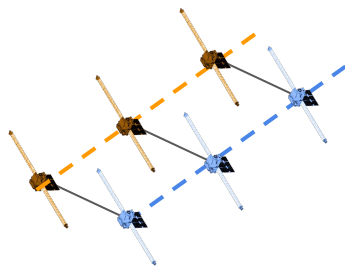


Pecora+, A&A, 708, A208 (2026)

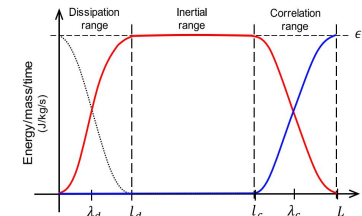
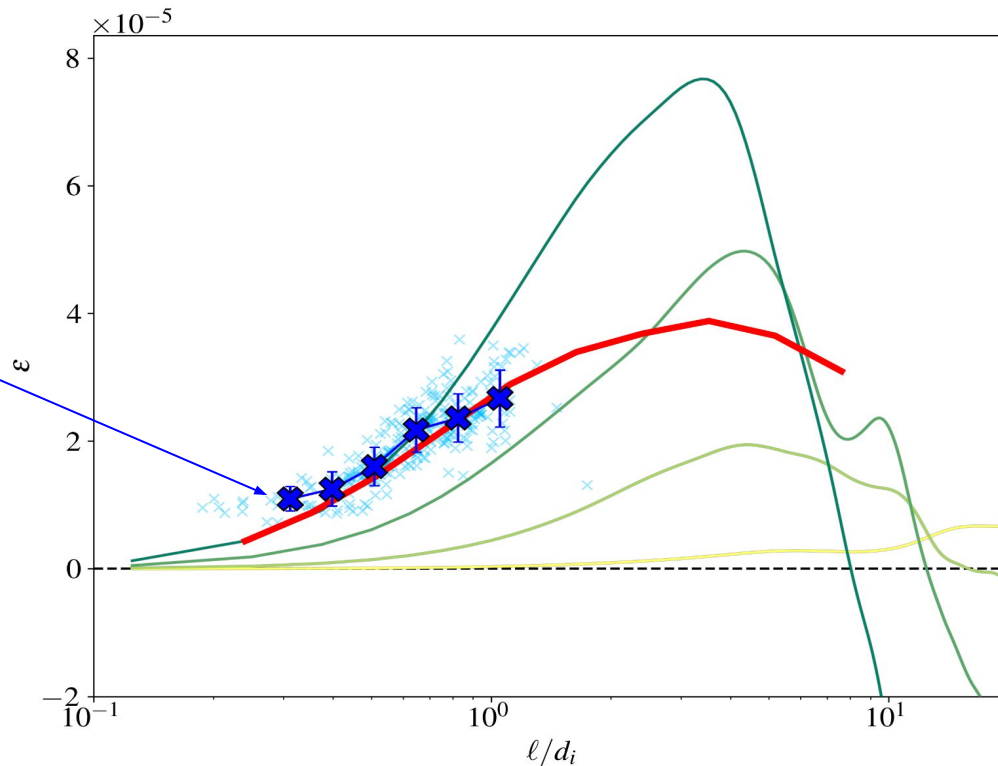
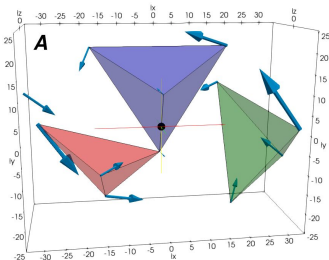


Kinetic Simulations

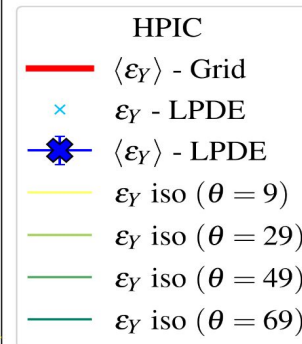
$$\nabla \cdot \mathbf{Y} = -4\epsilon$$



No Taylor
+
LPDE



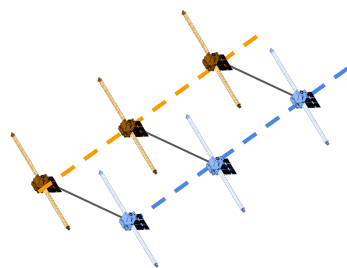
Adhikari+2023



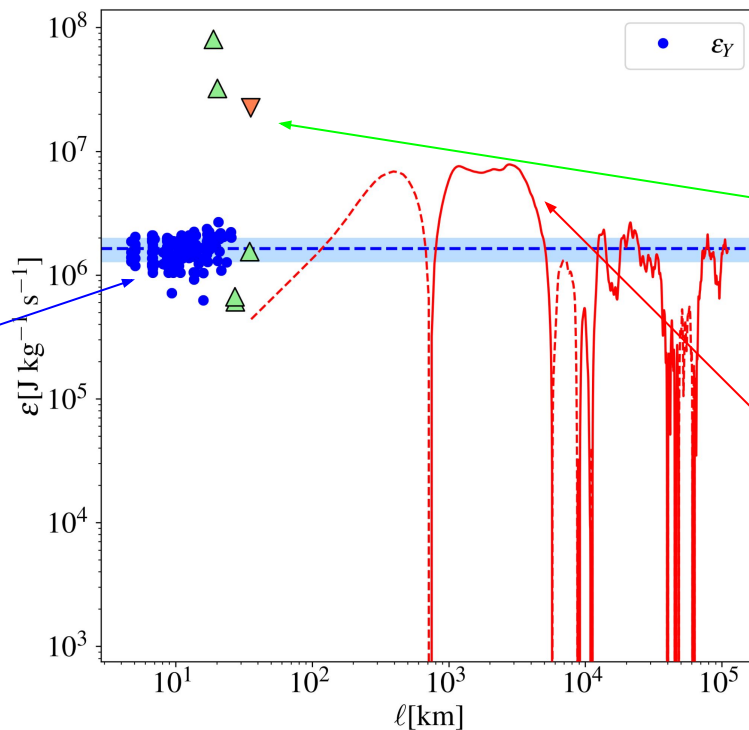
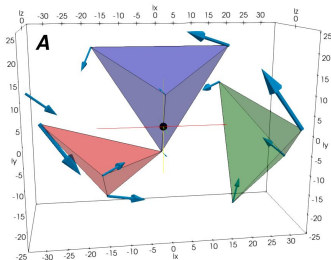
Pecora+, A&A, 708,
A208 (2026)



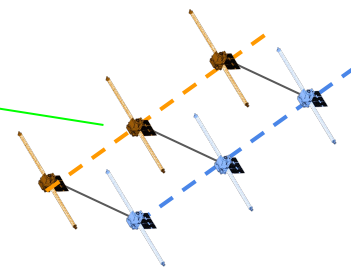
$$\nabla \cdot \mathbf{Y} = -4\epsilon$$



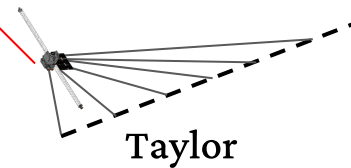
No Taylor
+
LPDE



$$\epsilon = -\frac{3Y}{4\ell}$$



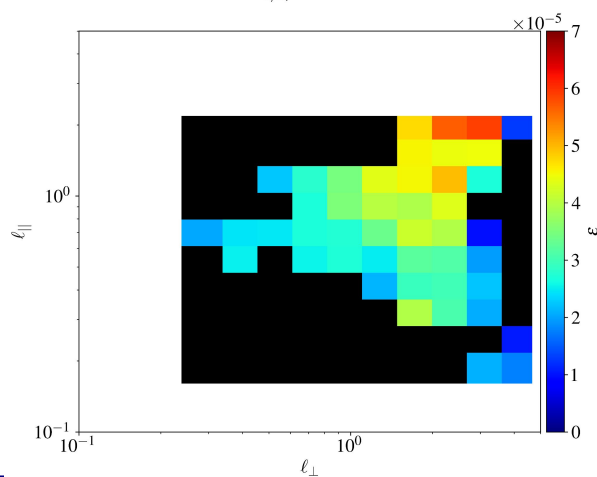
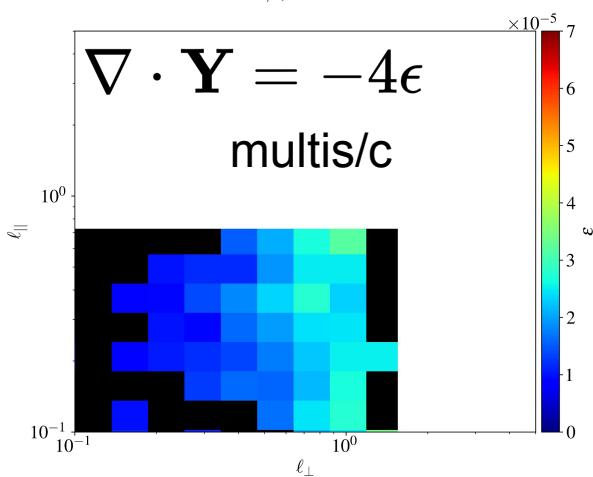
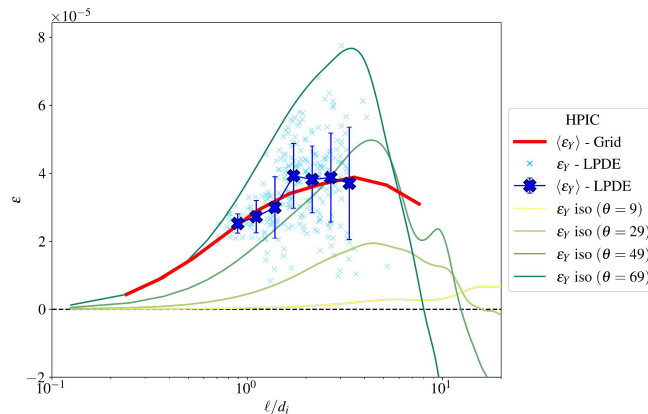
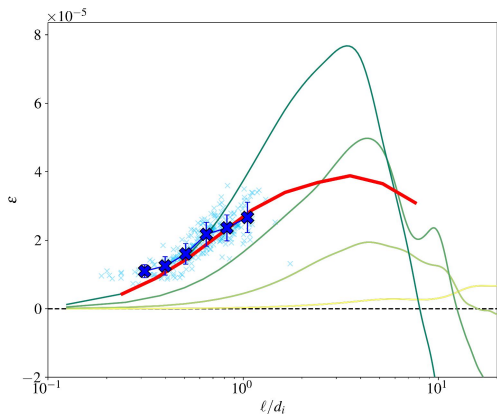
No Taylor



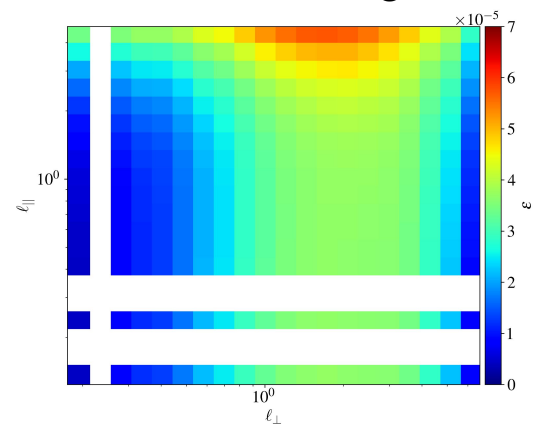
Taylor



Anisotropy of the Cascade with Virtual HS



$\nabla \cdot \mathbf{Y} = -4\epsilon$
finite differences
on the grid

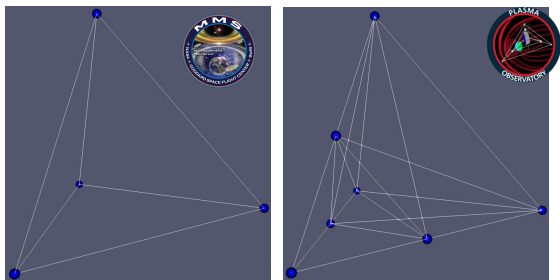


Pecora+, (2026)

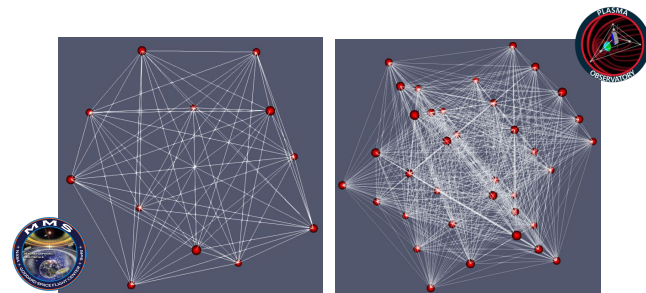


Improvements from MMS to Plasma Observatory

- ◆ More real-space analyses
- ◆ More accurate cascade rate evaluations
- ◆ **Scale-dependent estimates**



<i>Real space</i>	MMS	PO
spacecraft	4	7
baselines	6	21
tetrahedra	1	35



<i>Lag space</i>	MMS	PO
“spacecraft”	12	42
baselines	66	861
independent tetrahedra	240	55,860

(VII) Beyond 3D: Space-Time Statistics and Time-Decorrelation



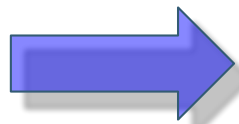
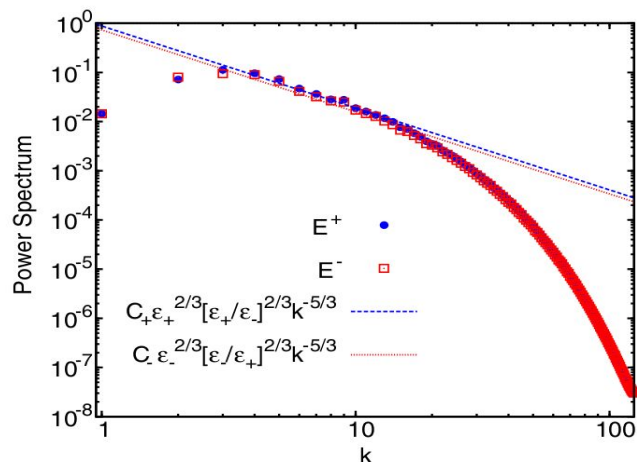
Space-Time Spectra: an Old, Unsolved Problem

$$\frac{\partial z^{\pm}}{\partial t} = - (z^{\mp} \cdot \nabla) z^{\pm} - \nabla P + \nu \nabla^2 z^{\pm}$$

$$z^{\pm}(x, y, z, t)$$

If we have a turbulent plasma and I average the spectrum in time:

$$z^{\pm}(\mathbf{x}, t) \rightarrow z^{\pm}(\mathbf{k}, t) \rightarrow P(k) = \langle P(k, t) \rangle$$



If I sit in different points in a plasma, without a mean flow, and I do many time spectra, what do I see?

Eulerian spectra

$$P(\omega) \sim \omega^{-5/3}$$

Sweeping

$$P(\omega) \sim \omega^{-2}$$

Local strain

What causes the decorrelation of modes?

Kolmogorov's local-in-scale
strain and distortion



Kolmogorov (1941)

Iroshnikov and Kraichnan's
sweeping



Iroshnikov (1964)
Kraichnan (1965)



Why is it important?

- Fundamental turbulence theory (closures EDQNM, DIA, LHDIA)
(Orszag 1970, Zhou et al. 2004)
- Particle diffusion (Matthaeus 2003)
- Correction to Taylor hypothesis (Hussain 1987)
- Dynamo theory (Pouquet et al. 1976, EDQNM)
- ...



Theoretical Background

$$R(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}, t + \tau) \rangle \quad \text{Two-point two-time autocorrelation function}$$



Fourier transform (only in space)

$$S(\mathbf{k}, \tau) = \frac{1}{(2\pi)^3} \int d\mathbf{r} e^{i\mathbf{k} \cdot \mathbf{r}} R(\mathbf{r}, \tau) \quad \text{Time-lagged power spectral density}$$



Can be decomposed as

$$S(\mathbf{k}, \tau) = S(\mathbf{k})\Gamma(\mathbf{k}, \tau)$$



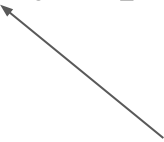
$$\Gamma(\mathbf{k}, \tau) = \frac{S(\mathbf{k}, \tau)}{S(\mathbf{k})} \quad \text{Decorrelation function or decorrelator}$$



Decorrelation Times

$$\Gamma(\mathbf{k}, \tau) = f[\gamma(\mathbf{k})\tau]$$

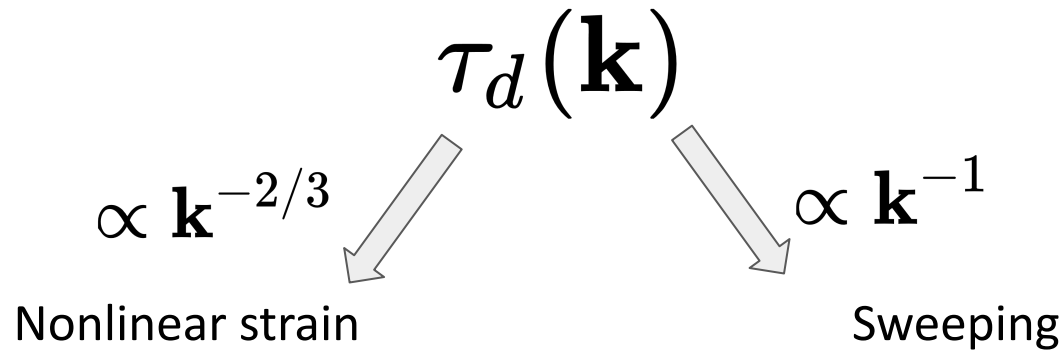
Decorrelation rates



$$\Gamma(\mathbf{k}, \tau) \sim e^{-\frac{\tau}{\tau_d(\mathbf{k})}} \quad \Gamma(\mathbf{k}, \tau) \sim e^{-\left(\frac{\tau}{\tau_d(\mathbf{k})}\right)^2}$$

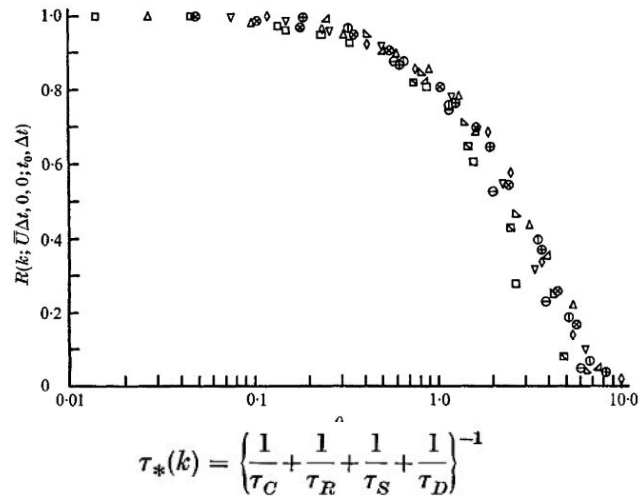


The Discriminant: The Decorrelation Time





Earlier Hydrodynamics Experiments

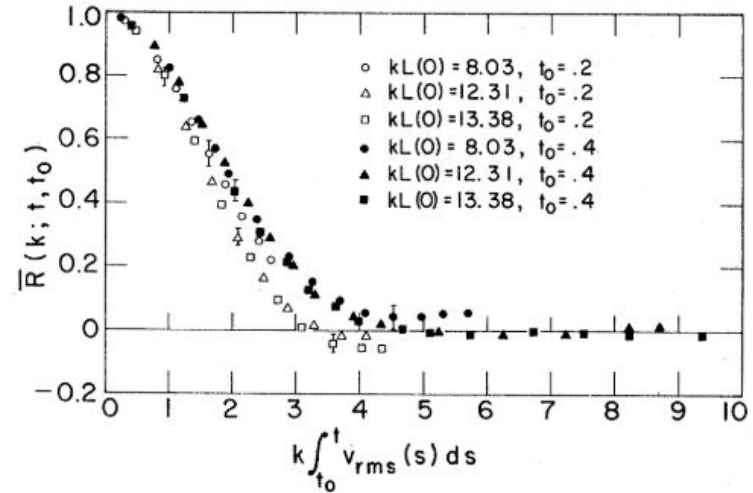


Comte-Bellot & Corssin 1971

grid turbulence

best collapse using a

combination of transfer times



Orszag & Patterson 1972

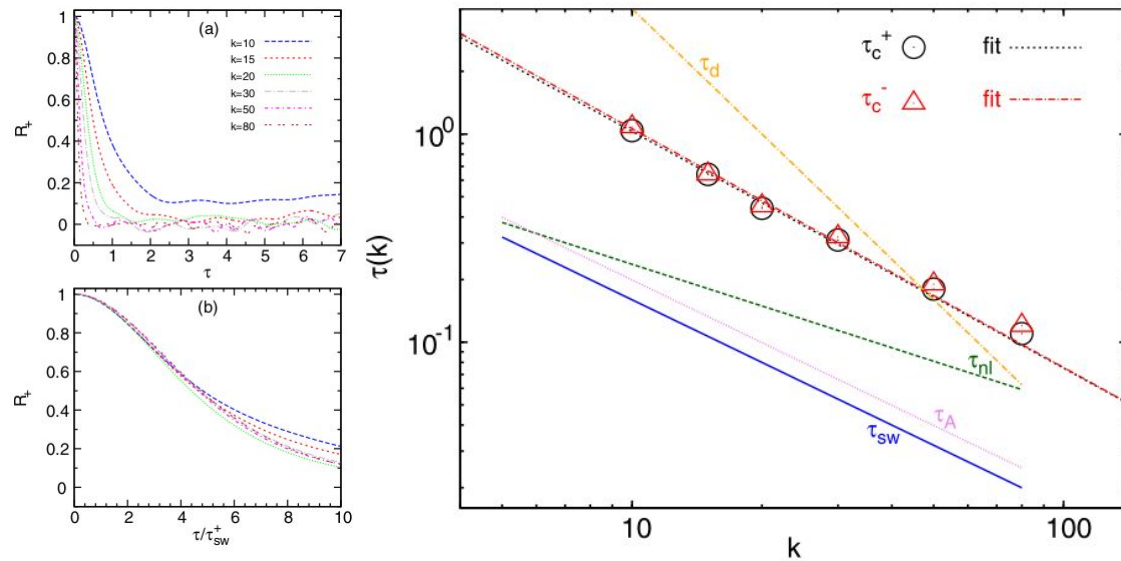
3D HD simulation

sweeping

McComb et al 1989: HD simulations Re dependence scaling (**moderate Re sweeping, larger Re strain**)

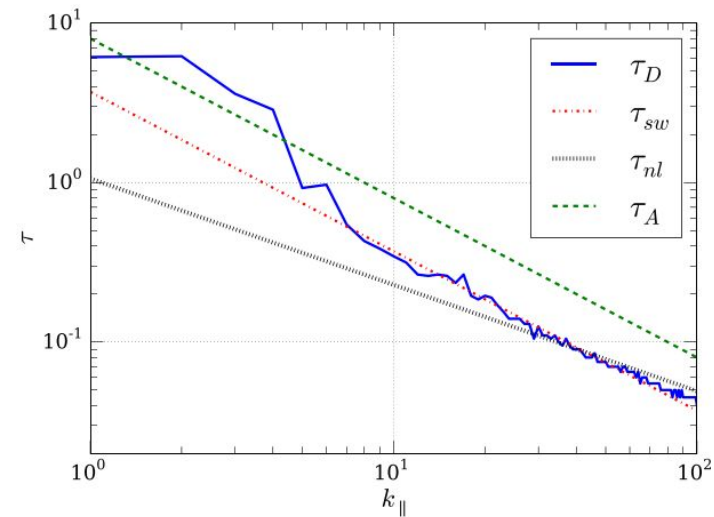


MHD Numerical Experiments



SS+, EPL 2011:

3D MHD simulation -
sweeping



Lugones et al. 2016, 2019:

3D MHD simulation -
sweeping



Solar Wind Measurements?

How can we measure this
with (multiple) spacecraft?

$$R(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}, t + \tau) \rangle$$

$$R^{ij}(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}^i - \mathbf{V}_{sw}\tau, t + \tau) \cdot \mathbf{b}(\mathbf{x}^j, t) \rangle$$

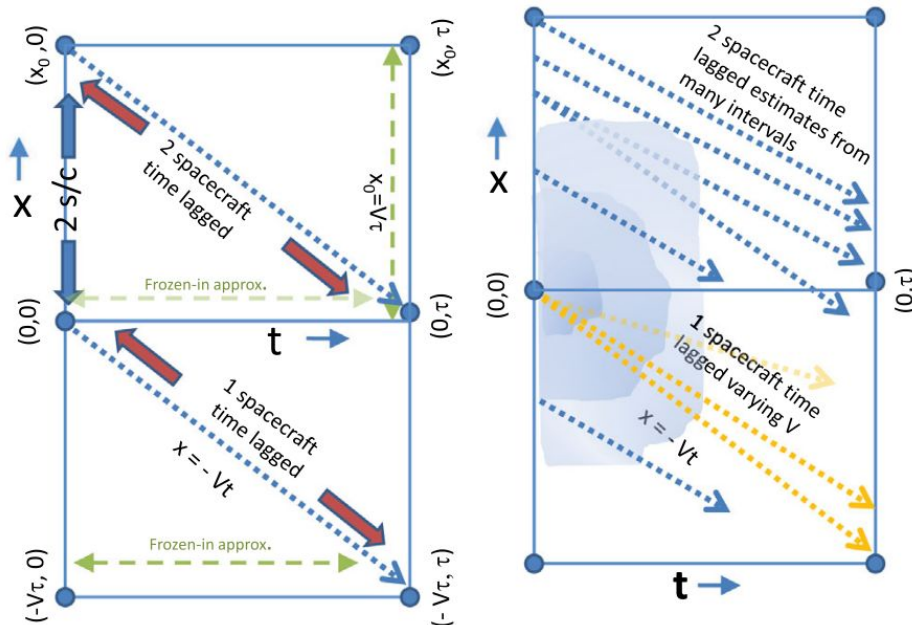
$$\mathbf{r} = \mathbf{x}^i - \mathbf{V}_{sw}\tau - \mathbf{x}^j$$



The Technique

$$R^{ij}(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}^i - \mathbf{V}_{sw}\tau, t + \tau) \cdot \mathbf{b}(\mathbf{x}^j, t) \rangle$$

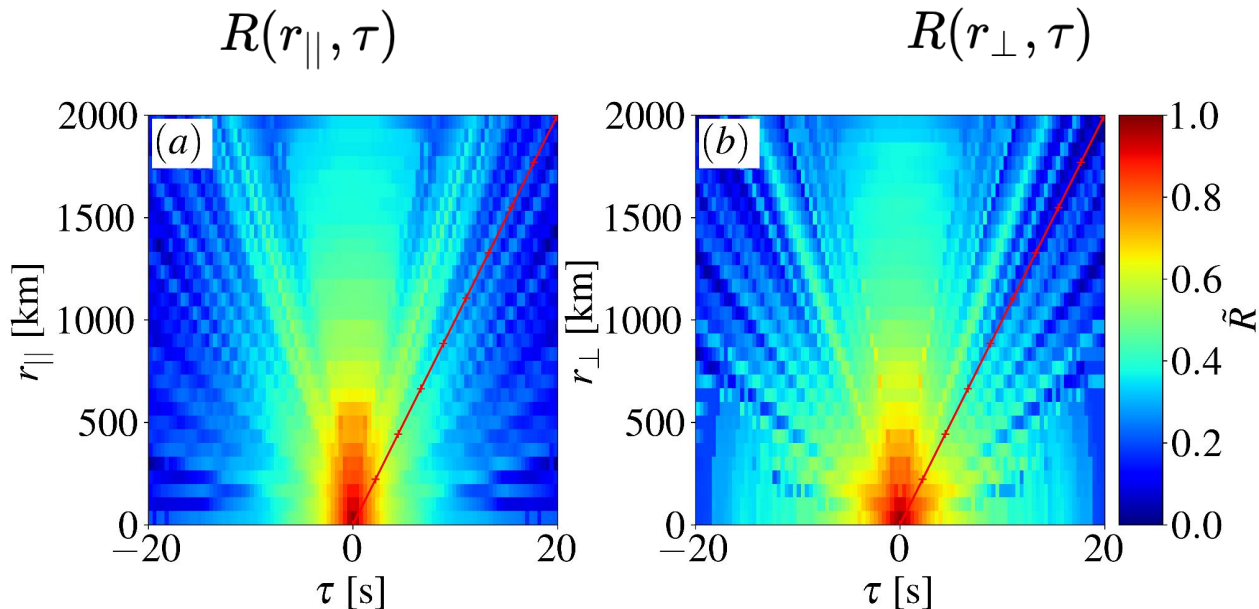
$$\mathbf{r} = \mathbf{x}^i - \mathbf{V}_{sw}\tau - \mathbf{x}^j$$



Matthaeus+, 2016



Space-Time from MMS

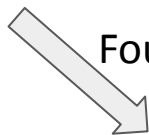
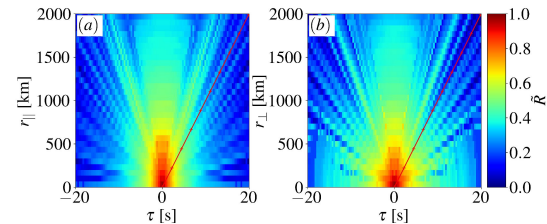


Roy et al. 2022

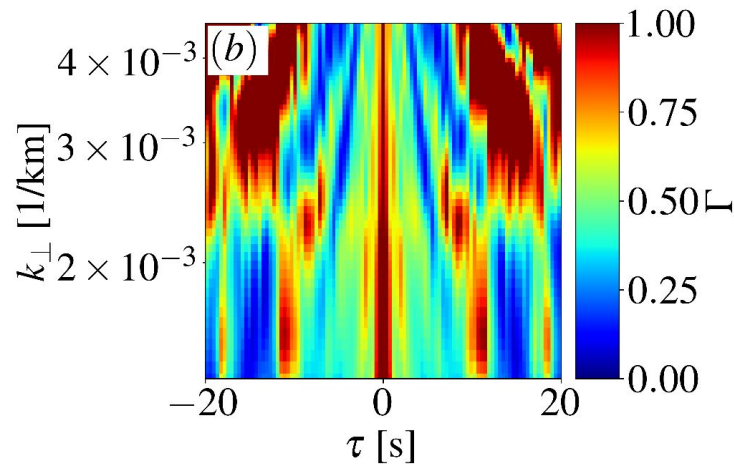
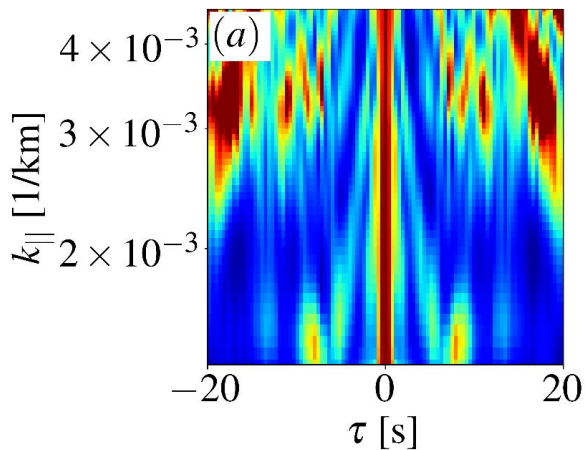
Using 1180 msheath intervals, and projecting $R(\mathbf{r}, \tau) \mapsto R(r_{\parallel}, r_{\perp}, \tau)$
then reducing $R(r_{\parallel}, \tau) = \int dr_{\perp} R(r_{\parallel}, r_{\perp}, \tau)$
and analogously $R(r_{\perp}, \tau) = \int dr_{\parallel} R(r_{\parallel}, r_{\perp}, \tau)$



k-tau Spectra

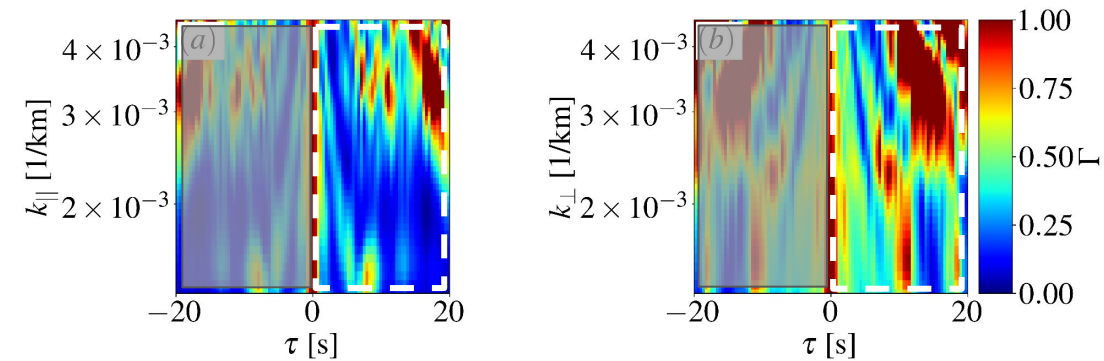


Fourier transform to obtain $\Gamma(\mathbf{k}, \tau) = \frac{S(\mathbf{k}, \tau)}{S(\mathbf{k})}$

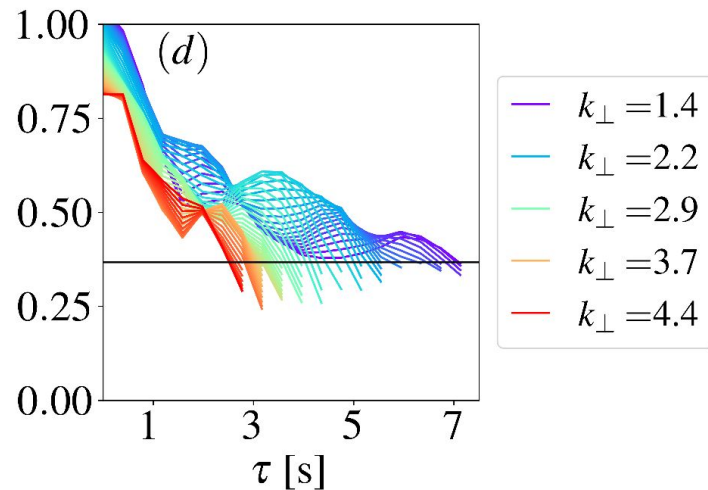
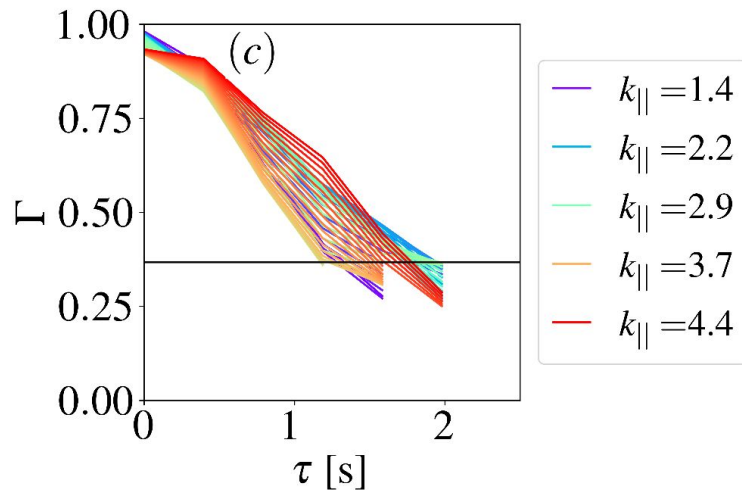




Parallel and Perpendicular Decorrelation

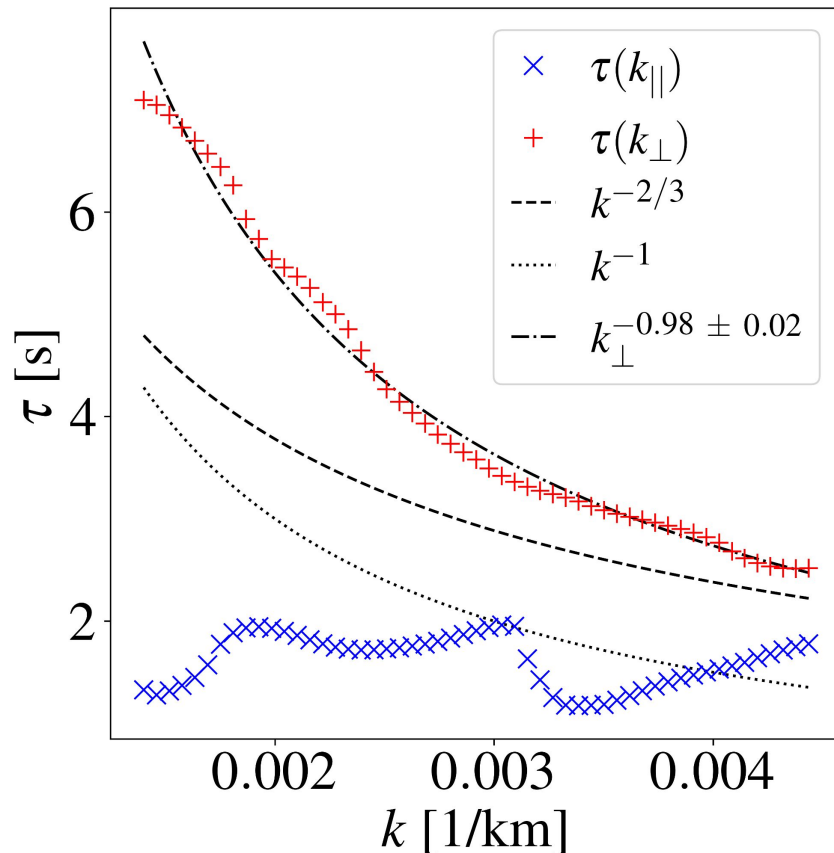
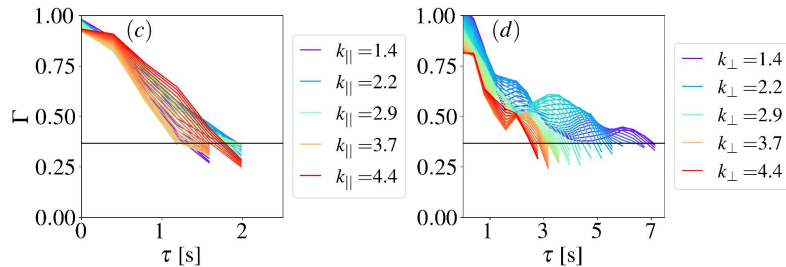


$$\Gamma(\mathbf{k}, \tau) \sim e^{-\frac{\tau}{\tau_d(\mathbf{k})}}$$





The Sweeping Effect



Turbulence in the terrestrial magnetosheath: Space-time correlation using the Magnetospheric Multiscale mission

Francesco Pecora¹, William H. Matthaeus^{1*}, Antonella Greco², Pablo Dmitruk^{3,4}, Yan Yang⁵, Vincenzo Carbone⁶, and Sergio Servidio⁶

Pecora et al. 2025 PNAS



A Summary of the Space-Time Problem

~60 years ago

$$R(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}, t + \tau) \rangle \quad \text{Main starti}$$

↓ by Fourier transforming

$$S(\mathbf{k}, \tau) = \frac{1}{(2\pi)^3} \int d\mathbf{r} e^{i\mathbf{k} \cdot \mathbf{r}} R(\mathbf{r}, \tau)$$

↓ Can be decomposed as

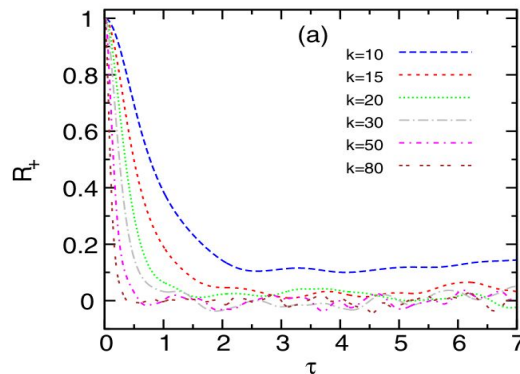
$$S(\mathbf{k}, \tau) = S(\mathbf{k}) \Gamma(\mathbf{k}, \tau)$$

$$\Gamma(\mathbf{k}, \tau) = \frac{S(\mathbf{k}, \tau)}{S(\mathbf{k})} \quad \text{de\text{c}orrelation function or "de\text{c}orrelator"}$$

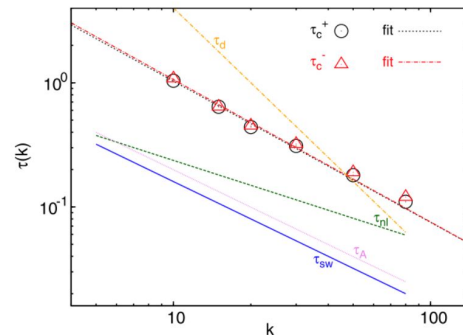
$$\Gamma(\mathbf{k}, \tau) \sim e^{-\frac{\tau}{\tau_d(\mathbf{k})}}$$

Kraichnan+, 1968

~15 years ago



MHD simulations



SS+, EPJ, 2011

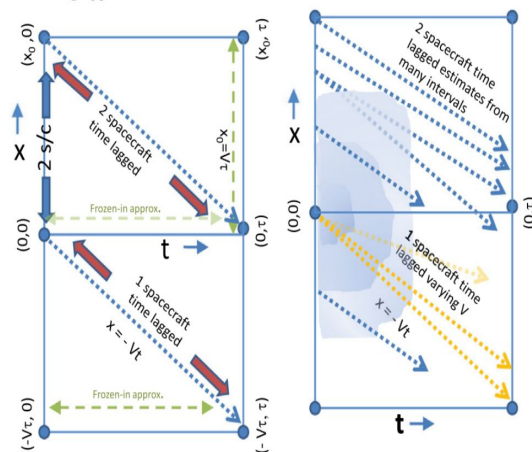


Space-Time Correlation from Satellites!

~9 years ago

$$R^{ij}(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}^i - \mathbf{V}_{sw}\tau, t + \tau) \cdot \mathbf{b}(\mathbf{x}^j, t) \rangle$$

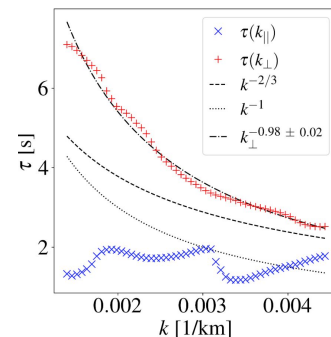
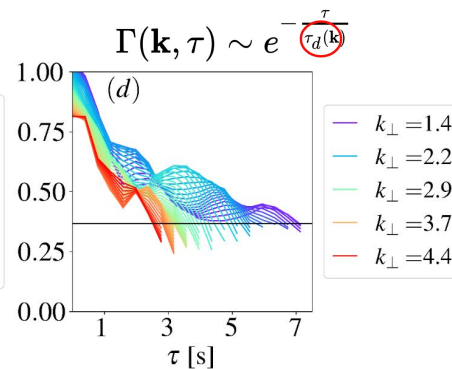
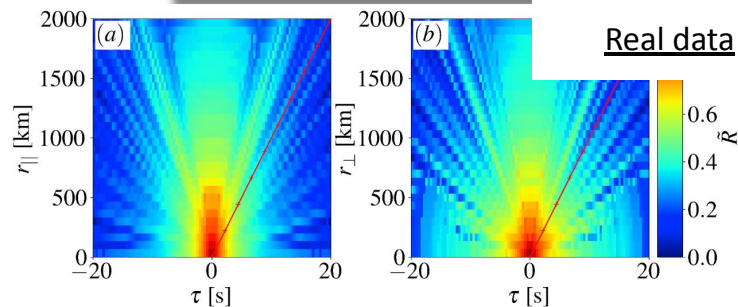
$$\mathbf{r} = \mathbf{x}^i - \mathbf{V}_{sw}\tau - \mathbf{x}^j$$



Matthaeus+, PRL, 2016

M:

~Today

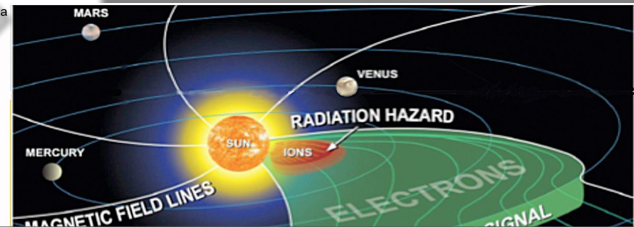
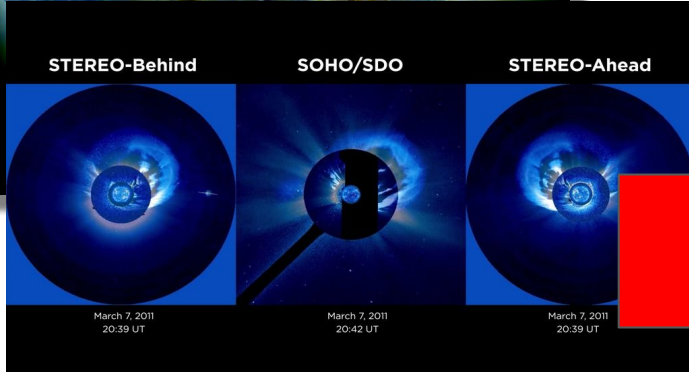
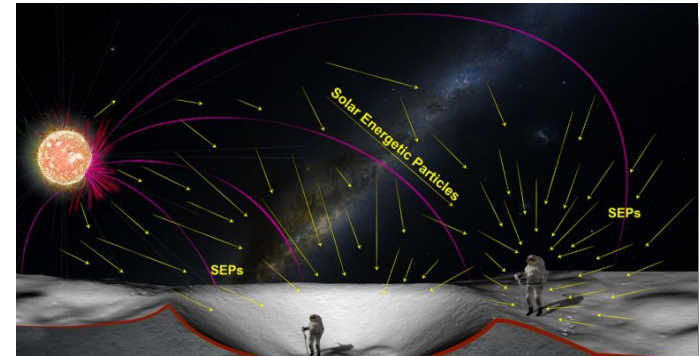
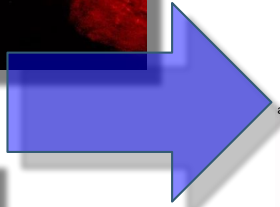
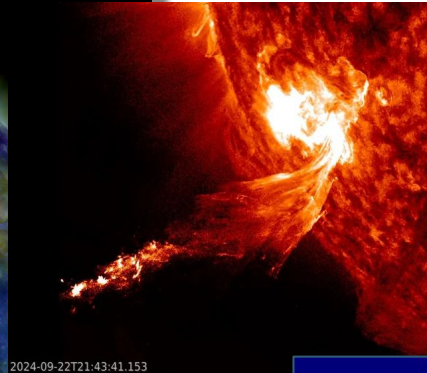
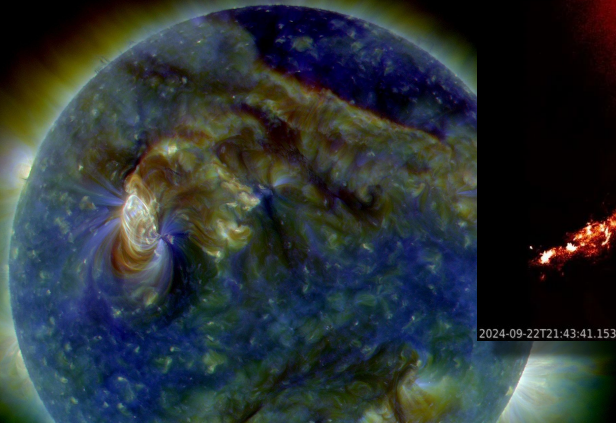


Pecora+, 2026

(VIII) Space Weather Applications and Conclusions

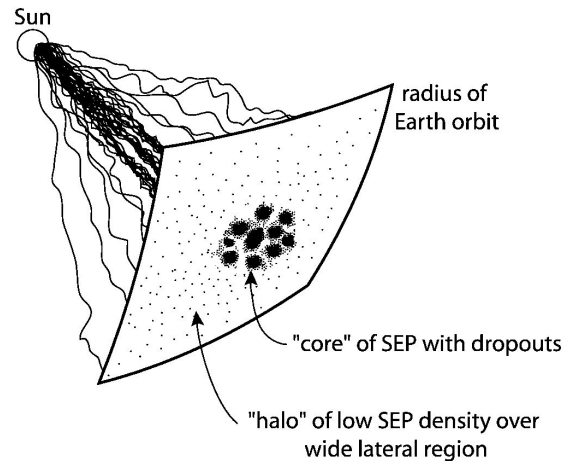
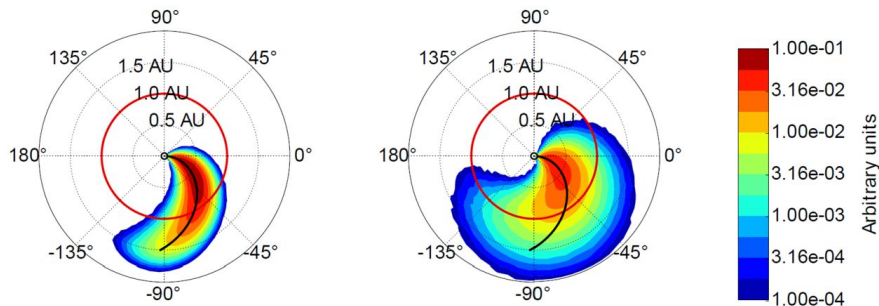
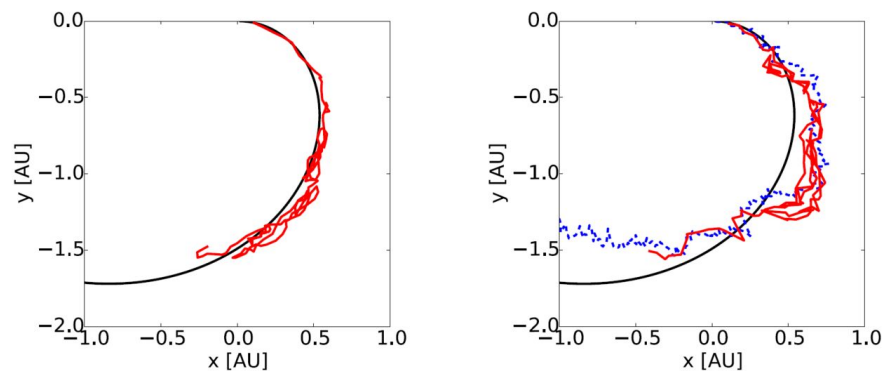
The Challenge of Space Weather

Catastrophic events

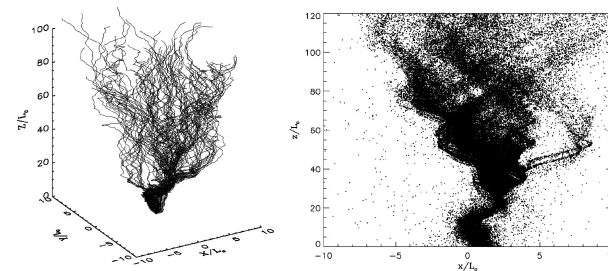


**Multipoint Analyses
are fundamental for space weather forecast**

Turbulence and SEP Diffusion



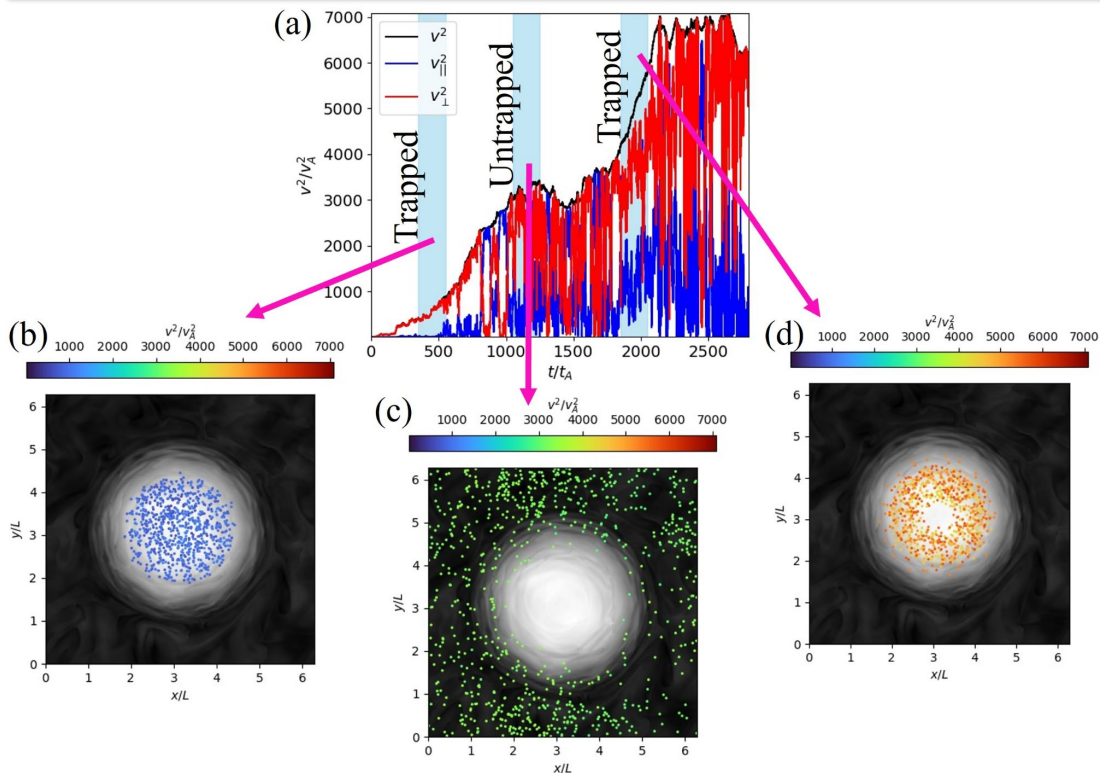
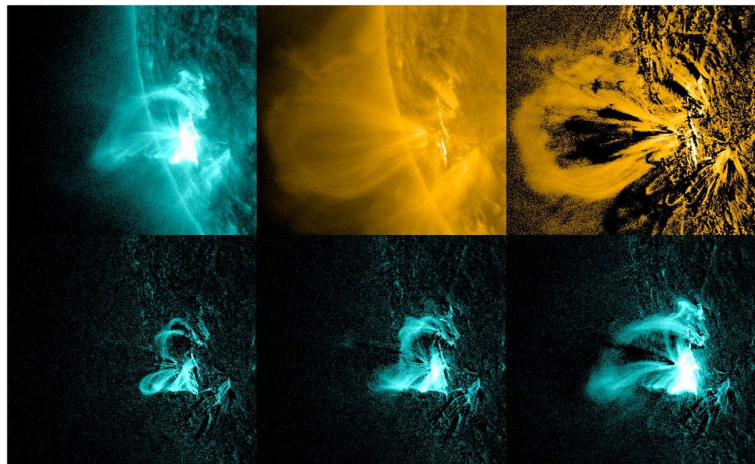
turbulence can
de-focus the
beams, can
decelerate/acc
elerate, deviate
particles... and
hence create
problems



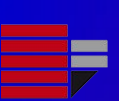


Twisting Structures on the Corona and Solar Wind

Solar Energetic Particles & Flux Tubes

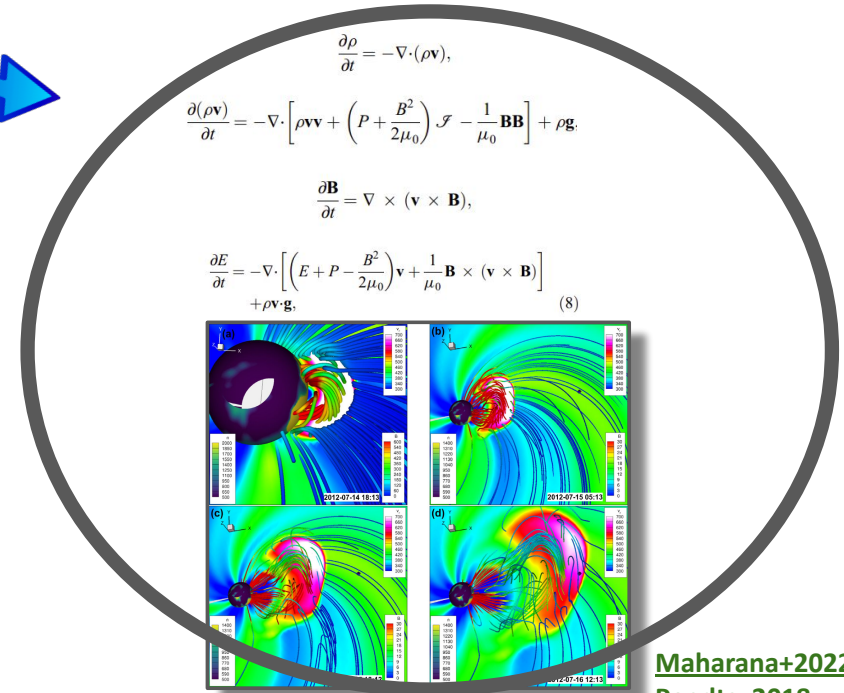
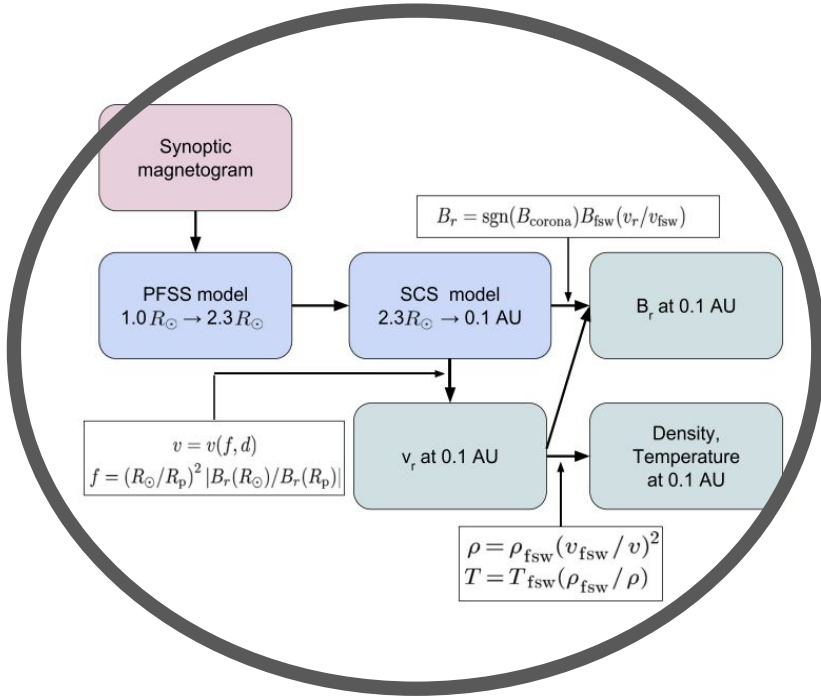


Pezzi+, A&A (2024)



From Wave-Emissions to Coronal Mass Ejections

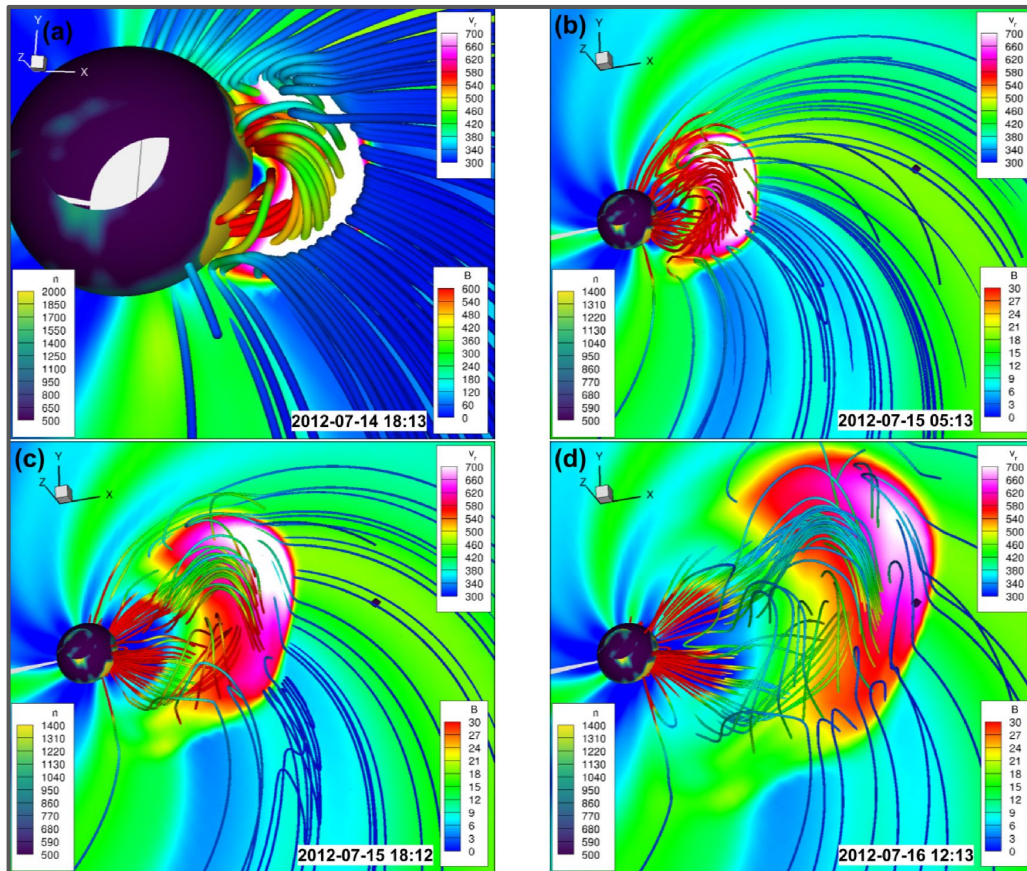
EUropean heliospheric **FOR**ecasting Information **AS**set



Maharana+2022
Poedts, 2018



"Numerical" Eruptions



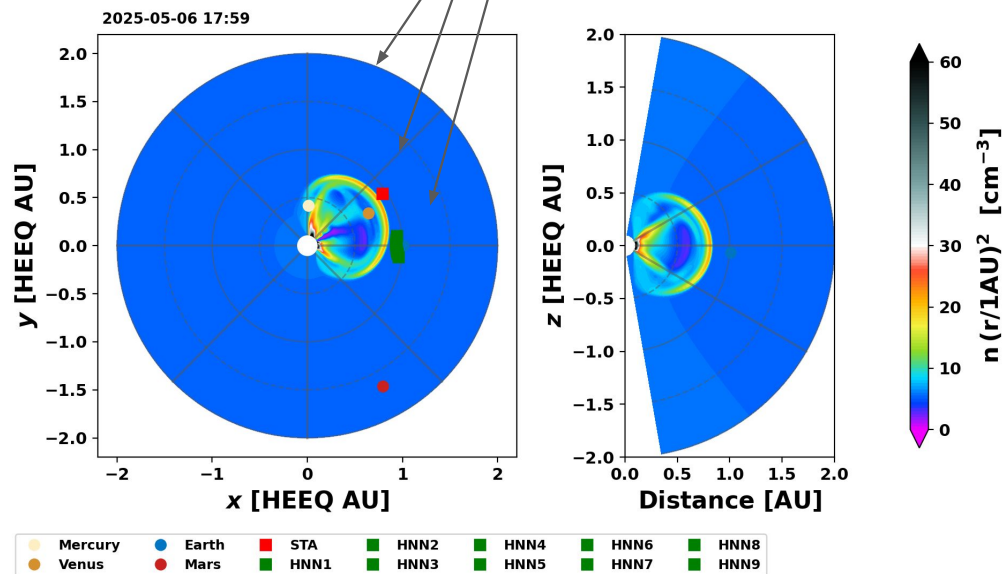
We can describe the
genesis of the flux rope,
and introduce test particles
at the shock

..simulations can run faster than nature

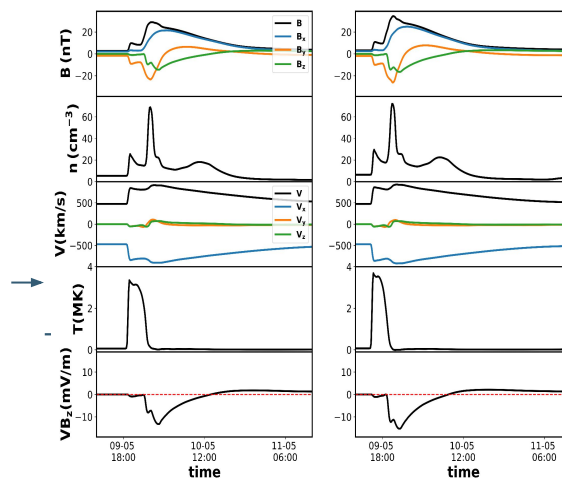


From the Sun to us!

Positions occupied by
real spacecraft



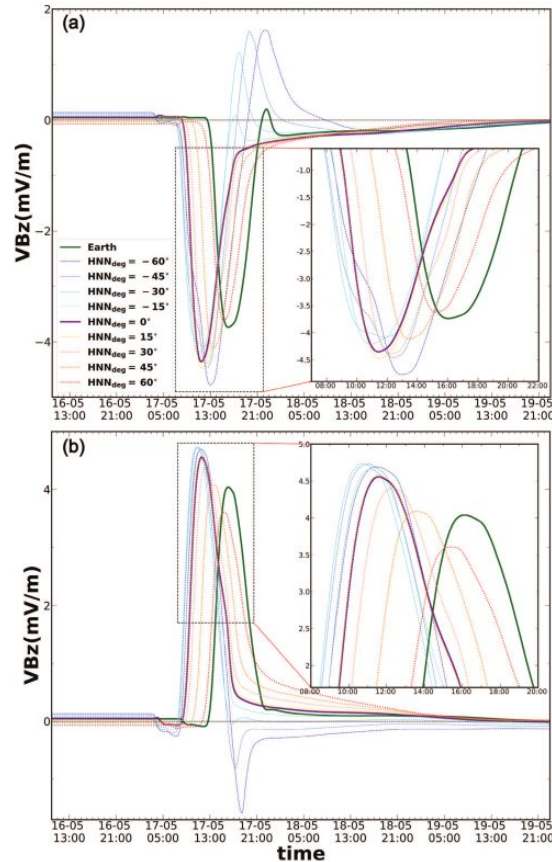
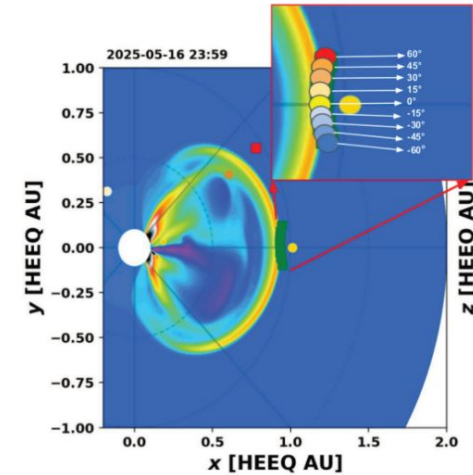
Virtual spacecraft used to make
space weather forecasting



Numerical
simulations of
optimal and
efficient in situ
spacecraft
positioning

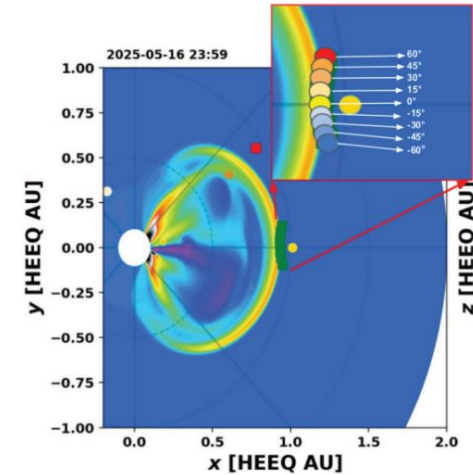
Prete+ (2025, 2026)

Multi-Spacecraft Space Weather



Prete+, J. Space Weather
Space Clim. 2026

Multi-Spacecraft Space Weather



Prete+, J. Space Weather
Space Clim. 2026

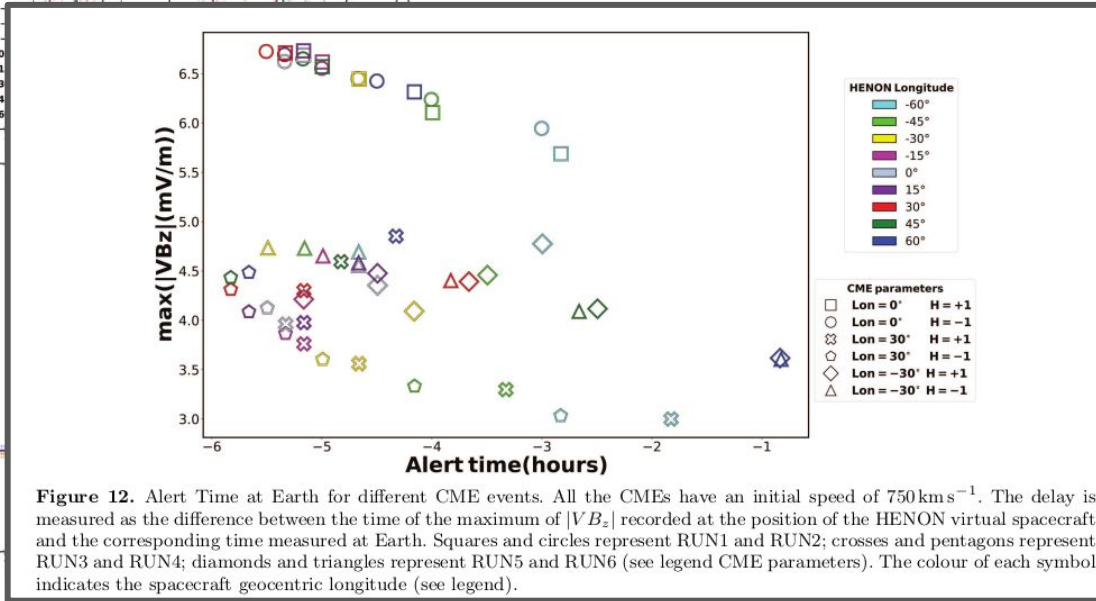
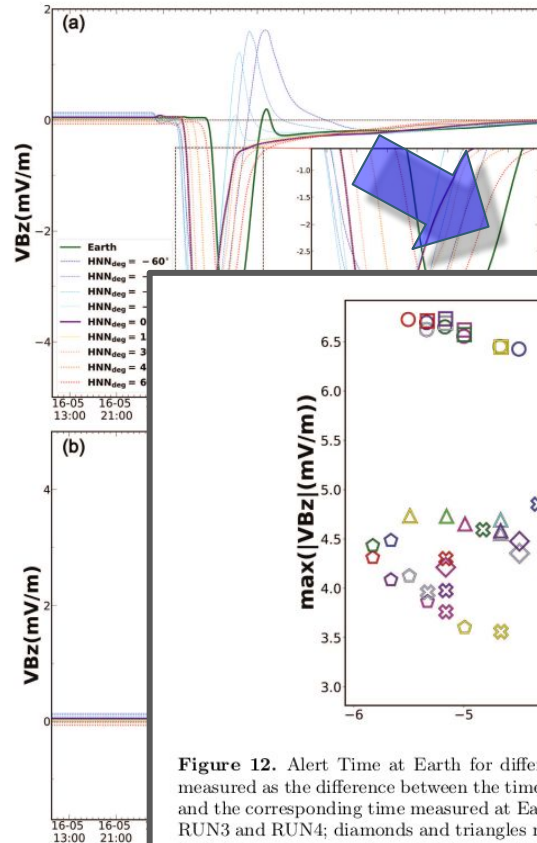


Figure 12. Alert Time at Earth for different CME events. All the CMEs have an initial speed of 750 km s^{-1} . The delay is measured as the difference between the time of the maximum of $|VB_z|$ recorded at the position of the HENON virtual spacecraft and the corresponding time measured at Earth. Squares and circles represent RUN1 and RUN2; crosses and pentagons represent RUN3 and RUN4; diamonds and triangles represent RUN5 and RUN6 (see legend CME parameters). The colour of each symbol indicates the spacecraft geocentric longitude (see legend).



Conclusions

- ◆ Numerical simulations are a powerful tool for interpreting observations
- ◆ Virtual spacecraft in turbulent plasmas have finally become a key strategy for designing new multi-spacecraft missions
- ◆ Second-order statistics are just the beginning. Third-order laws are essential to measure turbulence strength
- ◆ Multi-spacecraft techniques are fundamental to characterizing the heliosphere
- ◆ Much more is needed to understand plasma behavior via multi-spacecraft techniques, including the role of reconnection, particle energization, and — finally — the new concept of phase space cascade