# Anisotropies in the turbulent solar wind, perspective on multipoint observations from numerical simulations

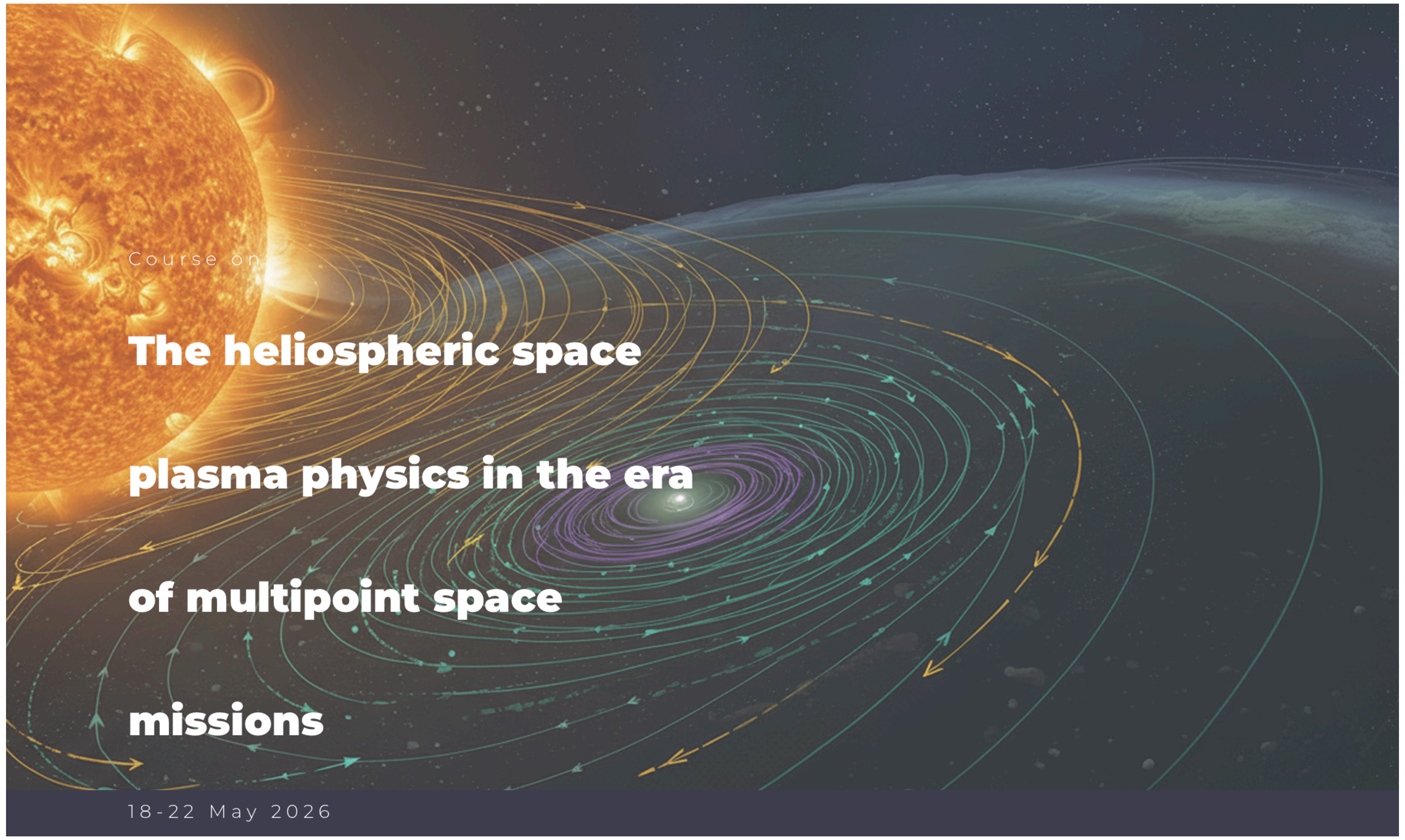
**Part 1**

Andrea Verdini
University of Firenze, Department of Physics & Astronomy

andrea.verdini@unifi.it

Arcetri Space & Astrophysical Plasma (ASAP) group

https://sites.google.com/unifi.it/asap



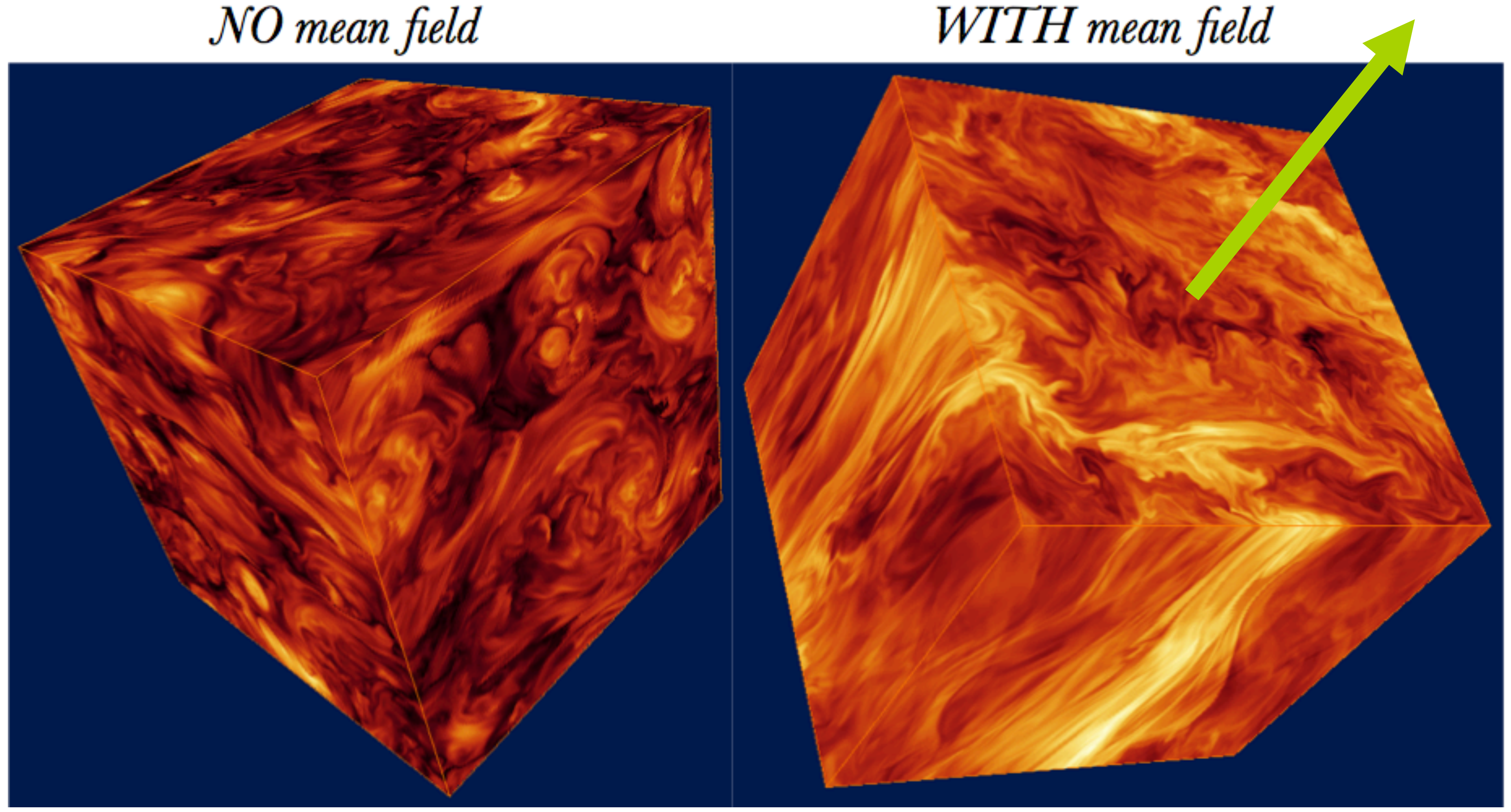
# Outline

- What anisotropy?  (spectral and scale dependent anisotropy)
- Why anisotropy ? (a semi-quantitative argument)
- How anisotropy is driven ? (anisotropy of the cascade)
- Measurements in the solar wind, some surprises:
- Special solar wind anisotropy (Maltese Cross) at different scales
- Scale dependent anisotropy

- Solar wind radial expansion: side effects
- some explanations for the observed solar wind anisotropies
- Why do we need multi-spacecraft ?
  - an example by revising spectral anisotropy
  - anisotropy of the cascade (the driver) revealed?

# What kind of anisotropy? Spectral Anisotropy

Muller & Biskamp 2005, PRE

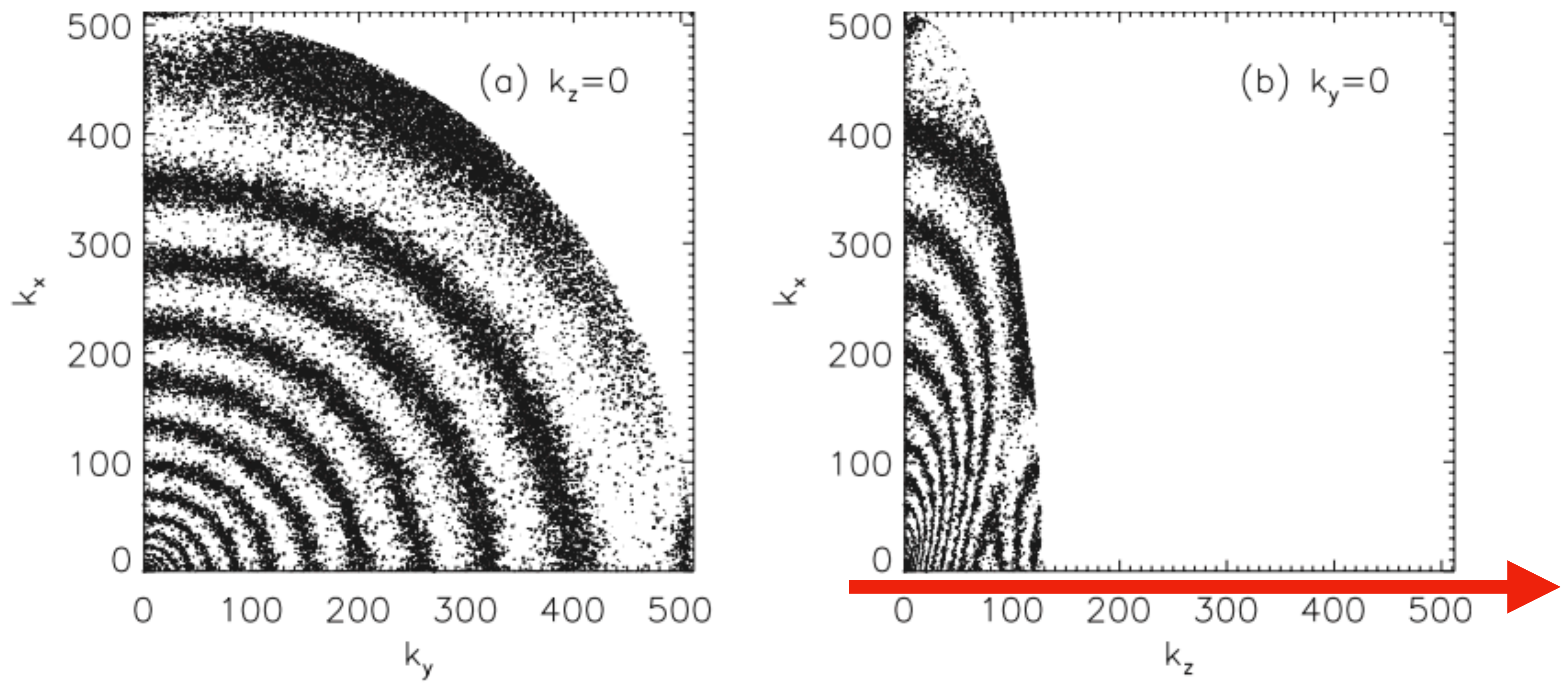



Uneven distribution of energy in the **wavevector** space

or

Uneven distribution of energy in the **increment** space

spectra




$$C(\boldsymbol{\ell}) = FT^{-1}[E(\mathbf{k})]$$

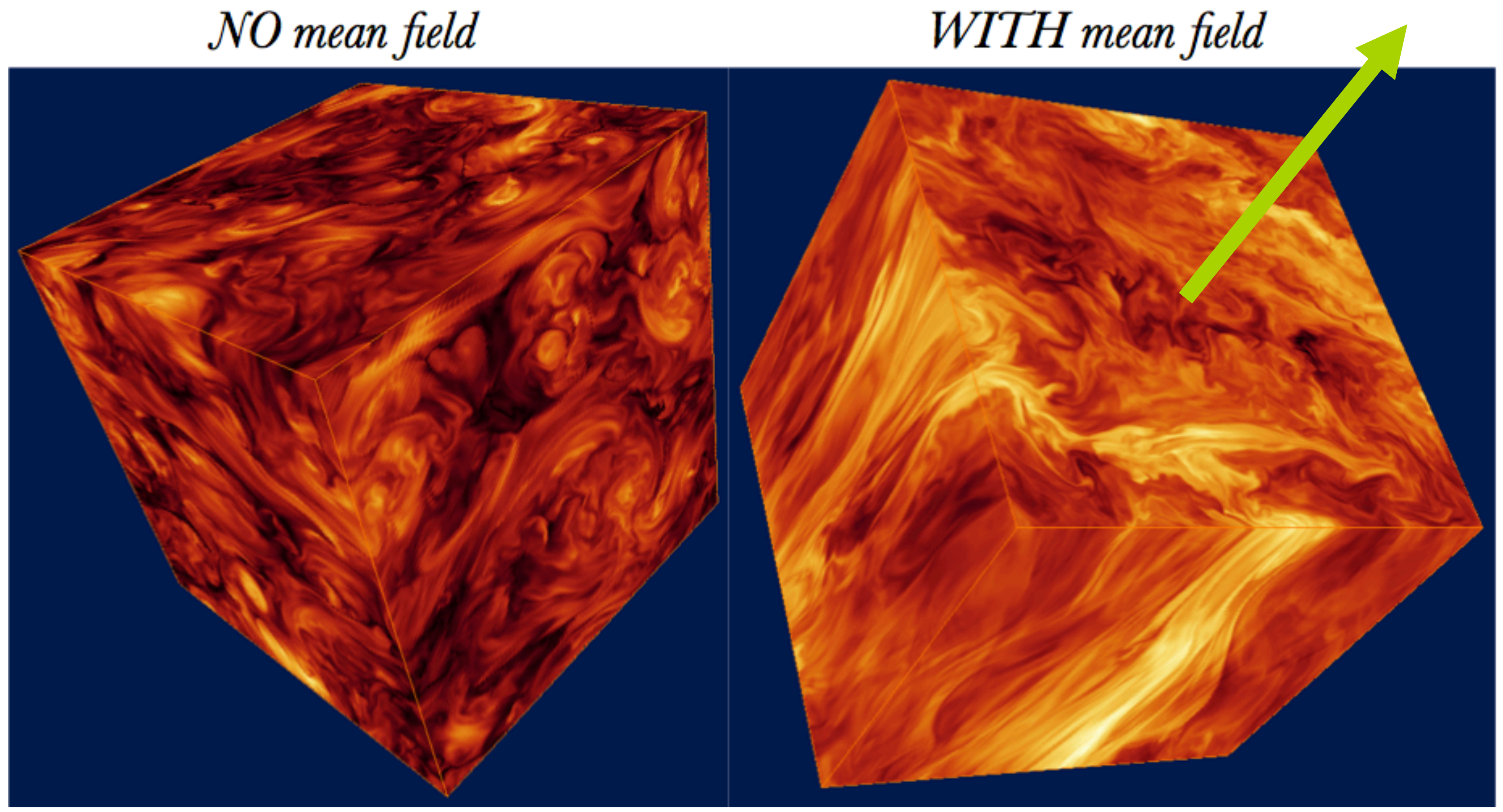
$$C(\boldsymbol{\ell}) = \langle u_i(\mathbf{x}+\boldsymbol{\ell})u_i(\mathbf{x})\rangle$$

$$SF(\boldsymbol{\ell}) = \langle |u_i(\mathbf{x}+\boldsymbol{\ell}) - u_i(\mathbf{x})|^2 \rangle$$

$$SF = 2[C(0) - C(\boldsymbol{\ell})]$$

# What kind of anisotropy? Spectral Anisotropy

Muller & Biskamp 2005, PRE



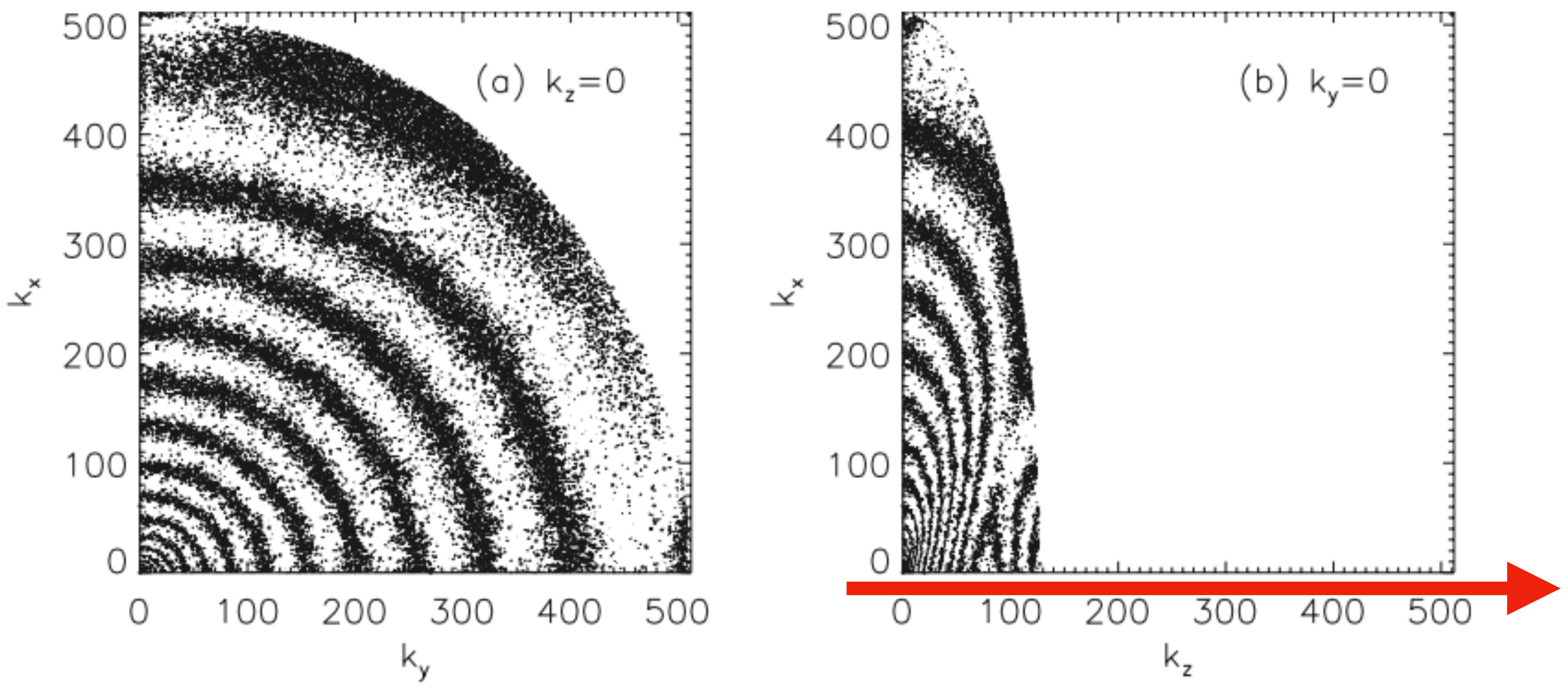

Uneven distribution of energy in the **wavevector** space

or

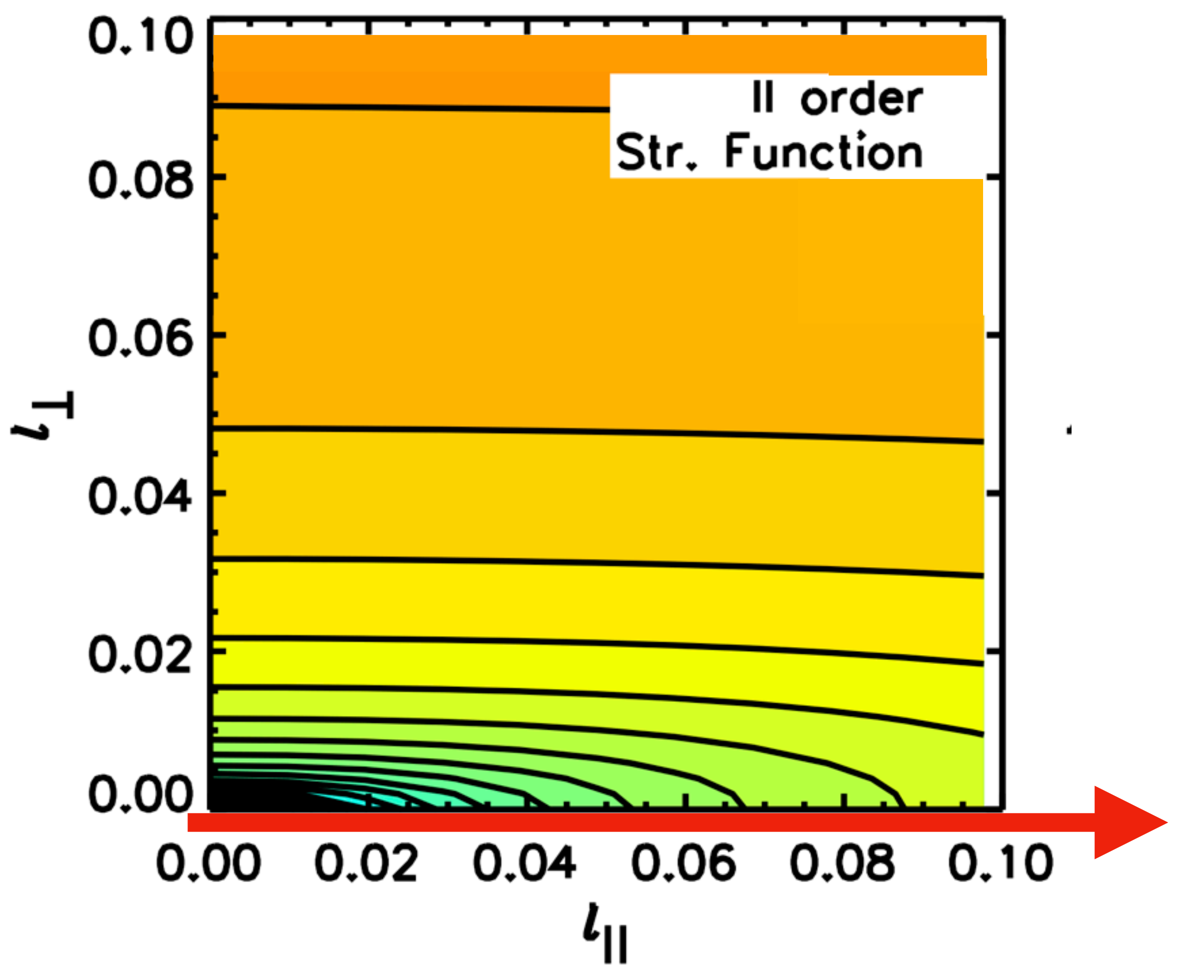
Uneven distribution of energy in the **increment** space

flipped

spectra




SF

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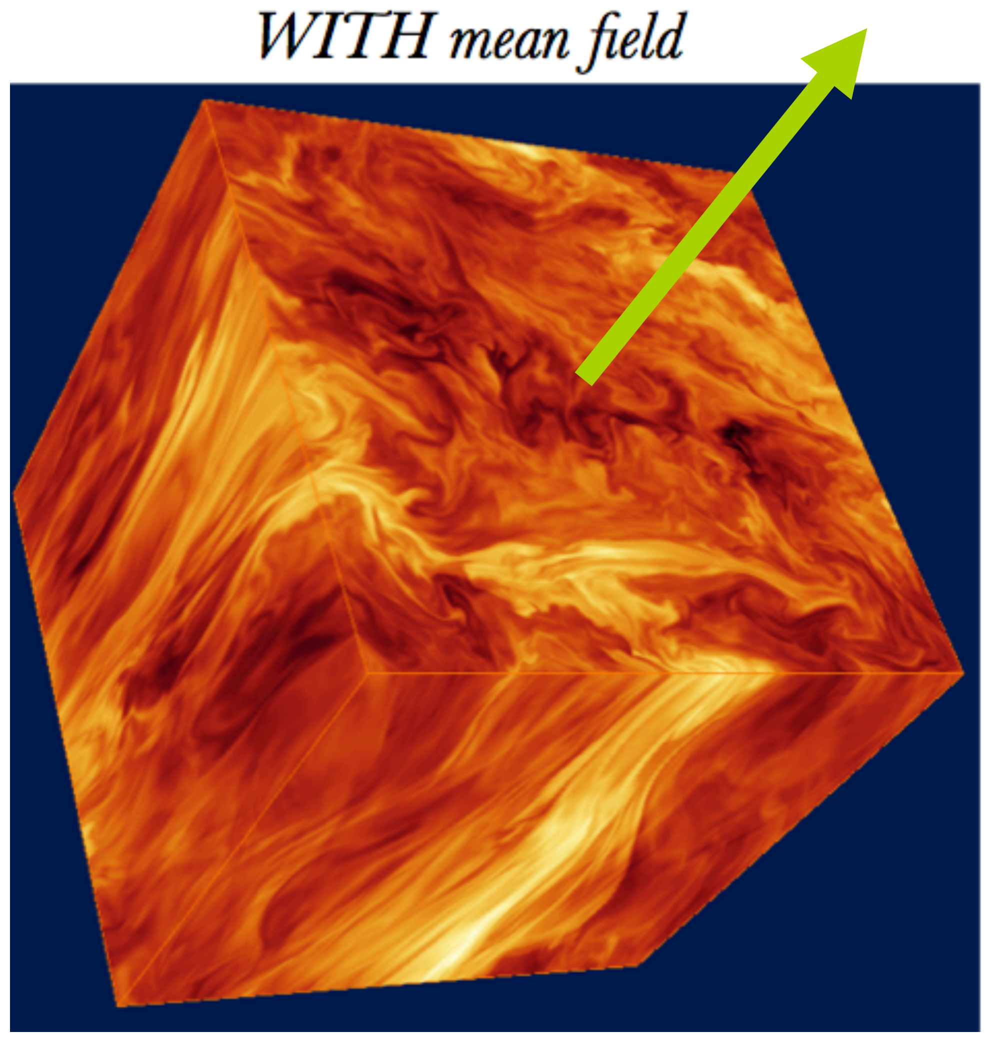
$$SF=2[C(0)-C(\boldsymbol{\ell})]$$

# Why Spectral Anisotropy?




Muller & Biskamp 2005, PRE

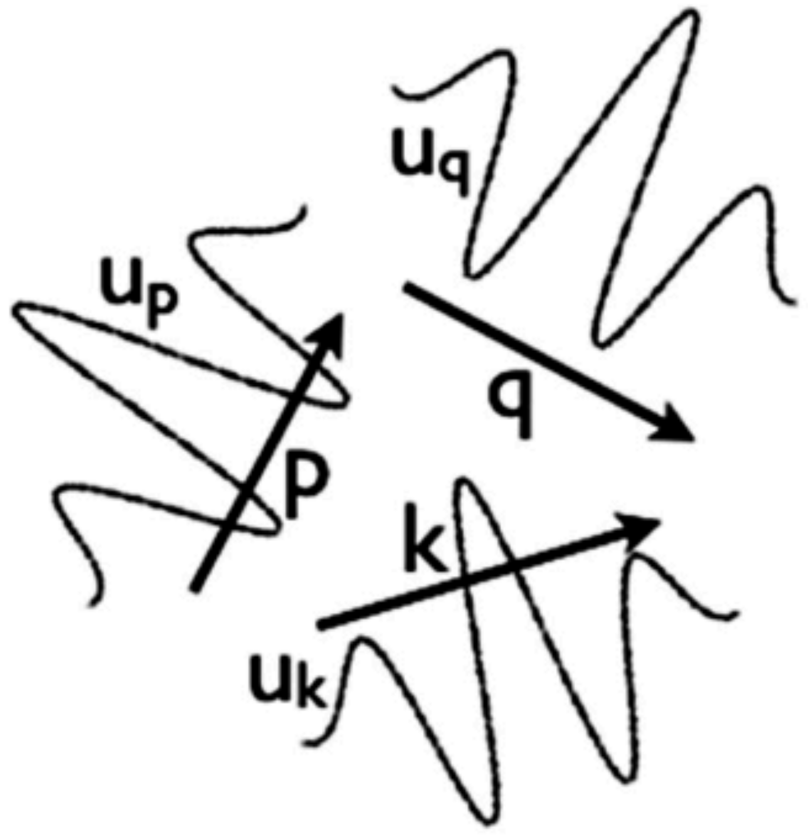
$$\partial_t \boldsymbol{z}^{\pm} \mp \boldsymbol{B}_0 \cdot \nabla \boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp} \cdot \nabla \boldsymbol{z}^{\pm} + \nabla P = 0 \qquad \mathbf{z}^{\pm} = \delta\mathbf{u} \mp \delta\mathbf{b}/\sqrt{4\pi\rho}$$

$$\partial_t \widehat{z_i^{\pm}}(k) \mp i k_{\|} B_0 \widehat{z_i^{\pm}}(k) + i P_{ijl}(k) \widehat{z_l^{\pm} z_j^{\mp}}(k) = 0 \qquad P_{ijl}(k) = k_j \left(\delta_{il} - k_l k_i / k^2\right)$$

$$\widehat{z^{\pm}}(k) = Z^{\pm}(k) e^{\pm i k_{\|} B_0 t}:$$

$$\partial_t Z_i^{\pm}(k) + i P_{ijl}(k) \int_{k=p+q} d^3p\, d^3q\, Z_l^{\pm}(p) Z_j^{\mp}(q) e^{\mp i\omega t} = 0$$

$$\omega = 2 q_{\|} B_0 \quad \sim 1/t_A$$




hydro

# Why Spectral Anisotropy?

$$\partial_t z^{\pm} \mp B_0 \cdot \nabla z^{\pm} + z^{\mp} \cdot \nabla z^{\pm} + \nabla P = 0 \qquad \mathbf{z}^{\pm} = \delta\mathbf{u} \mp \delta\mathbf{b}/\sqrt{4\pi\rho}$$

$$\partial_t \widehat{z_i^{\pm}}(k) \mp i k_{\parallel} B_0 \widehat{z_i^{\pm}}(k) + i P_{ijl}(k) \widehat{z_l^{\pm} z_j^{\mp}}(k) = 0 \qquad P_{ijl}(k) = k_j \left(\delta_{il} - k_l k_i / k^2\right)$$

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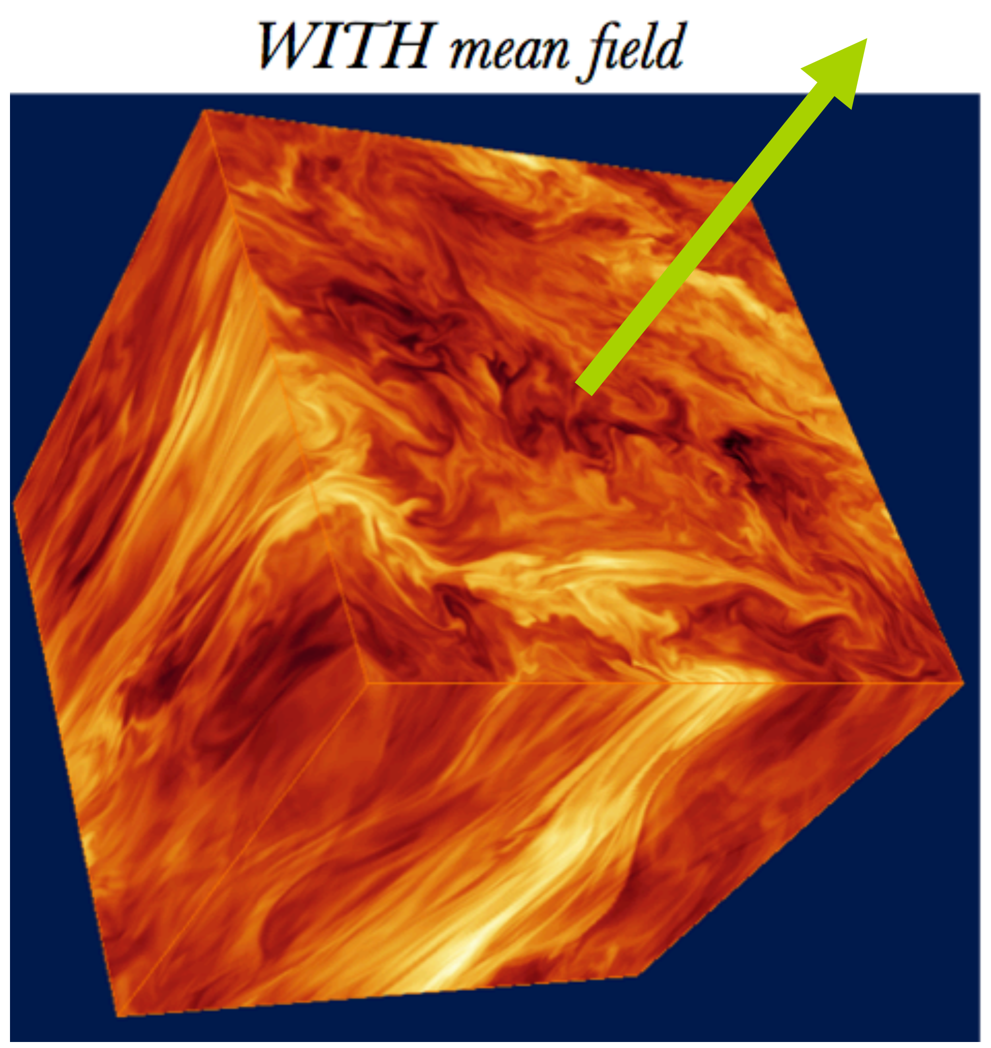
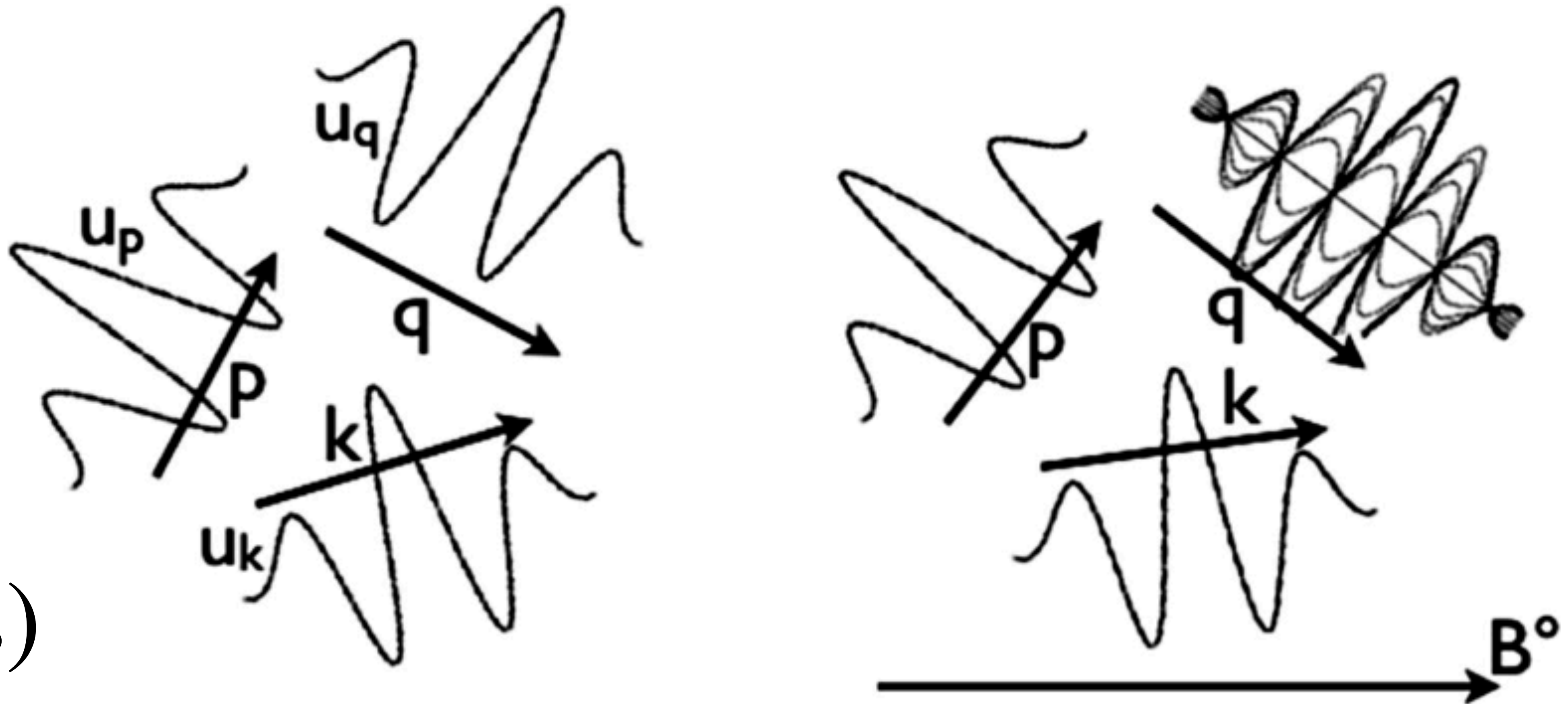



Muller & Biskamp 2005, PRE

If q || to B0 => no contribution to NL cascade

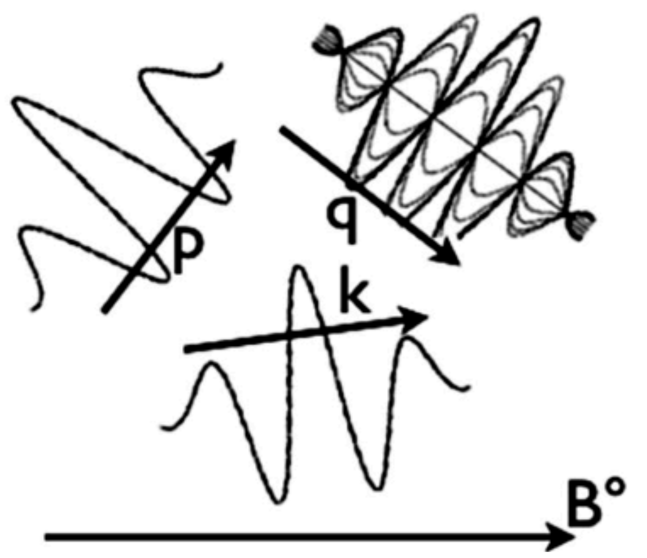
If q ⊥ to B0 => no oscillatory part (Hydro-like interactions)

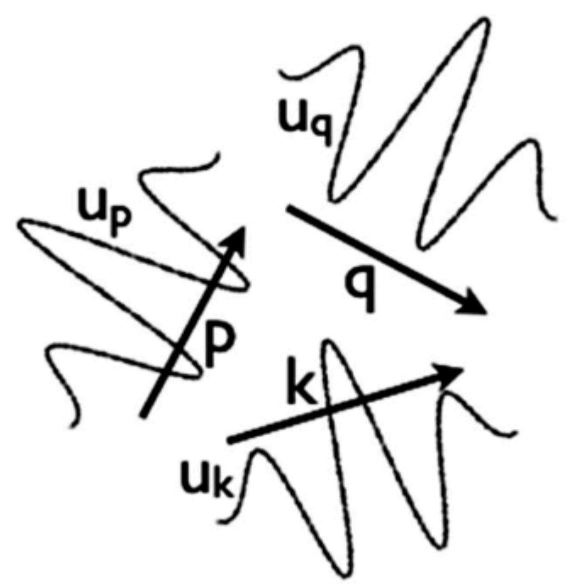


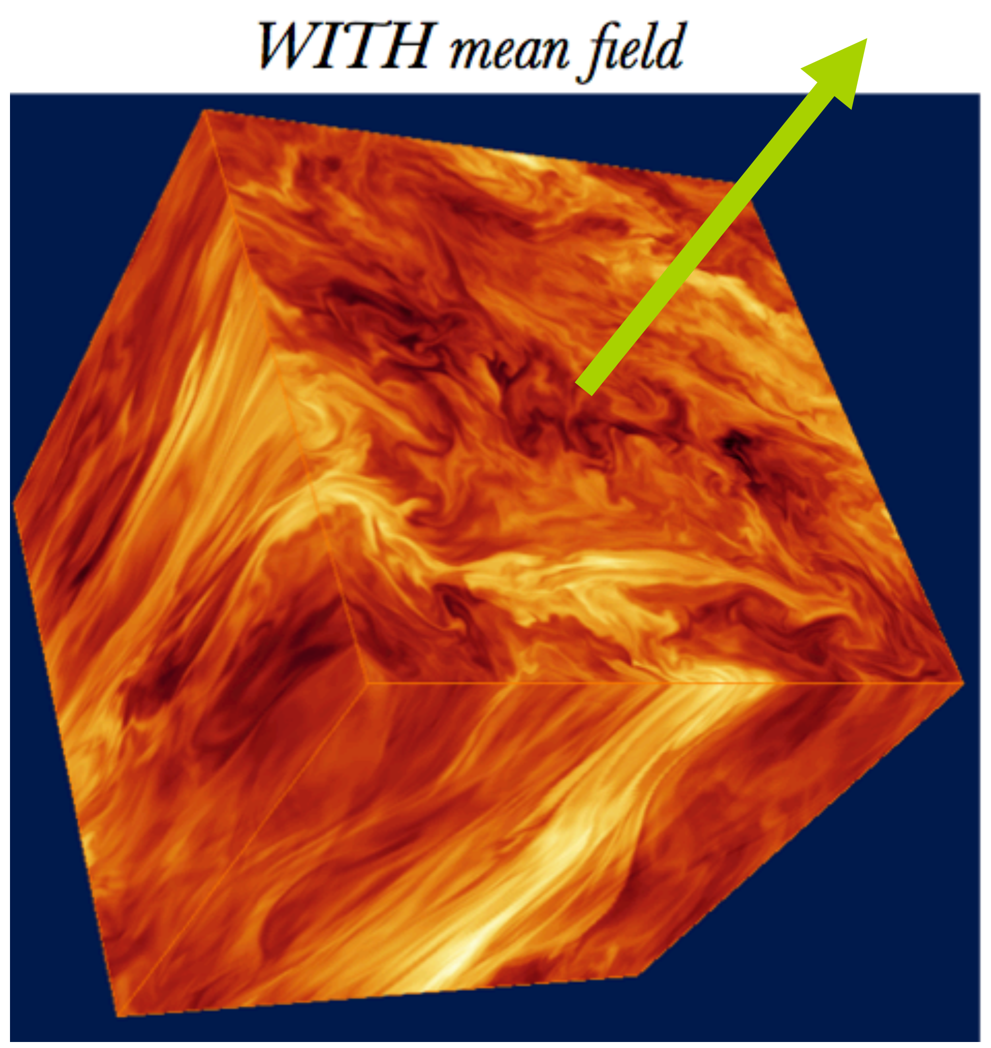

hydro

MHD with B°

# Why Spectral Anisotropy?



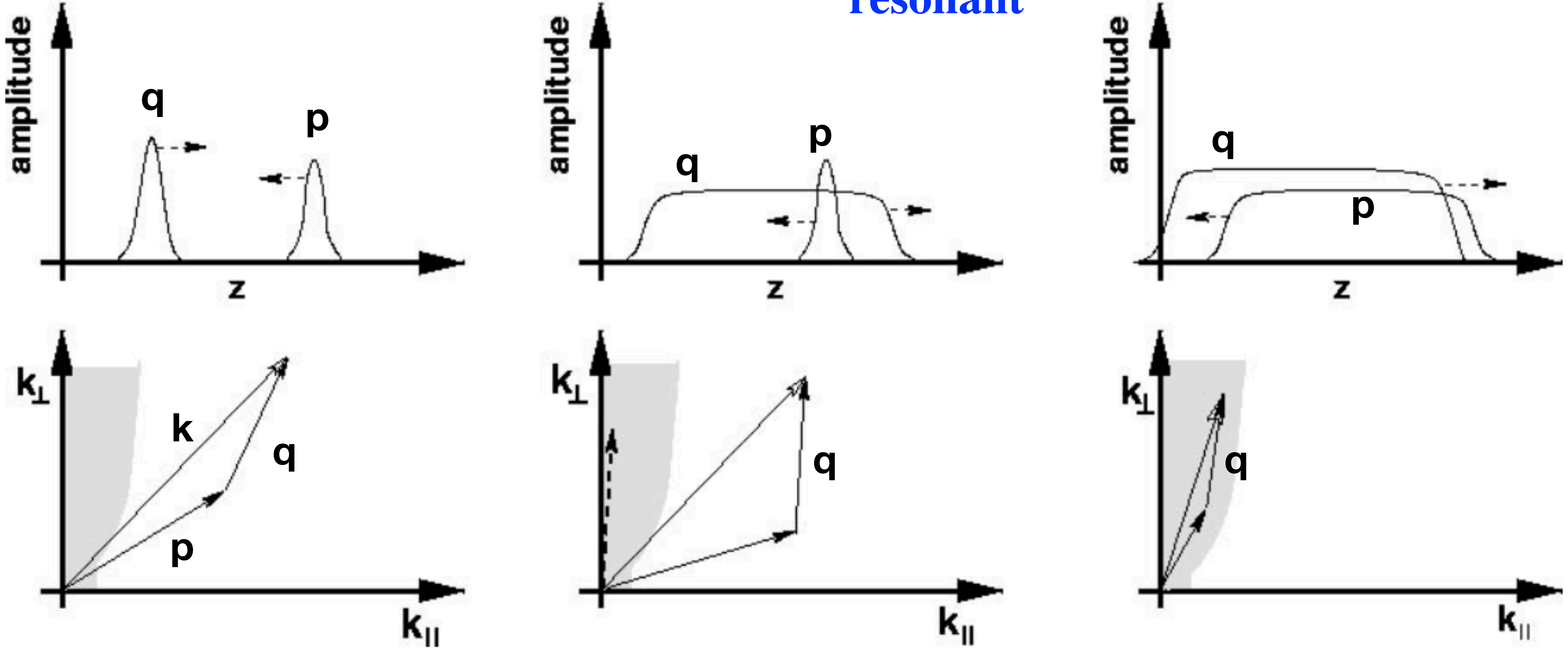




**Wave+Wave: negligible**

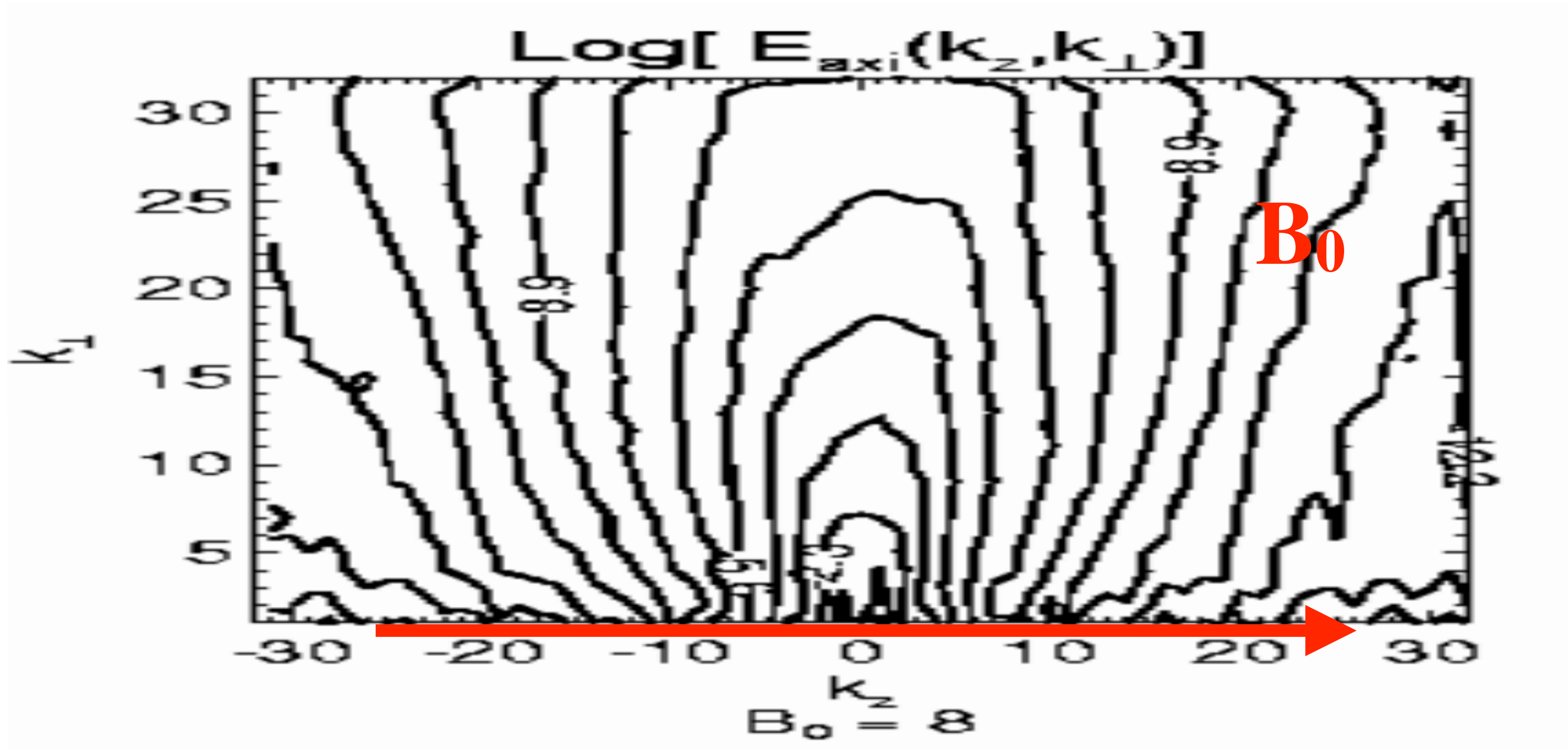
**Hydro+Wave : quasi resonant**

**Hydro+Hydro: resonant**

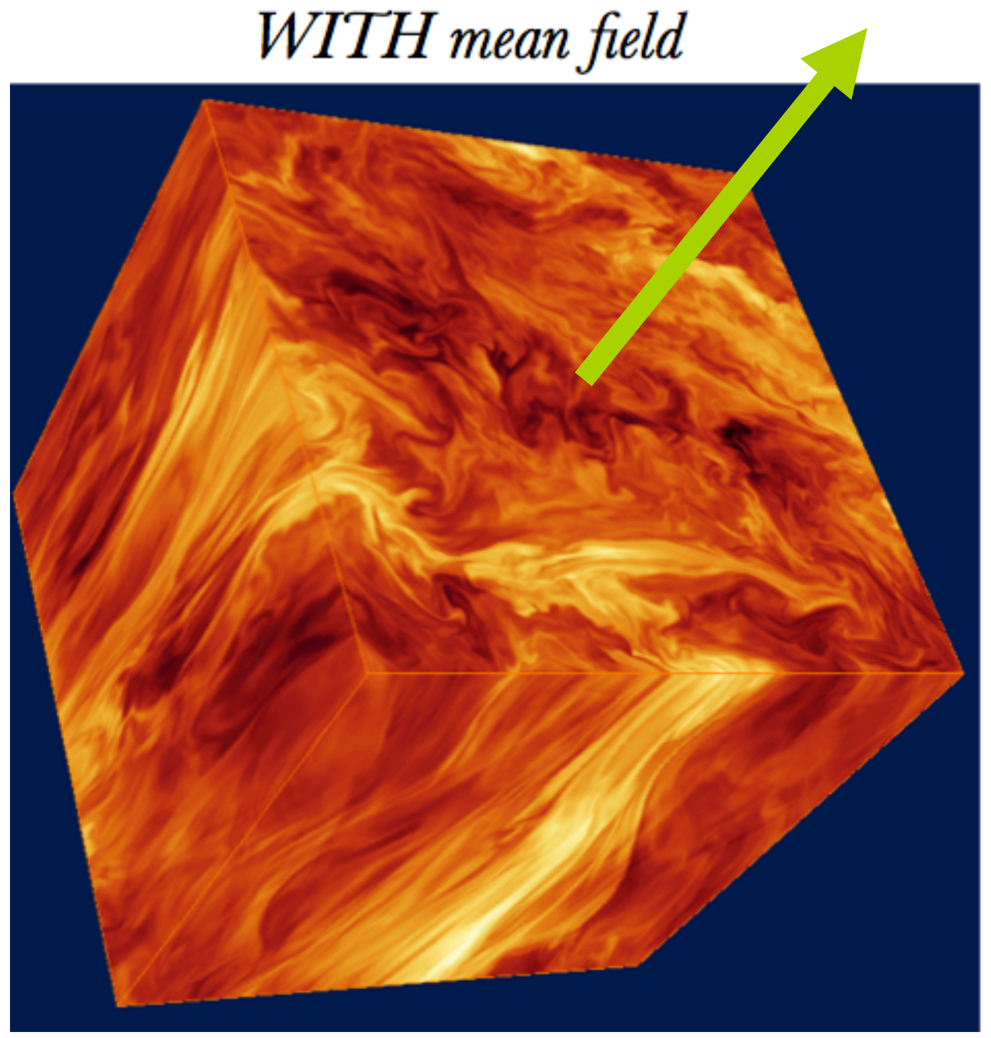



Oughton+ 2004 Phys. PoP

**spectrum develops preferentially along $k_\perp$**

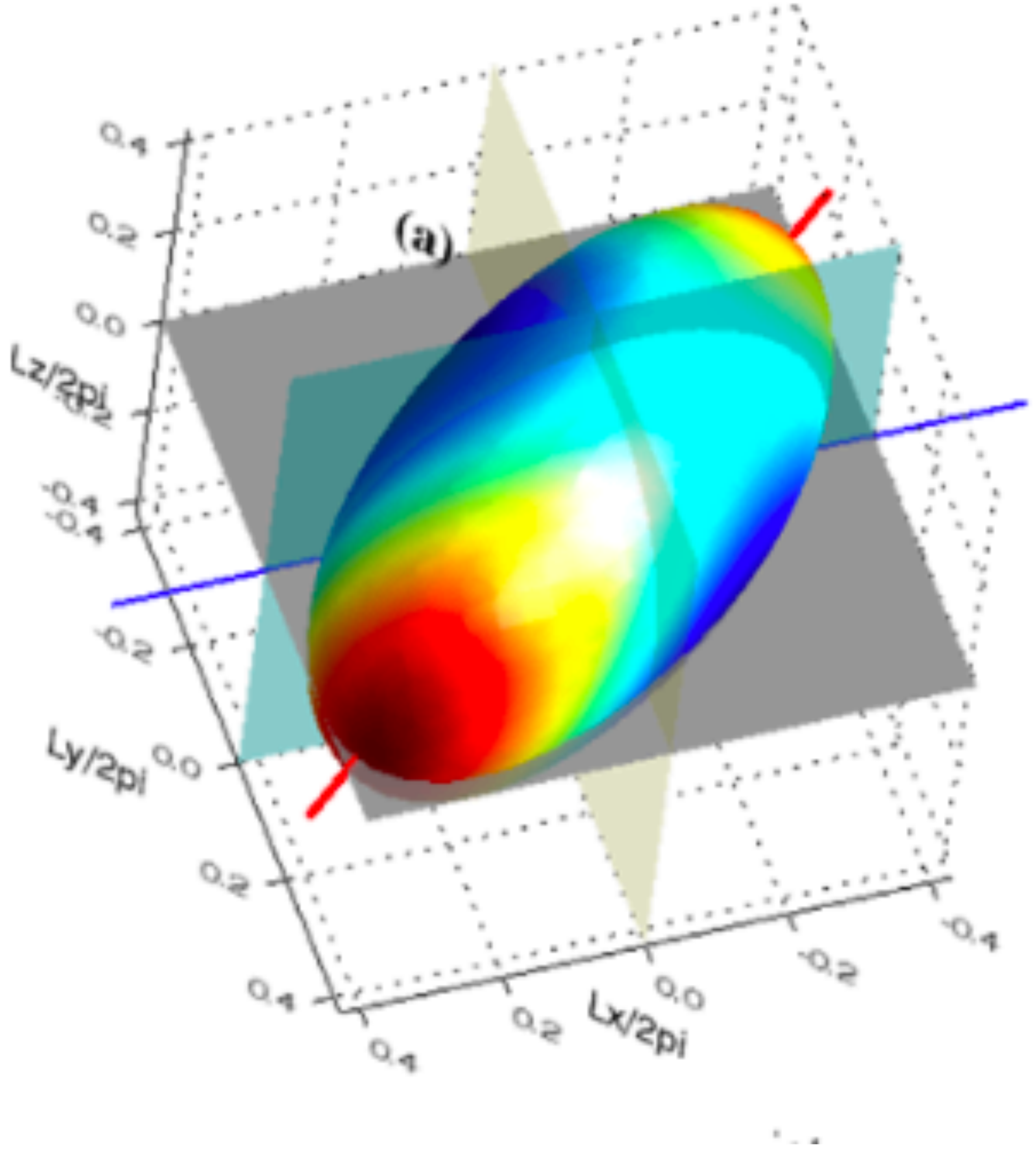


# Spectral Anisotropy

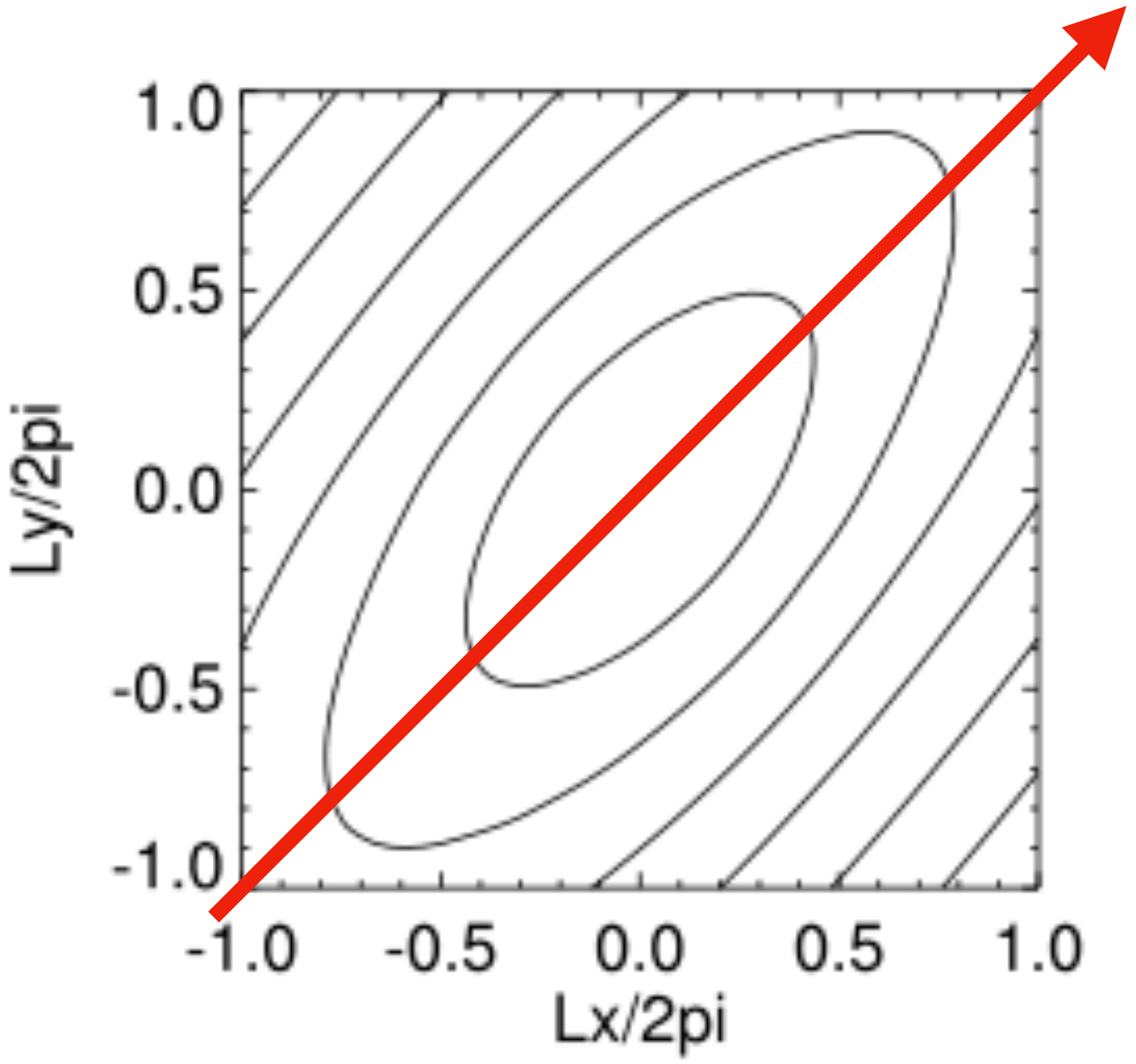



**Spectral Anisotropy** $C(\boldsymbol{\ell})=FT^{-1}[E(\mathbf{k})]$ **Driver?**

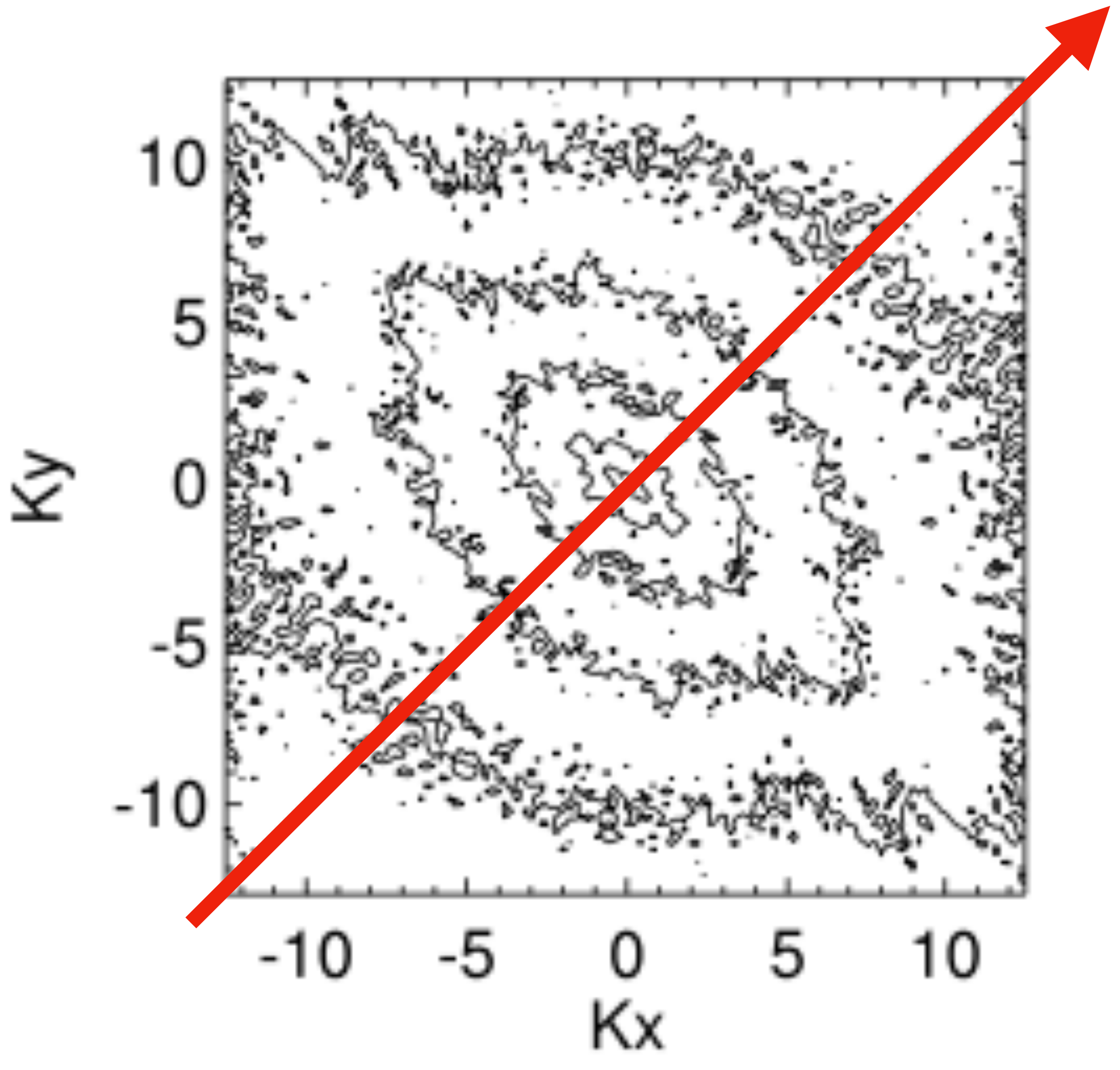
$C(\boldsymbol{\ell})$ 3D




$C(\ell_x,\ell_y)$




$E(k_x,k_y)$

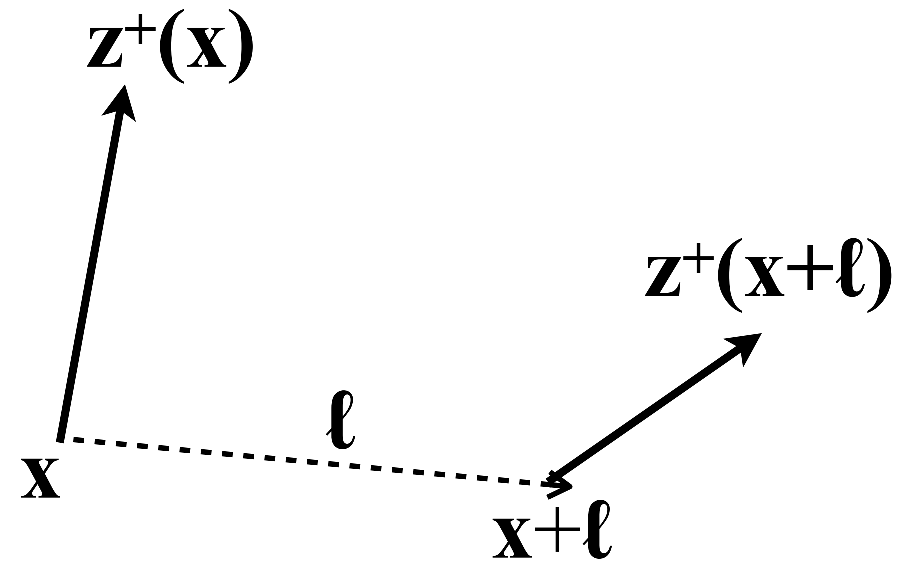



Montagud-Camps 2021 ApJ

# Isotropic cascade

$$\partial_t \boldsymbol{z}^{\pm} \mp \boldsymbol{B}_0 \cdot \nabla \boldsymbol{z}^{\pm} + \boldsymbol{z}^{\mp} \cdot \nabla \boldsymbol{z}^{\pm} + \nabla P = 0$$

$$\mathbf{z}^{\pm} = \delta \mathbf{u} \mp \delta \mathbf{b} / \sqrt{4\pi\rho}$$




$$\Delta \mathbf{z}^{\pm}(\mathbf{x}, \boldsymbol{\ell}) = \mathbf{z}^{\pm}(\mathbf{x} + \boldsymbol{\ell}) - \mathbf{z}^{\pm}(\mathbf{x})$$

- evaluate the eq. at **x** and **x**+**ℓ**,
- multiply by $\Delta\mathbf{z}^{\pm}$ ,
- average in the volume and obtain equations for $S^{\pm}$,
- sum them so you get total energy $S=S^{+}+S^{-}$

$$\partial_t S + \boldsymbol{\nabla}_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \boldsymbol{\nabla}_\ell^2 S$$

Conservation equation for II-order Structure function S (mean field is not here!)

$$S = 1/2 \left[ \langle |\Delta \mathbf{z}^{-}|^2 \rangle + \langle |\Delta \mathbf{z}^{+}|^2 \rangle \right]$$ **II-order Structure Function**

$$\epsilon = \nu/2 \left[ \langle \Sigma_i (\partial_{\mathbf{x}} z_i^{+})^2 \rangle + \langle \Sigma_i (\partial_{\mathbf{x}} z_i^{-})^2 \rangle \right]$$ **Dissipation rate**

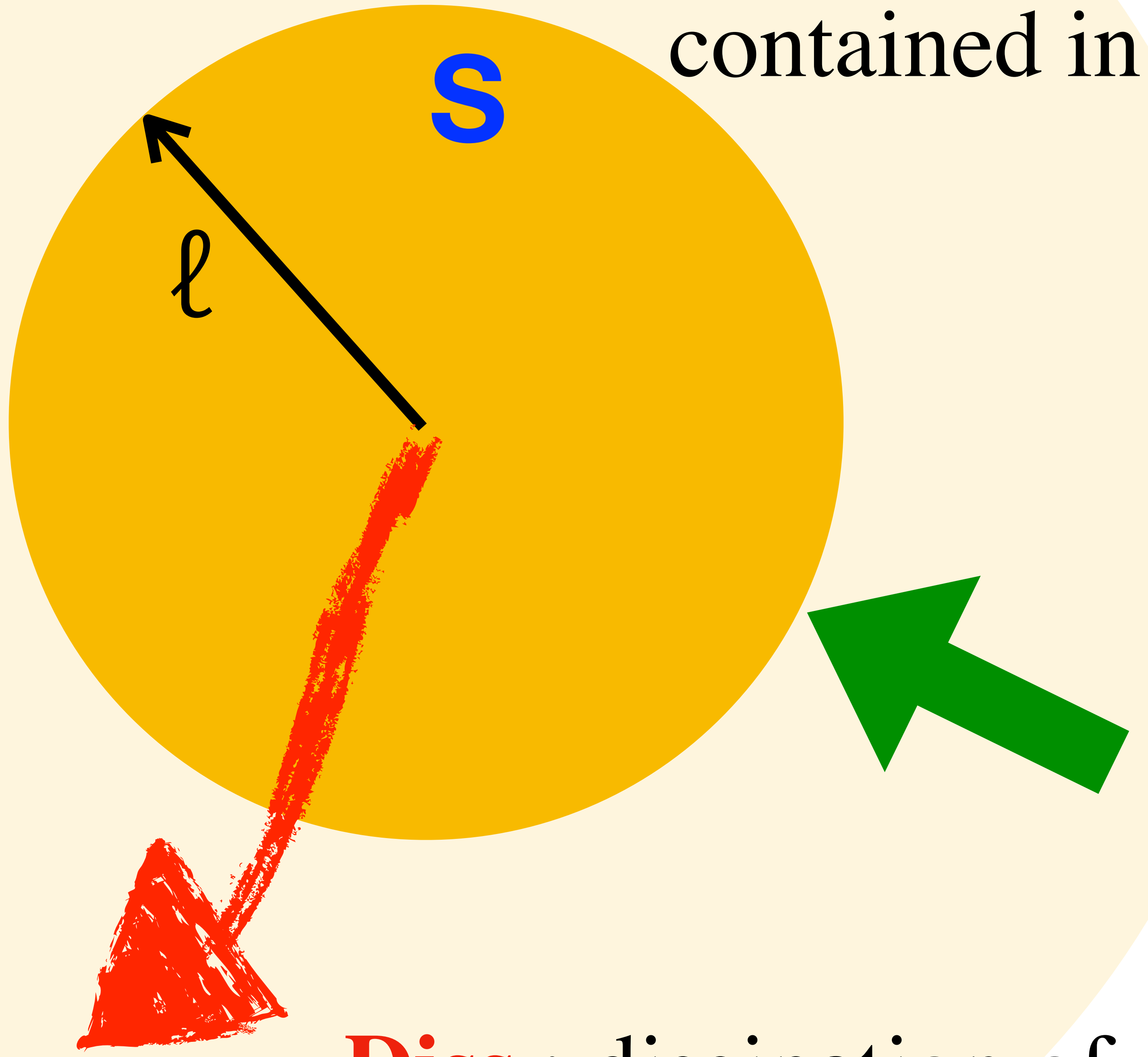
$$\mathbf{Y} = 1/2 \left[ \left\langle \Delta \mathbf{z}^{-} |\Delta \mathbf{z}^{+}|^2 \right\rangle + \left\langle \Delta \mathbf{z}^{+} |\Delta \mathbf{z}^{-}|^2 \right\rangle \right]$$ **III-order Structure Function**

# Isotropic cascade

Isotropic => can study its angular average (or a cut)

$$\partial_t S + \boldsymbol{\nabla}_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \boldsymbol{\nabla}_\ell^2 S$$

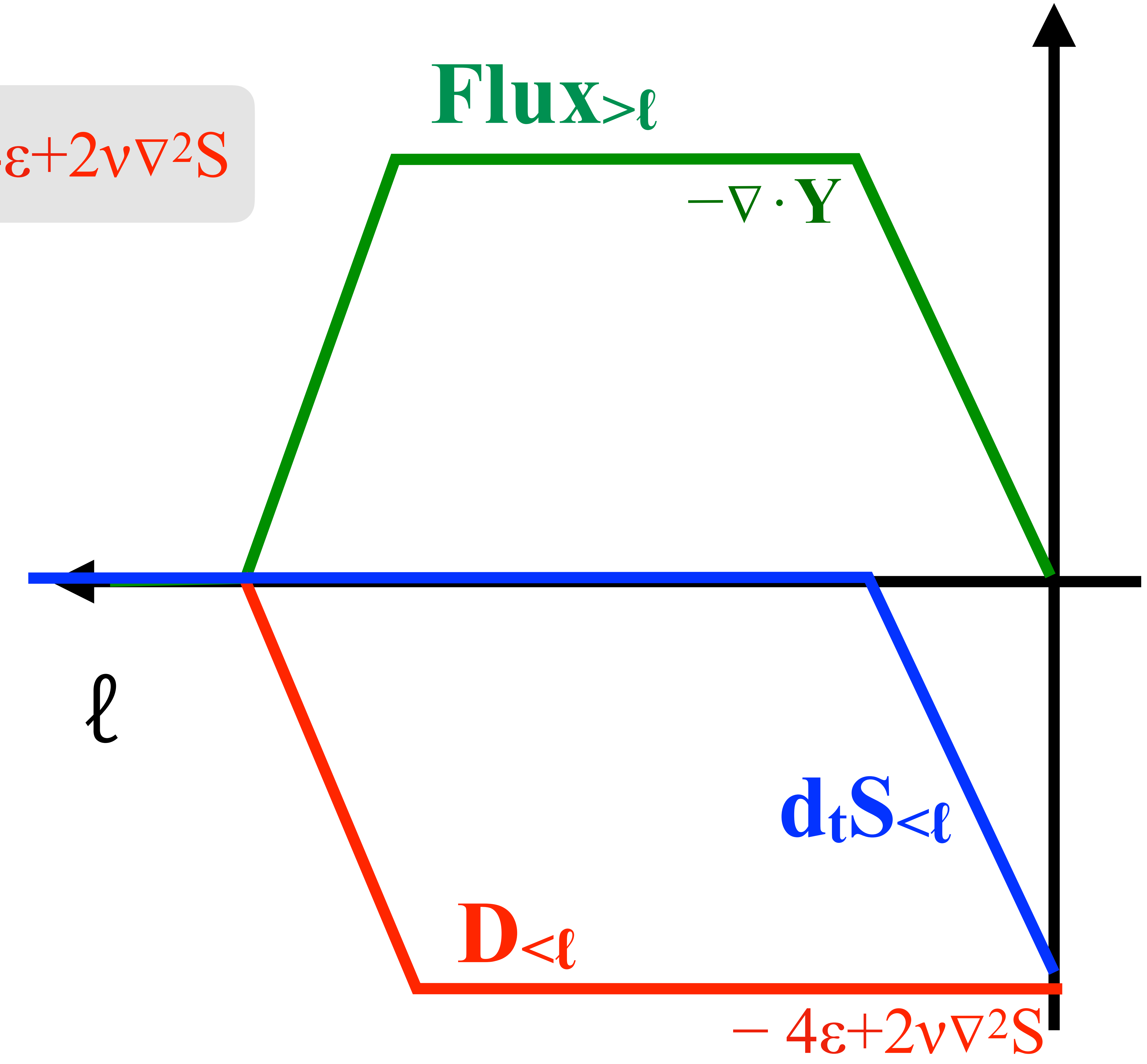
$$dS/dt \;\; = -\nabla \cdot \mathbf{Y} - 4\varepsilon + 2\nu\nabla^2 S$$




**dS/dt** variation of energy contained in scales up to ℓ

**Flux** : energy flux from scales larger than ℓ

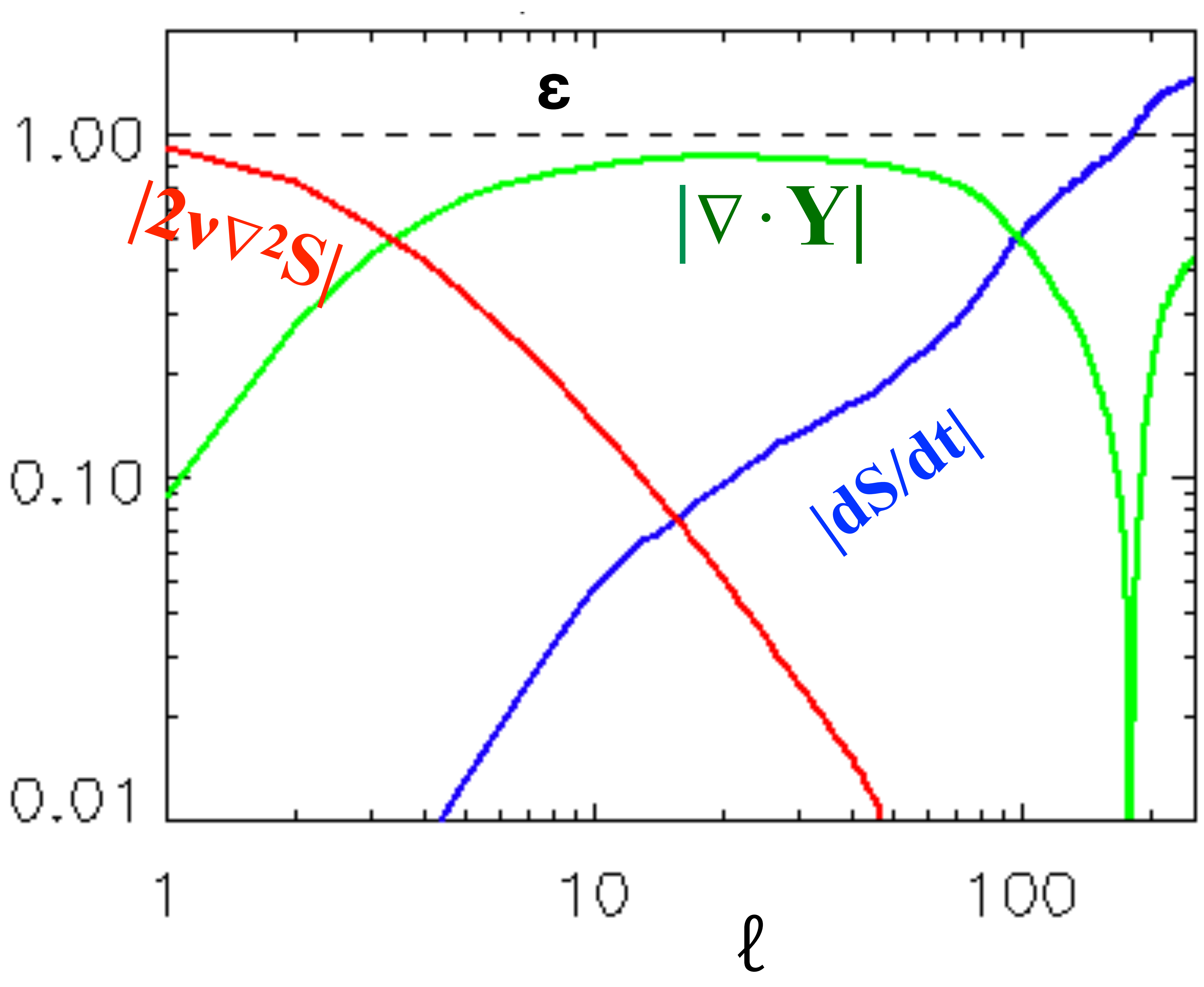
**Diss** : dissipation of energy contained in scales up to ℓ



# Isotropic cascade

## More popular representation

$dS/dt + \nabla \cdot \mathbf{Y} - 2\nu\nabla^2 S = -4\varepsilon$



# Isotropic cascade

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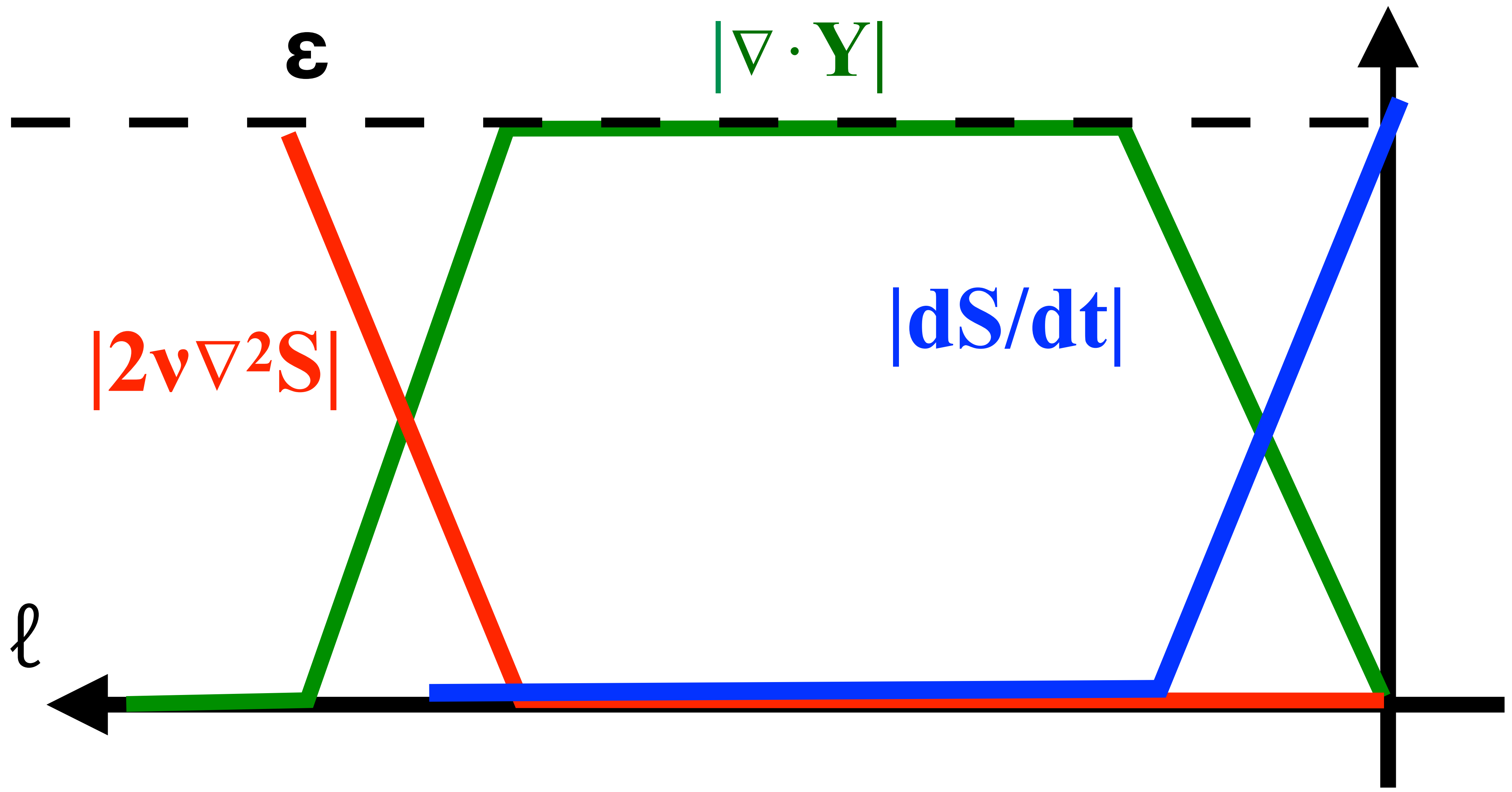
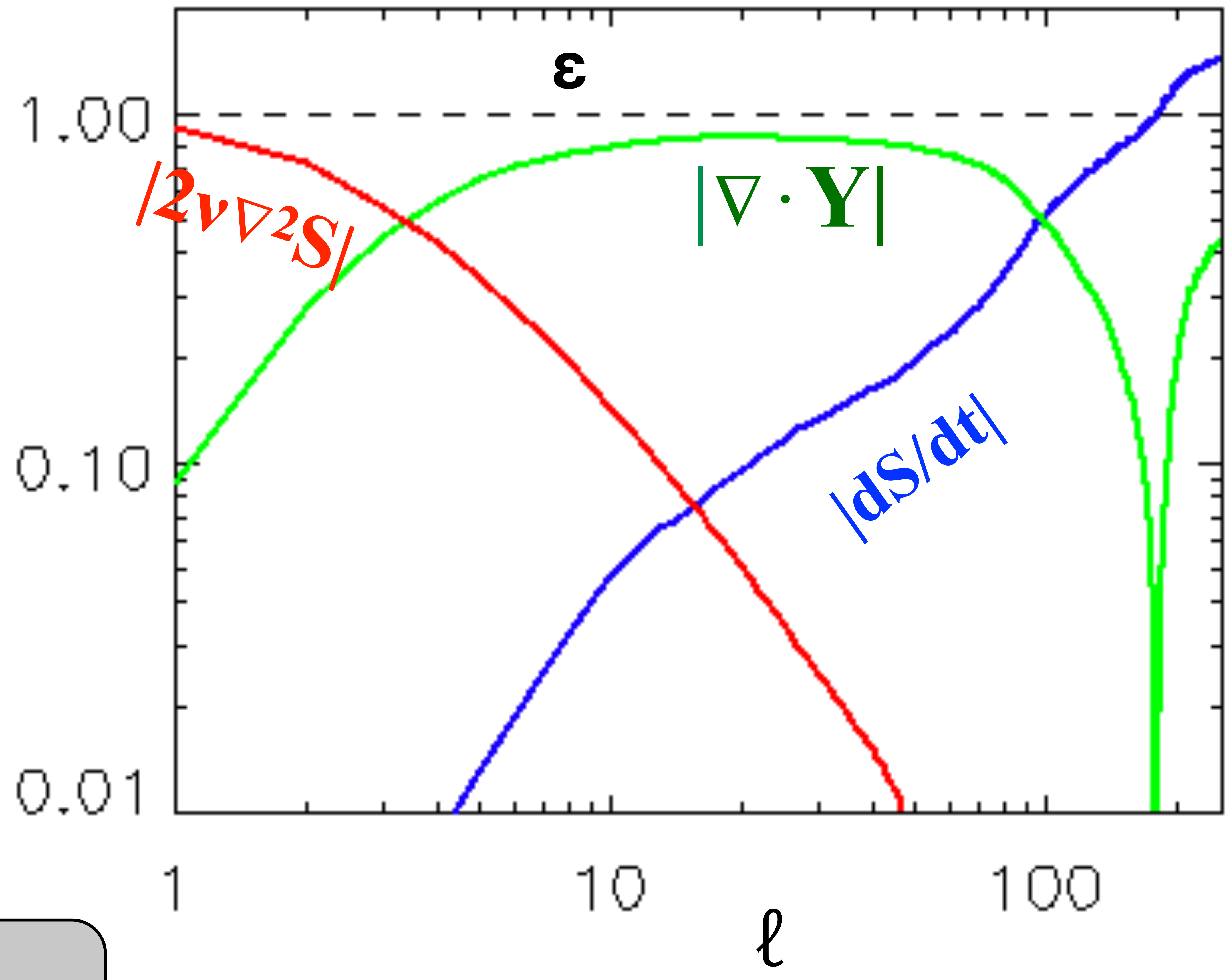



**Ideally : when injection and dissipation are well separated**

Politano Pouquet 1996 PRE

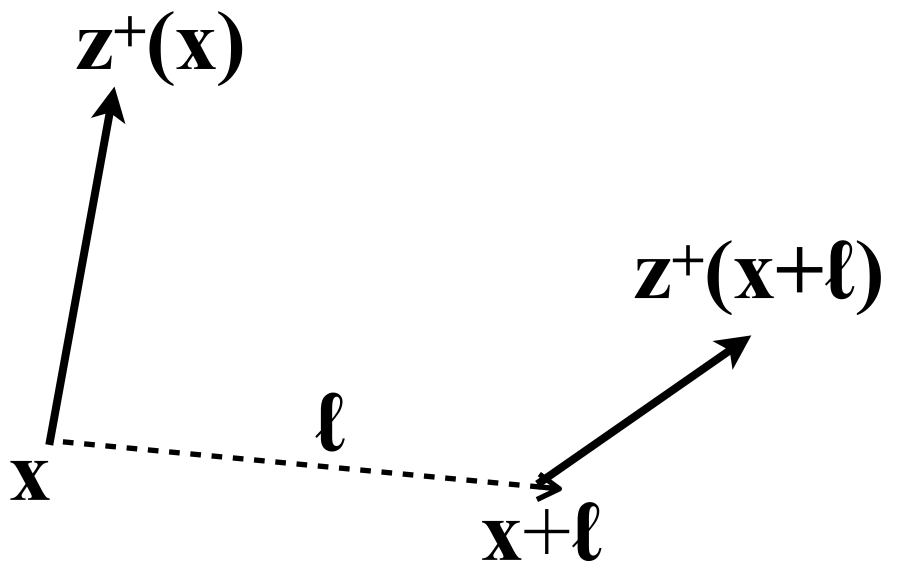
$$\nabla_\ell \cdot \mathbf{Y} = -4\epsilon$$




**Cascade <=> $\nabla \cdot \mathbf{Y}$**

# Isotropic cascade

$$\partial_t S + \boldsymbol{\nabla}_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \boldsymbol{\nabla}_\ell^2 S$$




$$\Delta \mathbf{z}^{\pm}(\mathbf{x}, \boldsymbol{\ell}) = \mathbf{z}^{\pm}(\mathbf{x} + \boldsymbol{\ell}) - \mathbf{z}^{\pm}(\mathbf{x})$$

**II-order Structure Function**

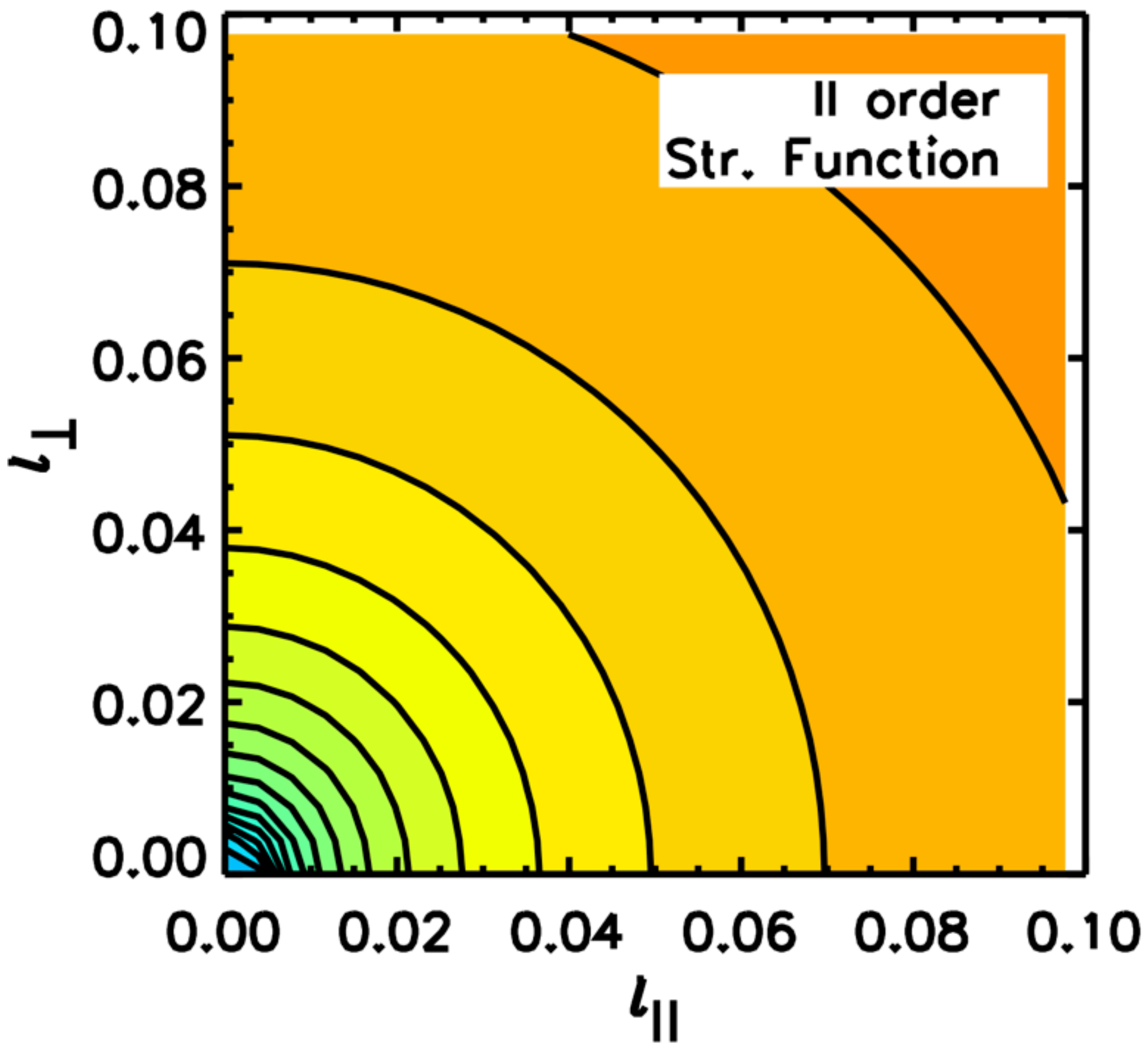
$$S = 1/2 \left[ \langle |\Delta \mathbf{z}^-|^2 \rangle + \langle |\Delta \mathbf{z}^+|^2 \rangle \right]$$

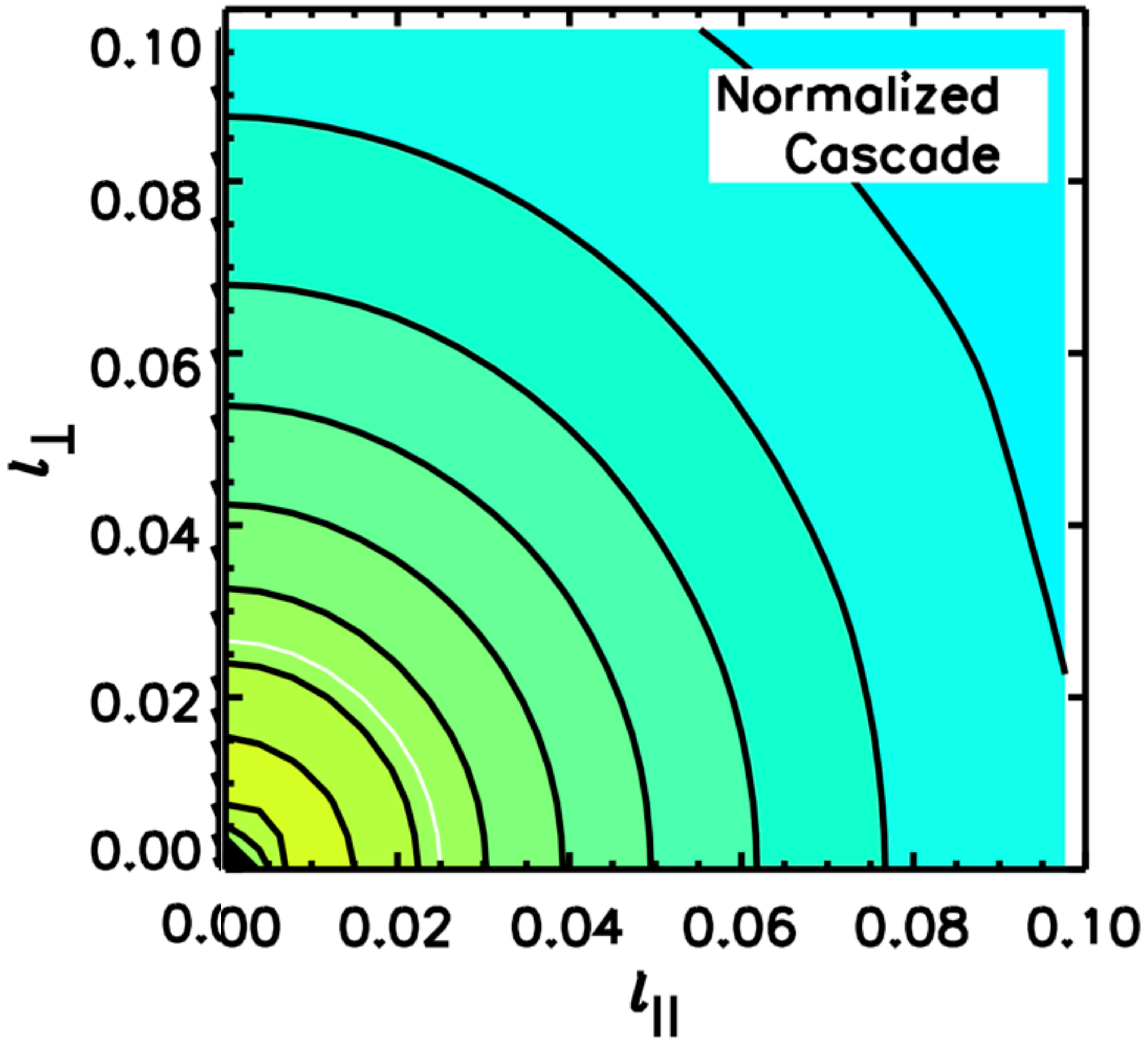
**Divergence of the flux**

$$-\nabla \cdot \mathrm{Y} \,/\, 4\varepsilon$$

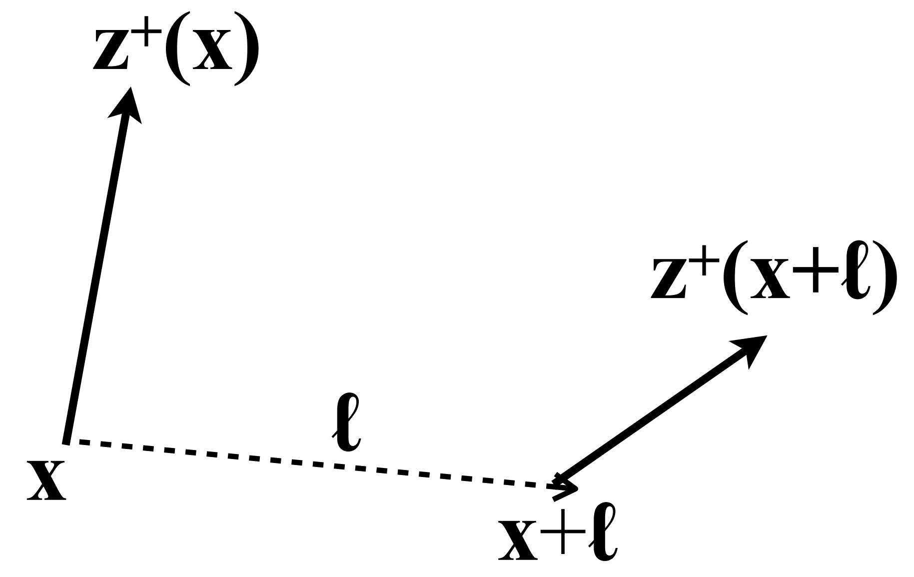
**III-order Structure Function**

$$\mathbf{Y} = 1/2 \left[ \langle \Delta \mathbf{z}^- |\Delta \mathbf{z}^+|^2 \rangle + \langle \Delta \mathbf{z}^+ |\Delta \mathbf{z}^-|^2 \rangle \right]$$



# Isotropic cascade




$$\partial_t S + \nabla_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \nabla^2_\ell S$$

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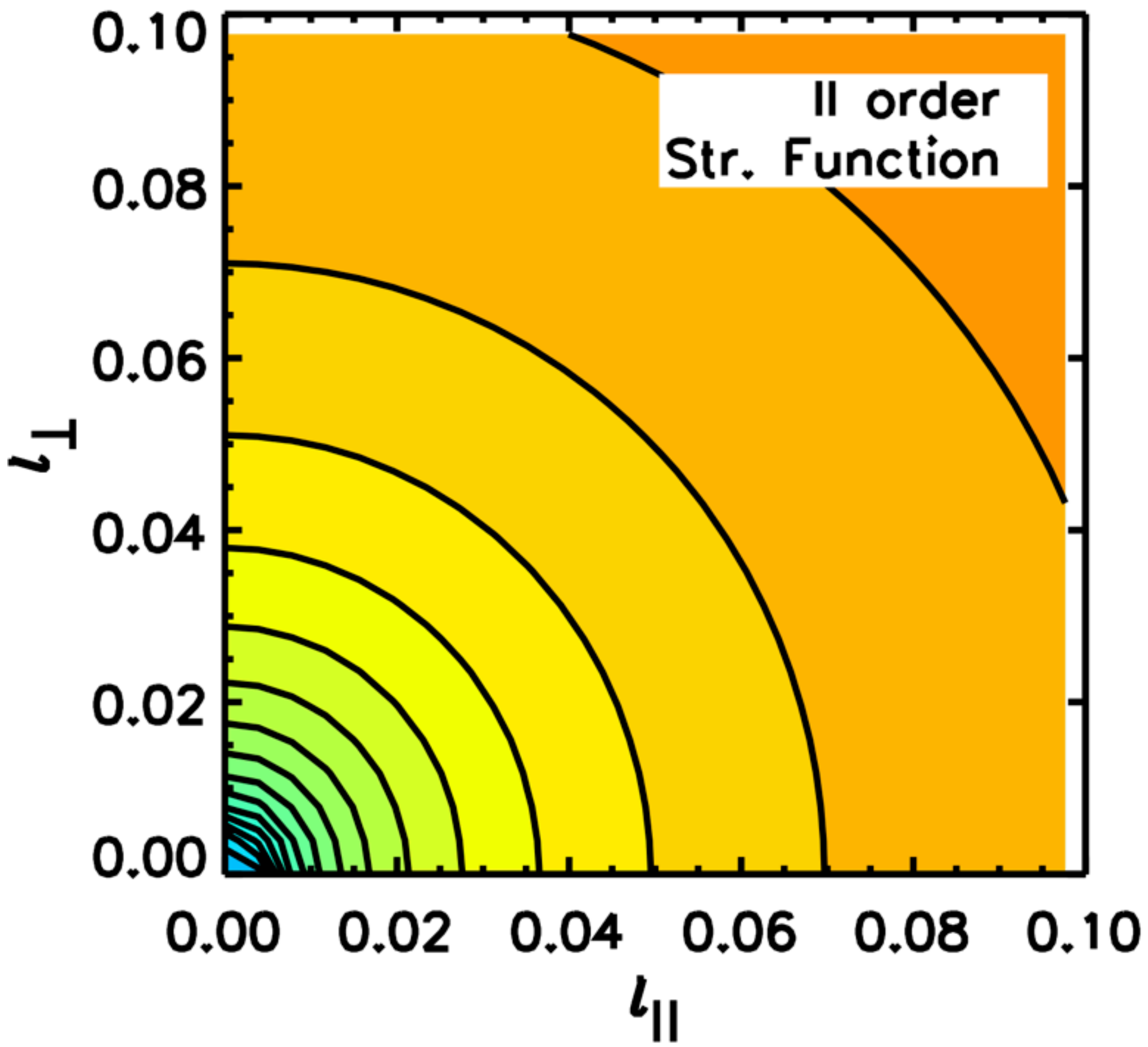
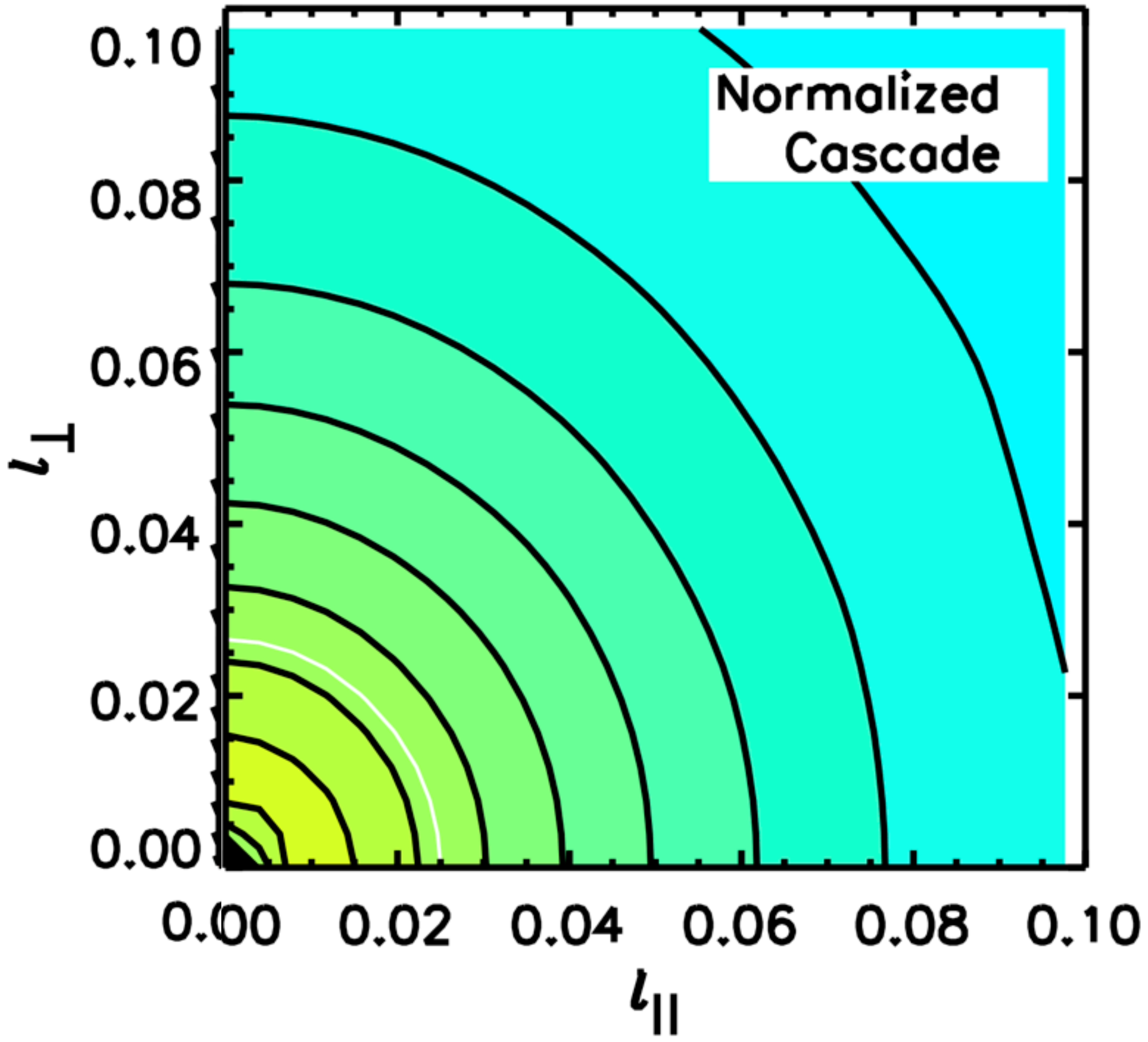



**Divergence of the flux**

$$-\nabla \cdot \mathrm{Y} / 4\varepsilon$$




**III-order Structure Function**

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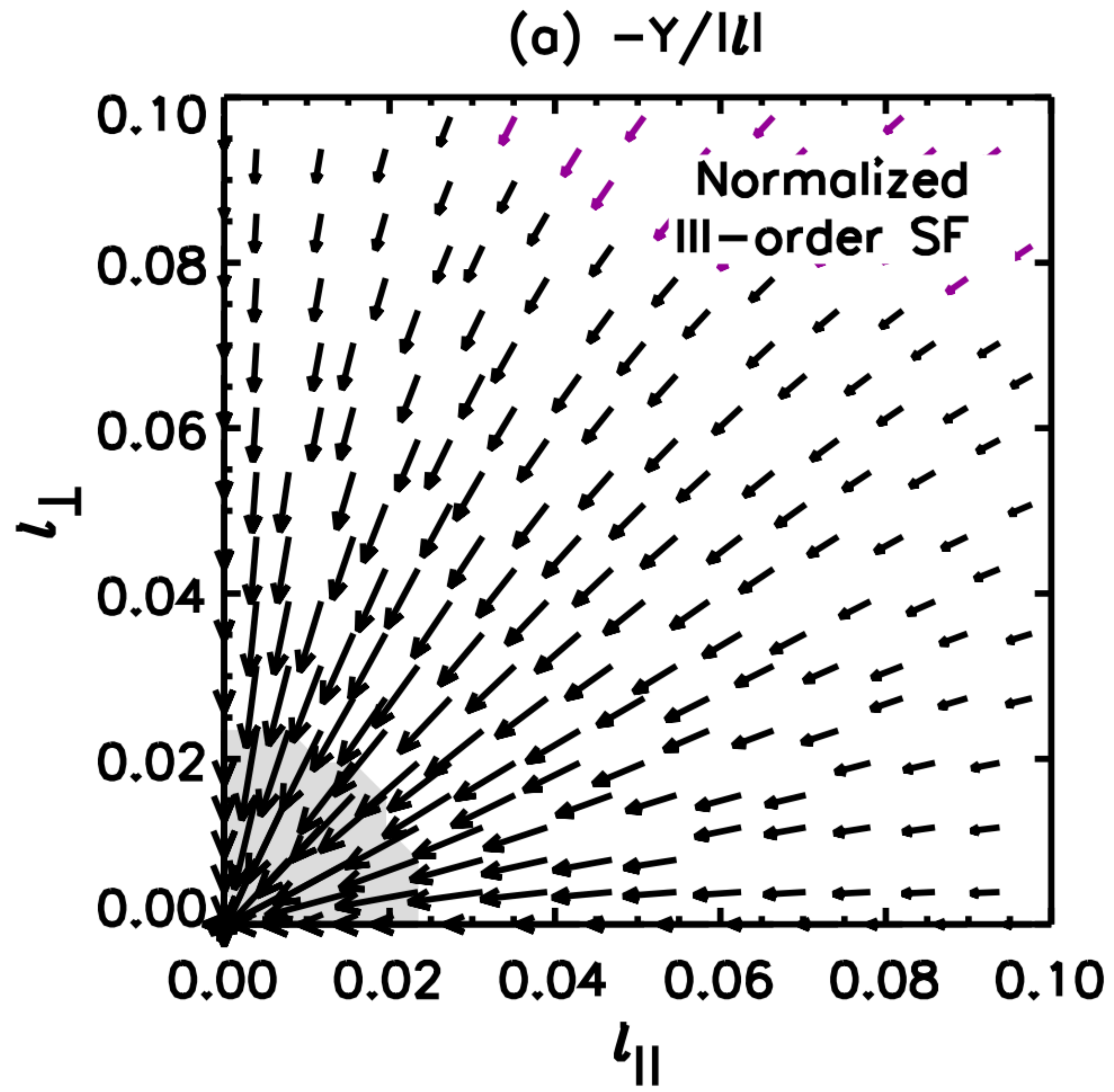



Verdini et al. 2015 ApJ

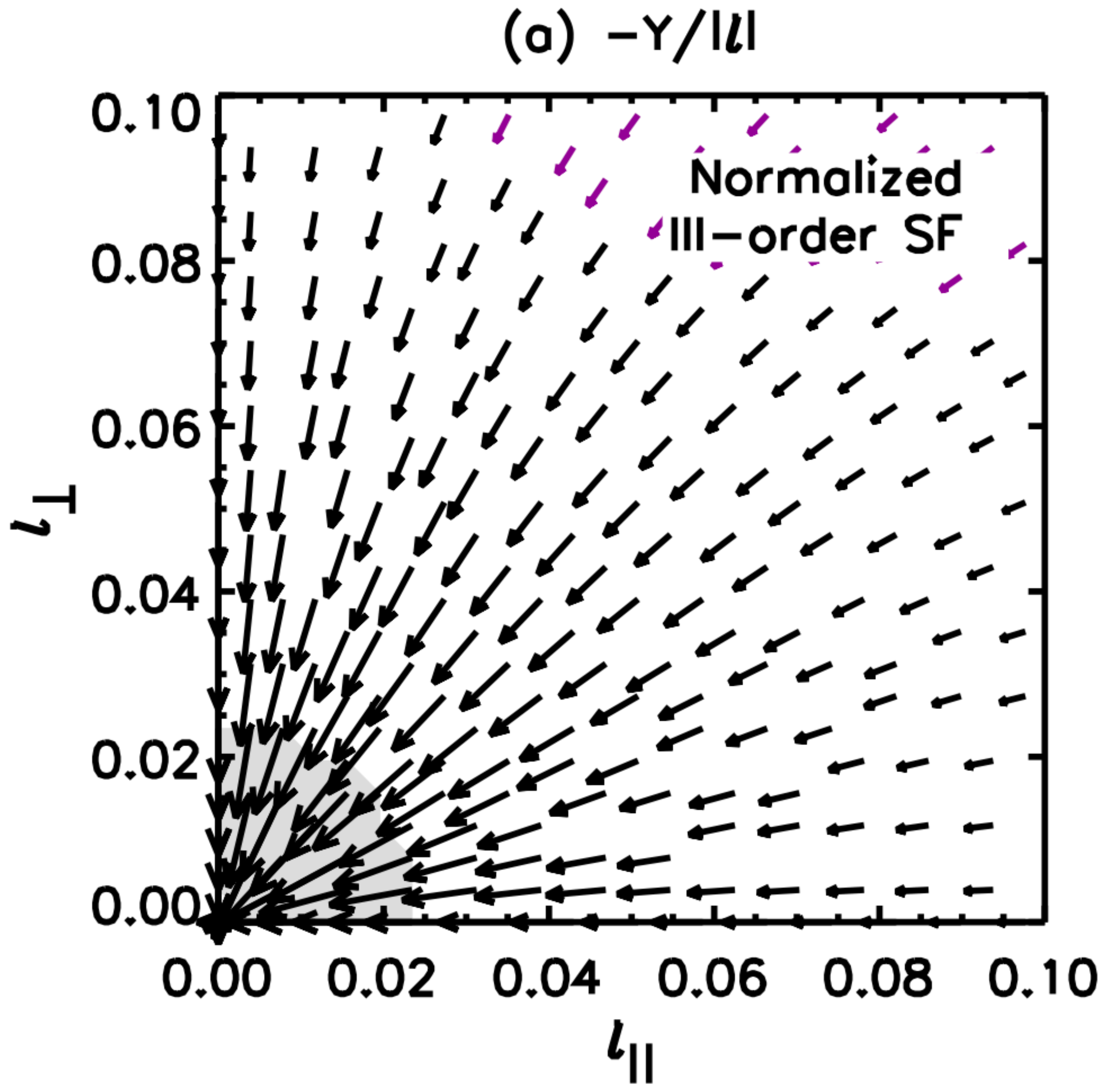
# Meaning of Y

$$\partial_t S + \nabla_\ell \cdot \mathbf{Y} = -4\epsilon + 2\nu \nabla_\ell^2 S$$

**WE DO NOT KNOW**

**III-order Structure Function**

$$\mathbf{Y} = 1/2\left[\langle \Delta\mathbf{z}^- |\Delta\mathbf{z}^+|^2\rangle + \langle \Delta\mathbf{z}^+ |\Delta\mathbf{z}^-|^2\rangle\right]$$



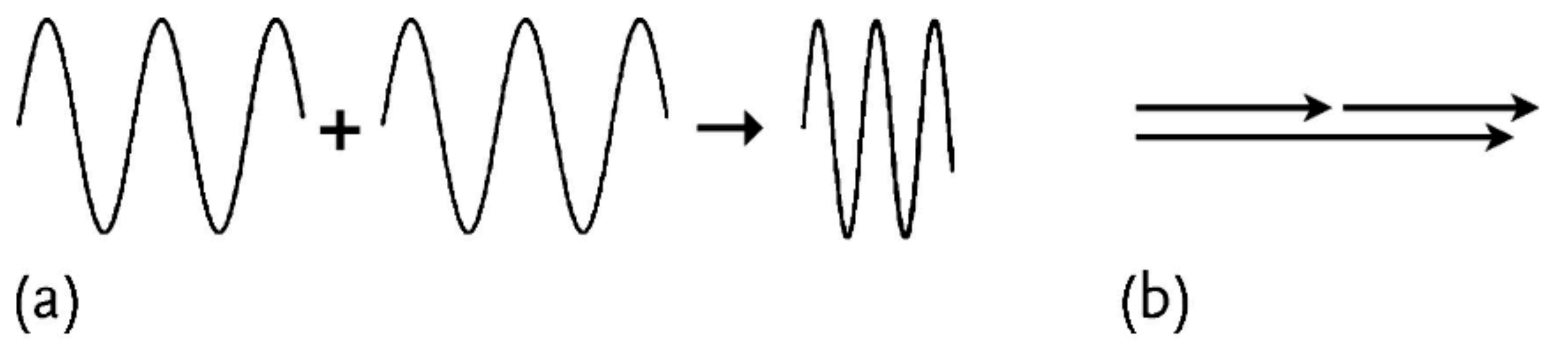

$C(\boldsymbol{\ell}) = \langle u_i(\mathbf{x}+\boldsymbol{\ell}) u_i(\mathbf{x})\rangle$

$SF(\boldsymbol{\ell}) = \langle |u_i(\mathbf{x}+\boldsymbol{\ell}) - u_i(\mathbf{x})|^2\rangle$

$SF = 2[C(0) - C(\boldsymbol{\ell})]$

$$\partial_t C - \nabla_\ell \cdot Y = -2\nu \nabla_\ell^2\, C,$$

physical space   wavenumber



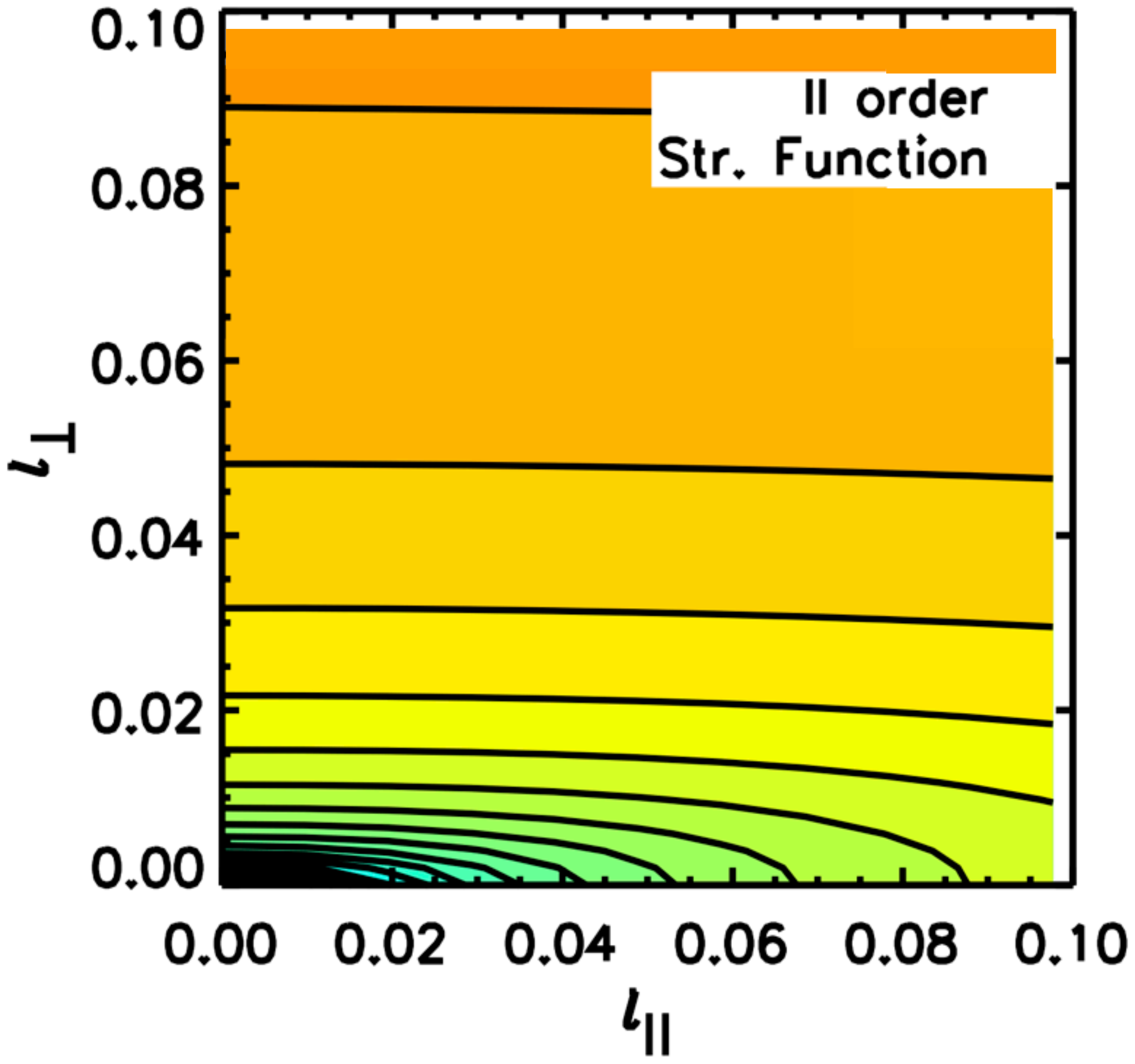

**Y brings negative correlations toward small scales**

through spherical surfaces (iso surfaces of $\nabla \cdot Y$)

# Anisotropic cascade

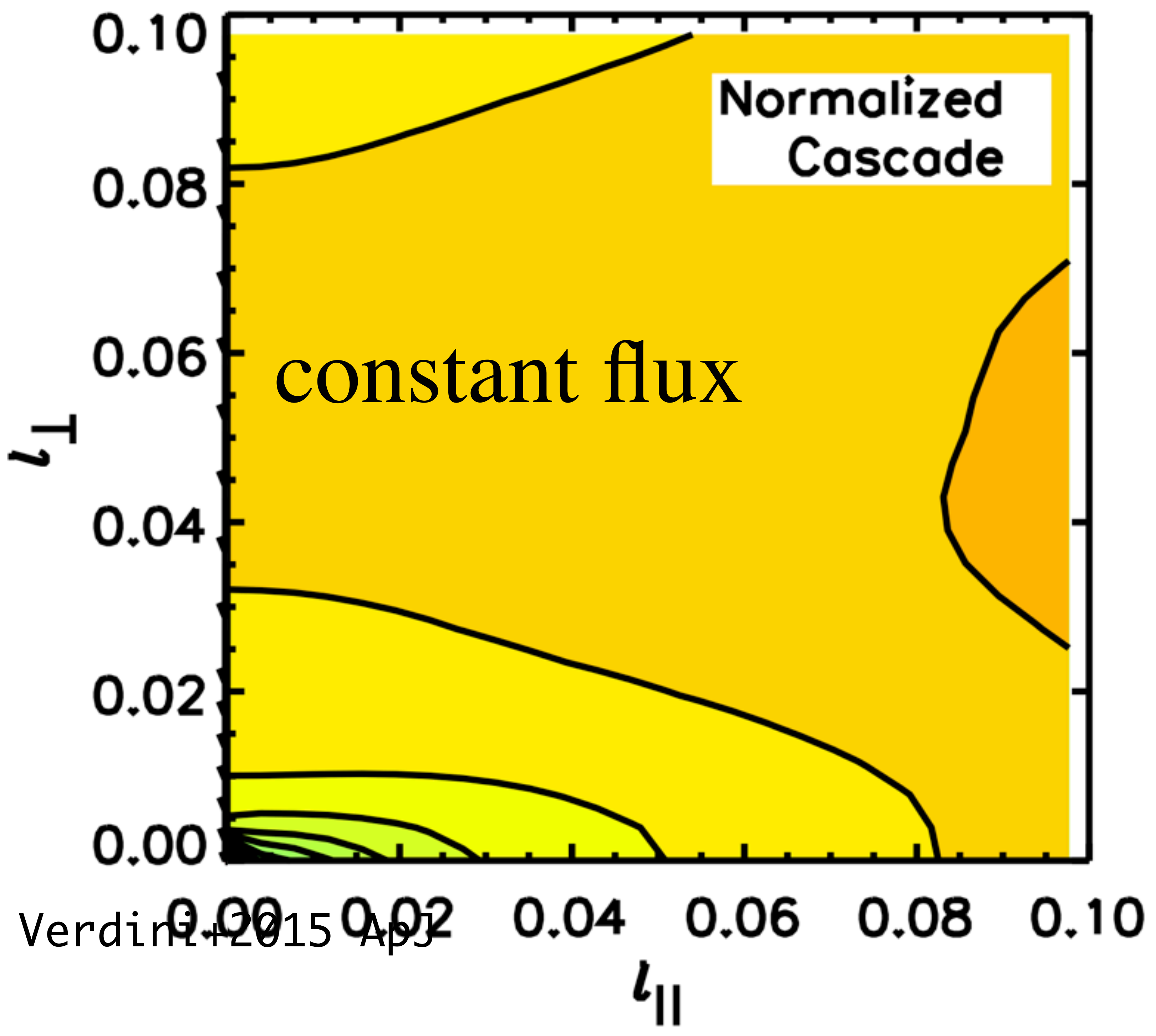
Anisotropy shows up in the difference in ǁ and ⊥ ranges for which $\nabla \cdot Y$=const

**II-order Structure Function**




**Divergence of the flux**

$-\nabla \cdot Y / 4\varepsilon$



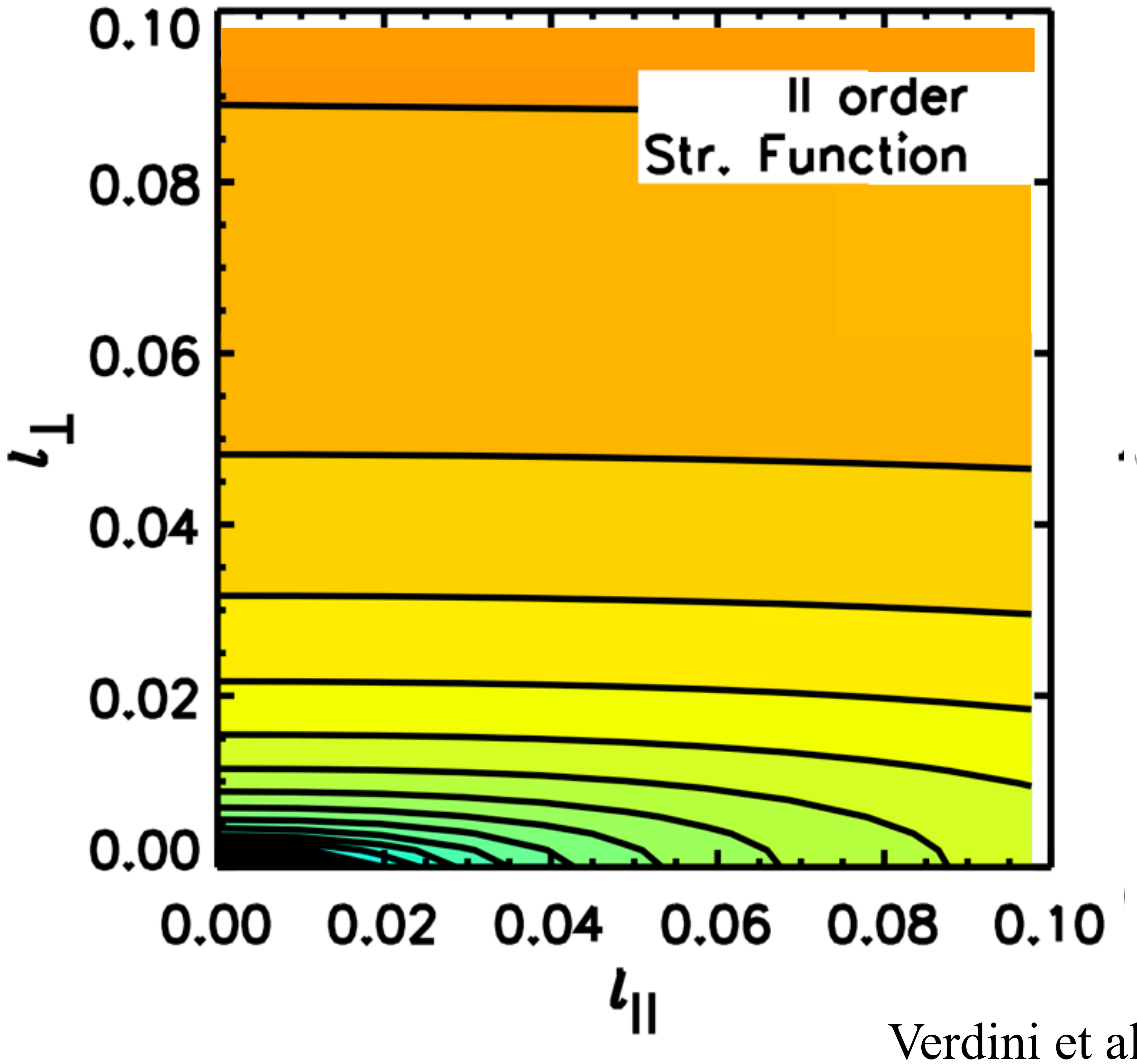

Verdini+2015 ApJ

# Anisotropic cascade

Anisotropy shows up in the difference in ∥ and ⊥ ranges for which $\nabla\cdot Y$=const

**Y** is difficult to interpret without knowing the surfaces of constant divergence

**II-order Structure Function**

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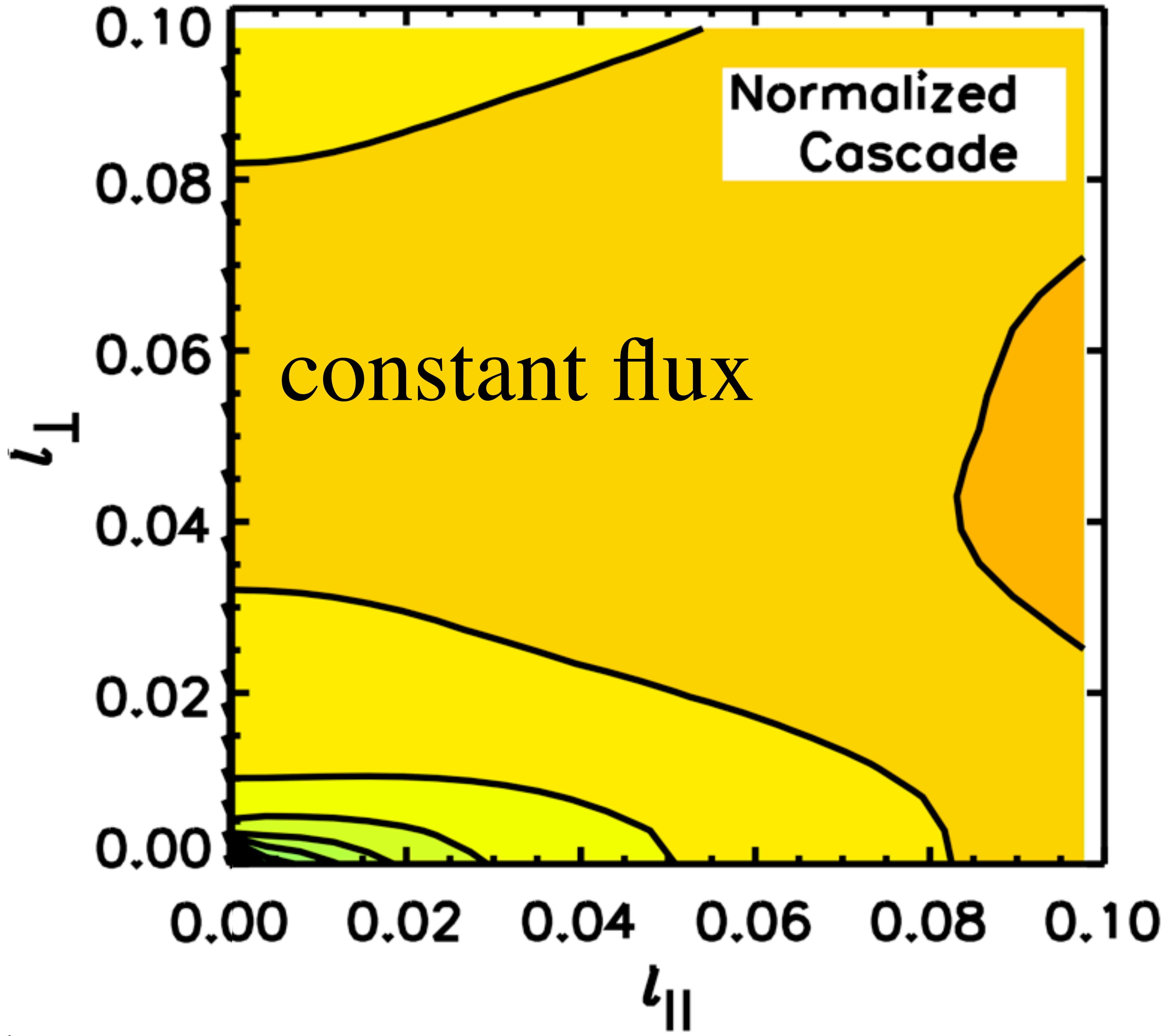
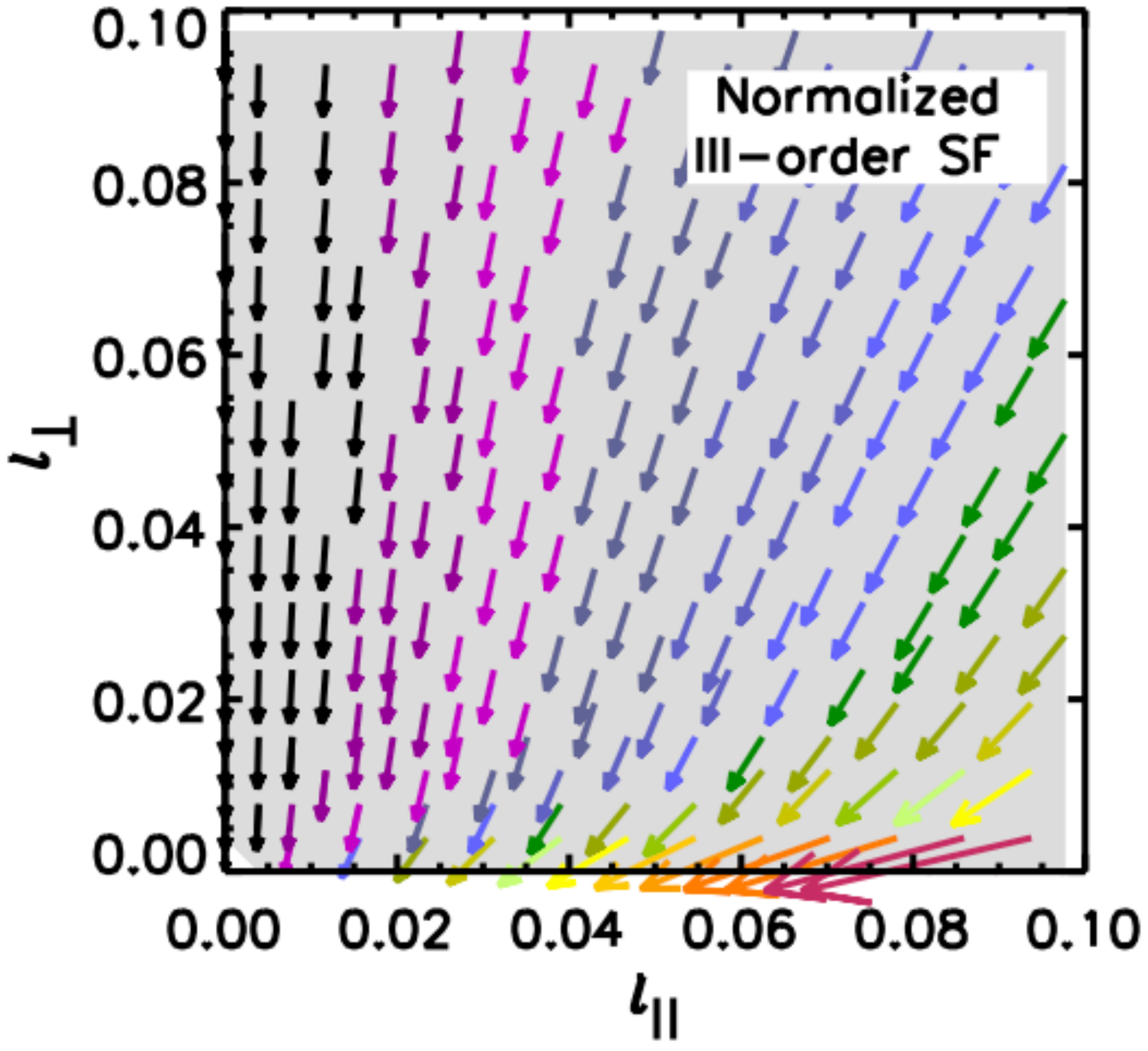


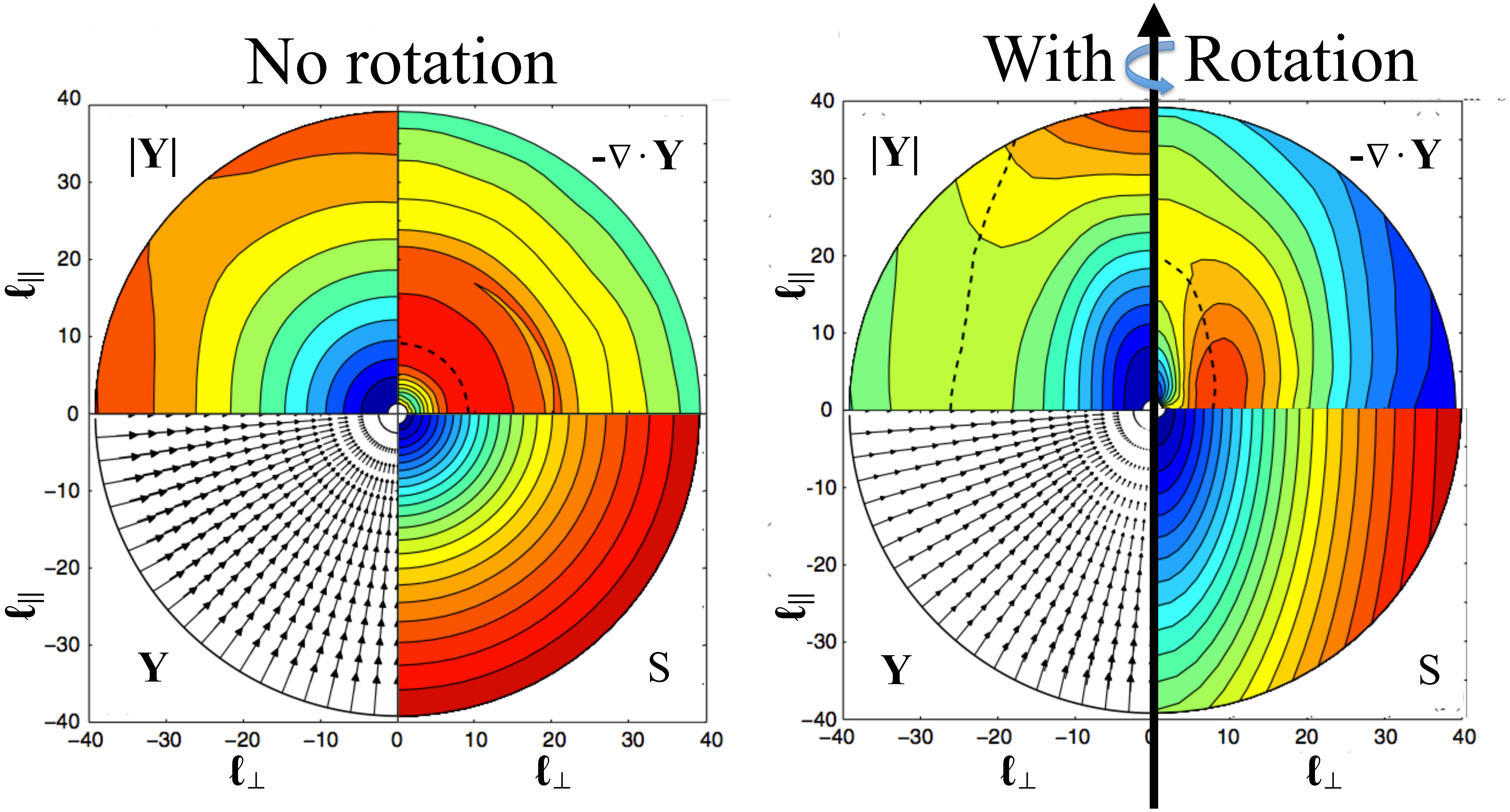

**III-order Structure Function**

(a) $-Y/|l_\perp|$




Verdini et al. 2015 ApJ

# Hydrodynamics with Rotation




Lamriben et al 2011, PRL

# Hydrodynamics with Rotation


Lamriben et al 2011, PRL

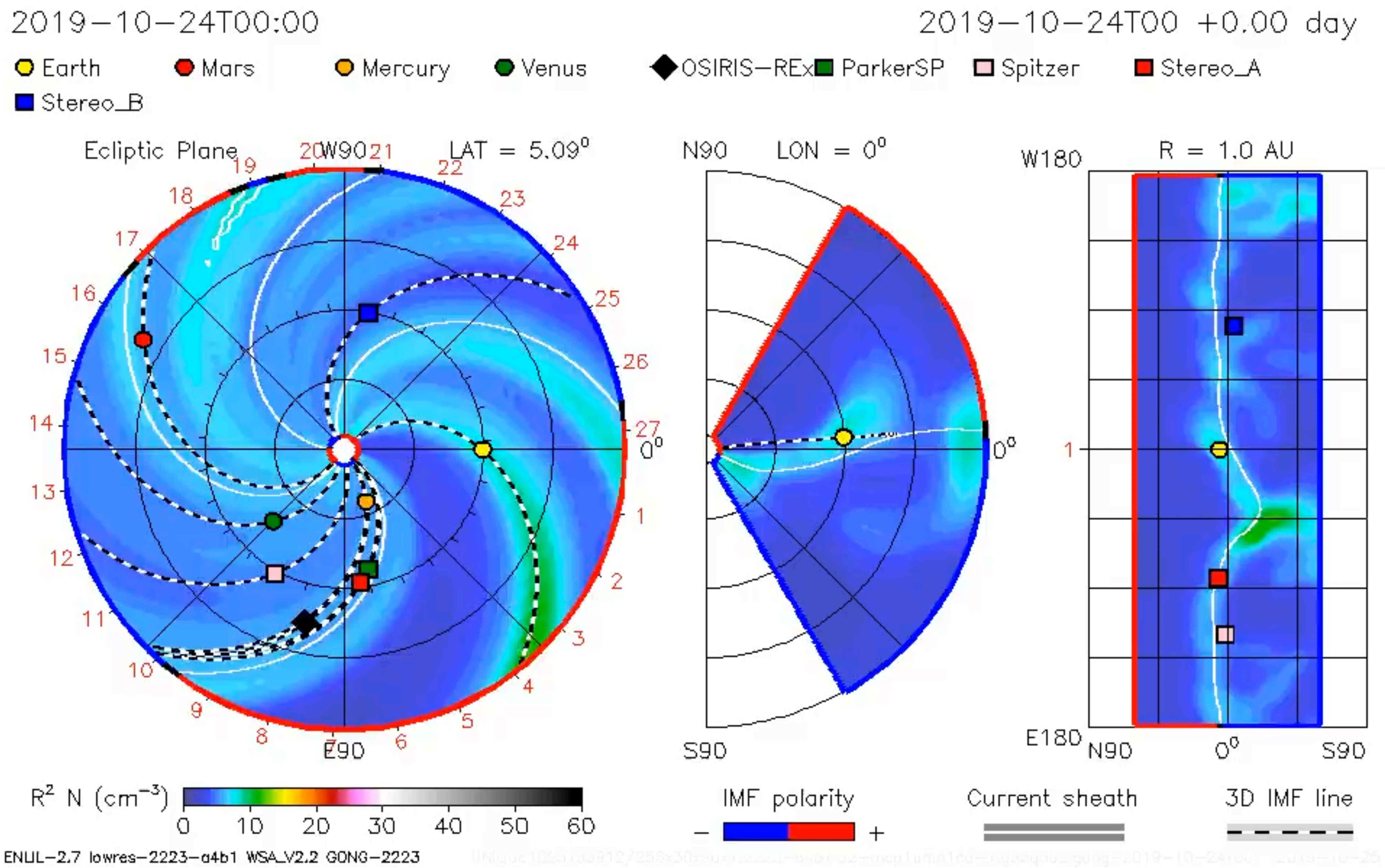
# Solar Wind Measurement: Exploits Variability

Solar wind flows almost radially (at least from 0.3 au)

Magnetic field follows a spiral whose inclination depends on the speed (faster, more radial)

On the ecliptic opposite sectors cross each other due to mis-alignment of magnetic and rotation axis

Many samples to cover different $\theta_{BV}$



# Measure of anisotropy

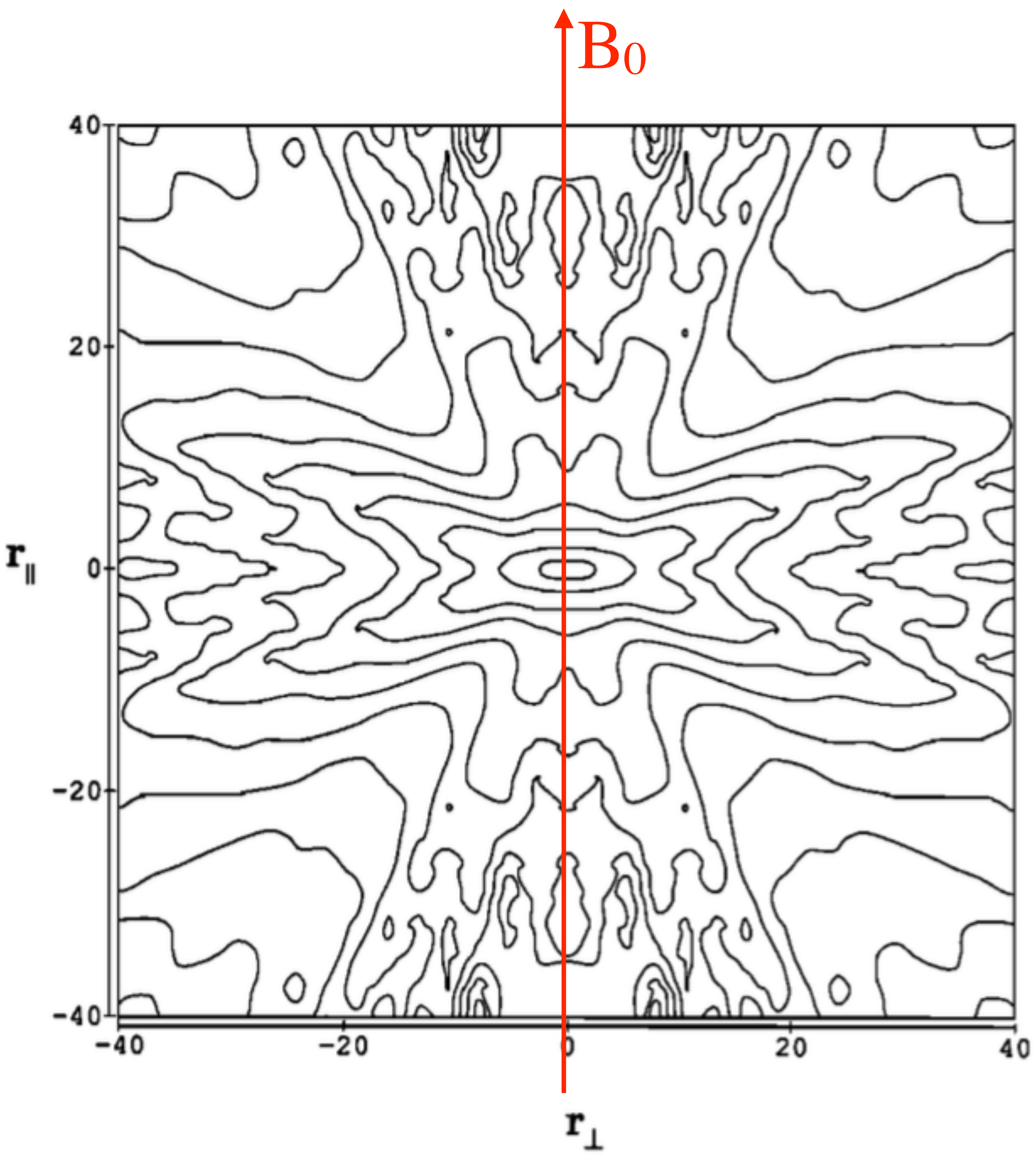
Consider a timeseries in which $B_0$ has direction w.r.t. to the sampling direction R≡Vsw , i.e. a given $\theta_{BV}$

Remove Average
Compute autocorrelation

$\ell = V_{sw} \cdot \tau$

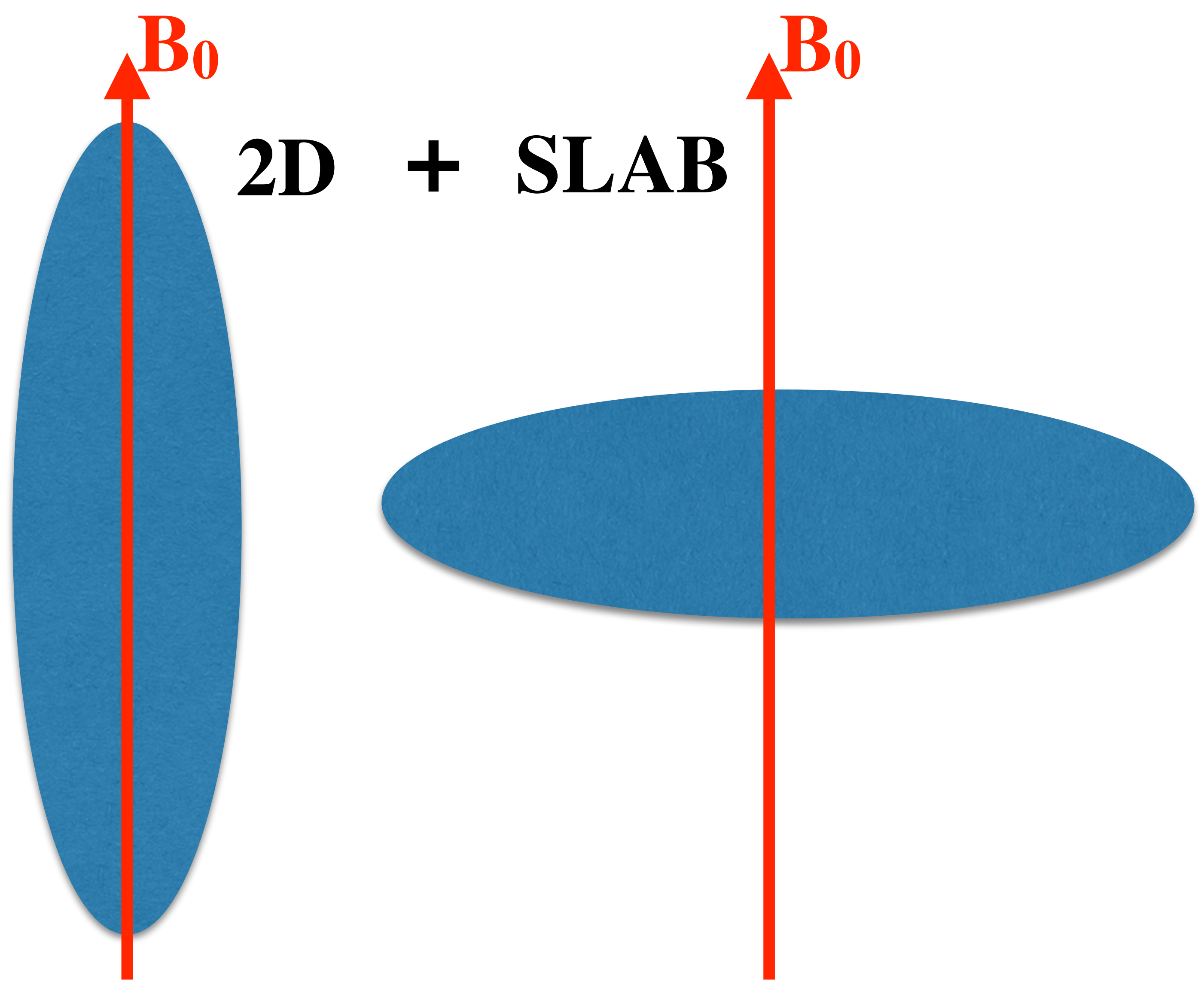
Normalize by C(0)=δu^2

Bin AC according to $\theta_{BV}$

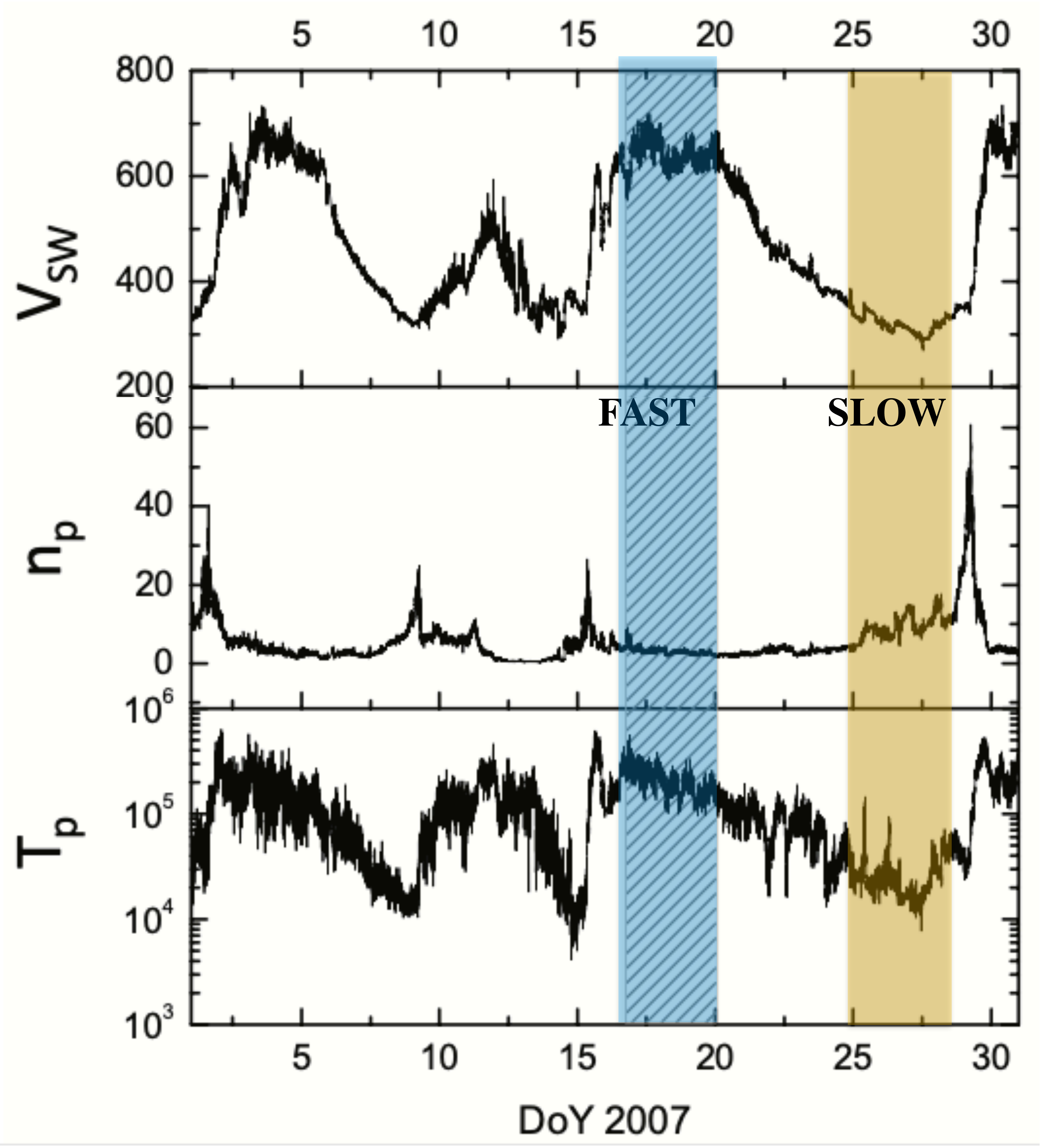



Matthaeus Goldstein Roberts 1990

Maltese Cross



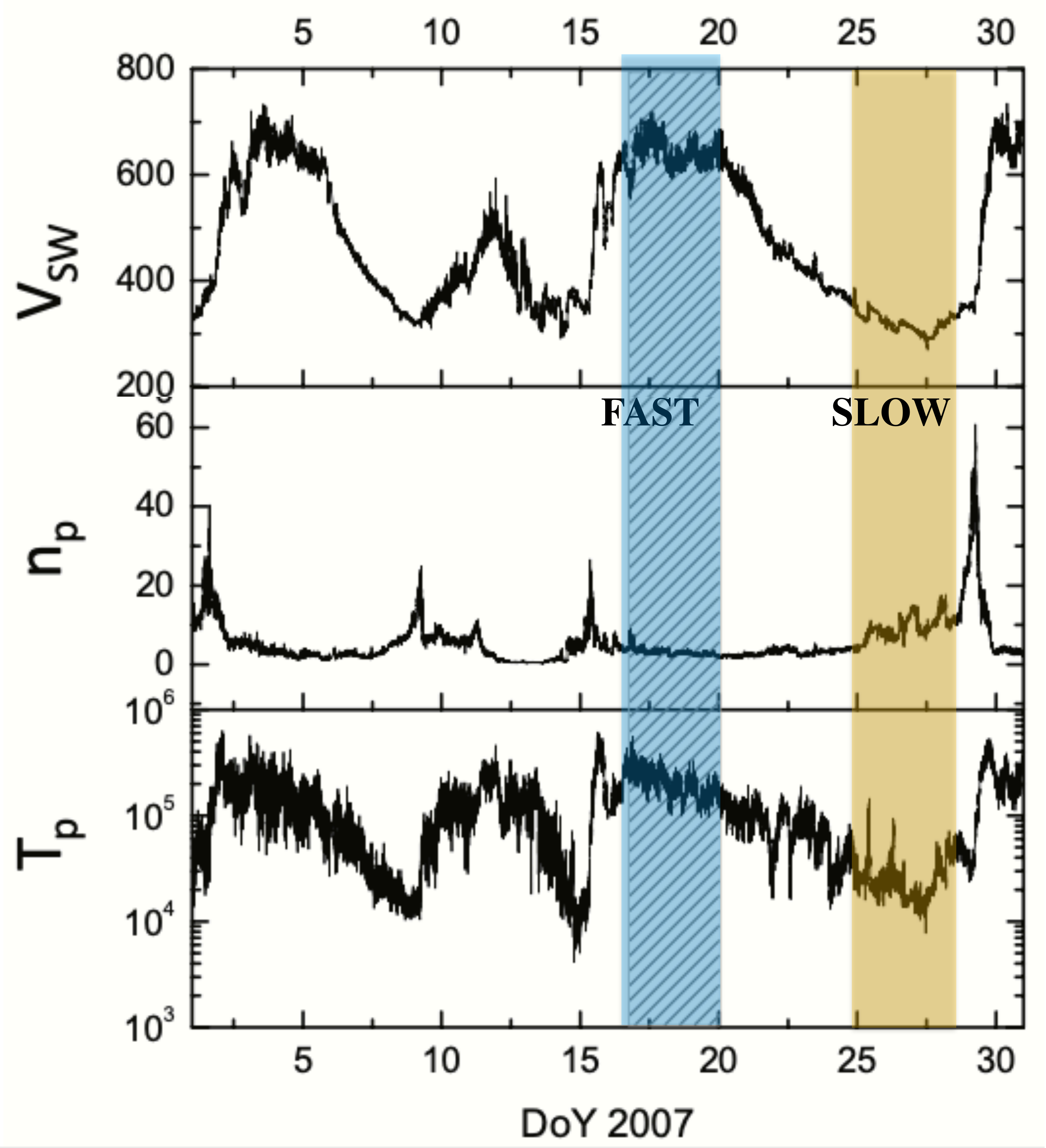
# Wind Speed and Turbulence




**Fast, Hot, Rarefied**

**Slow, Cool, Dense**

# Wind Speed and Turbulence

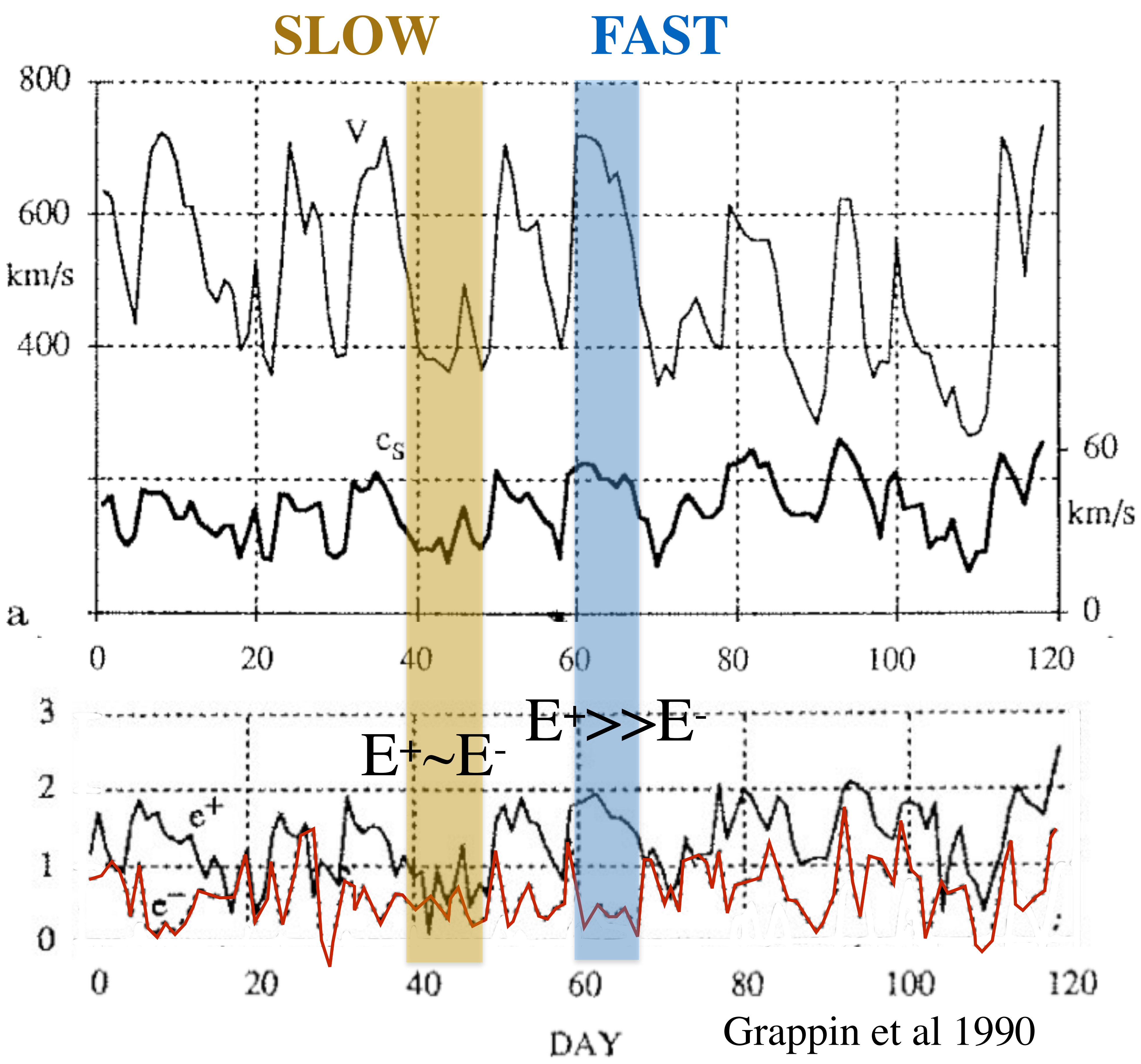



**Fast, Hot, Rarefied**

Alfvénic and almost incompressible

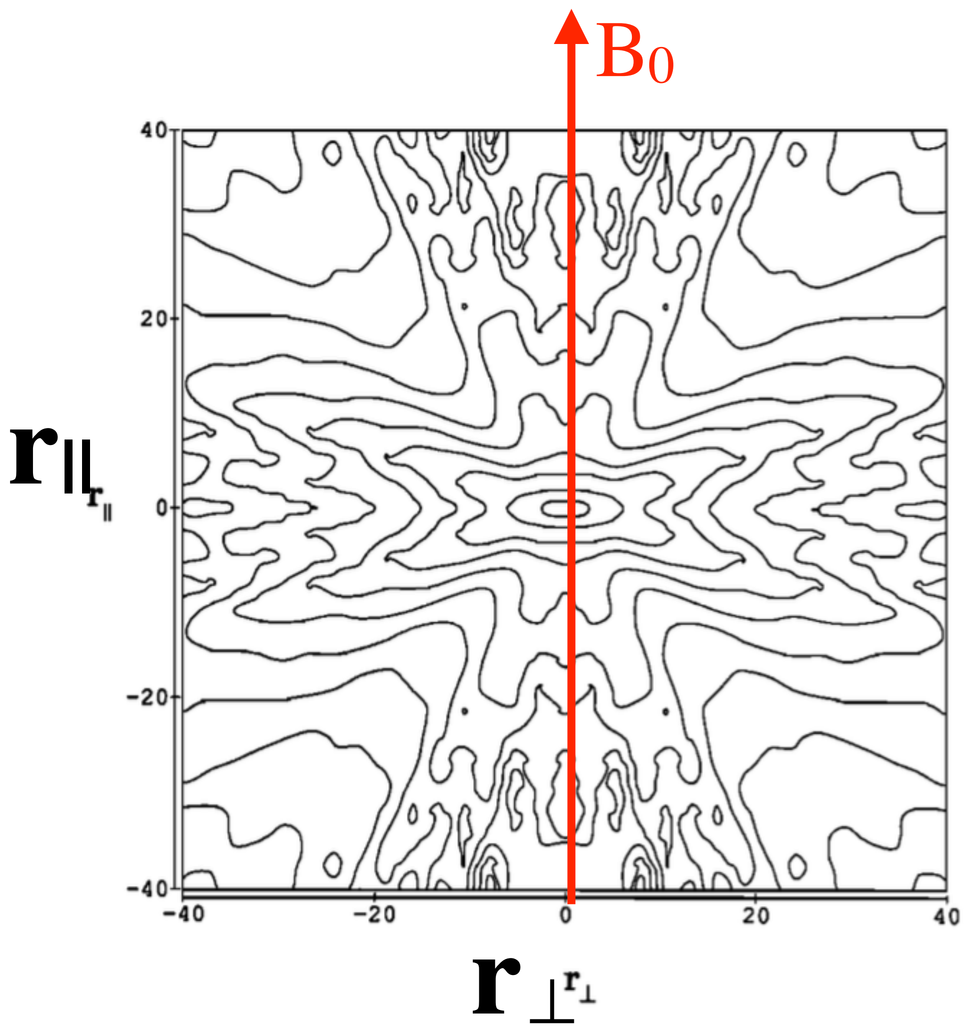
**Slow, Cool, Dense**

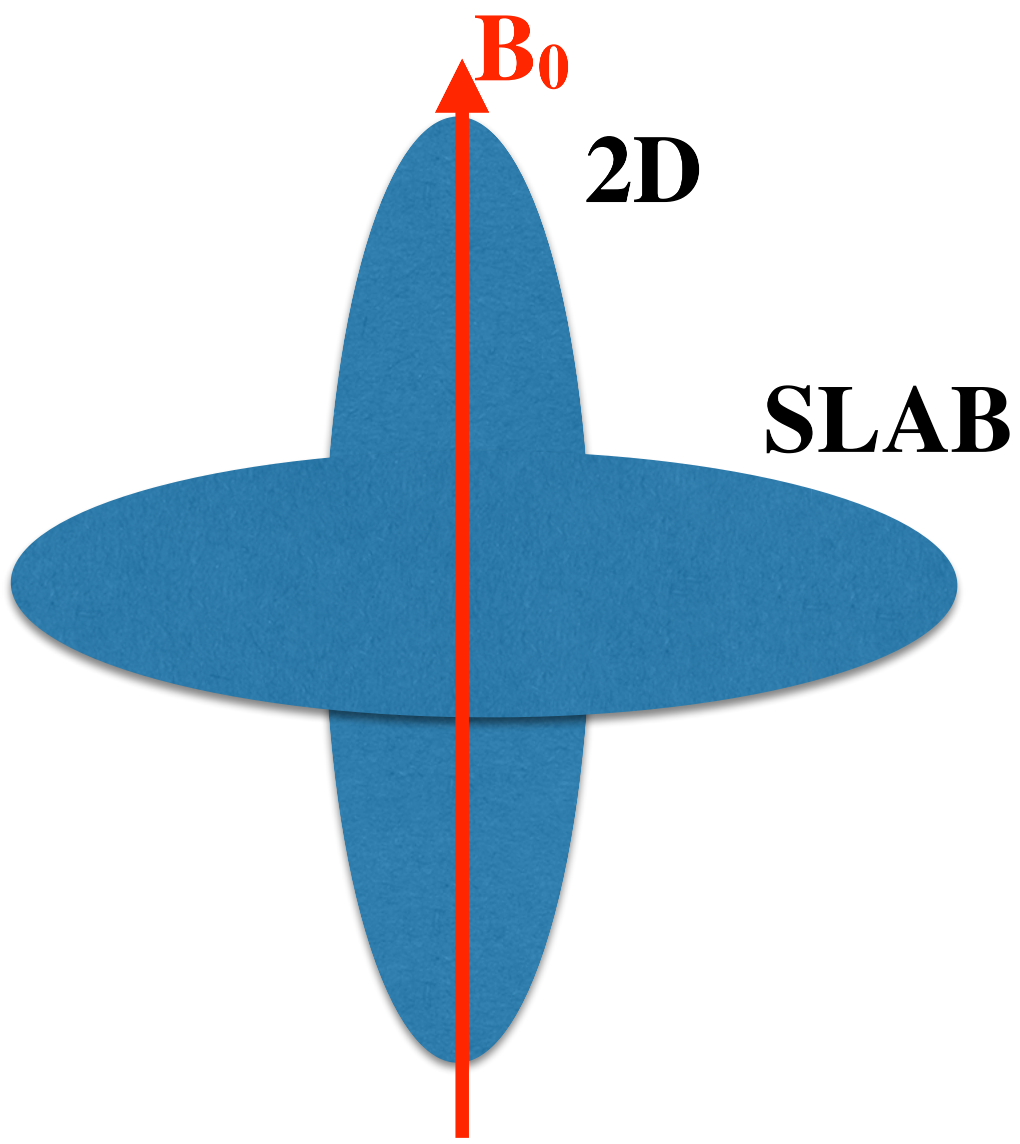
Non Alfvénic and compressible




Grappin et al 1990

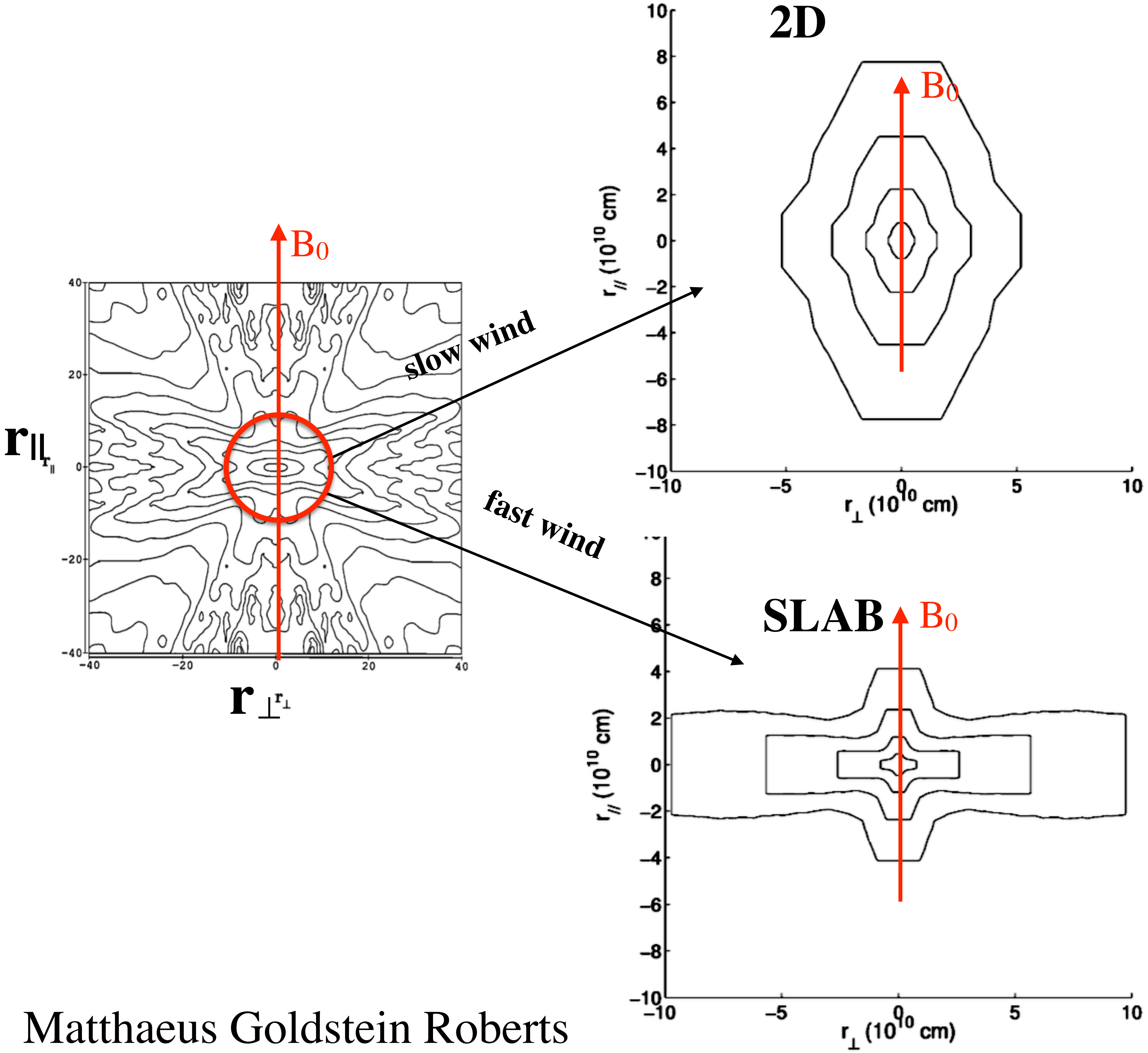
# Anisotropy at smaller scale




Matthaeus Goldstein Roberts
1990

# Anisotropy at smaller scale




Matthaeus Goldstein Roberts 1990

Dasso et al 2005

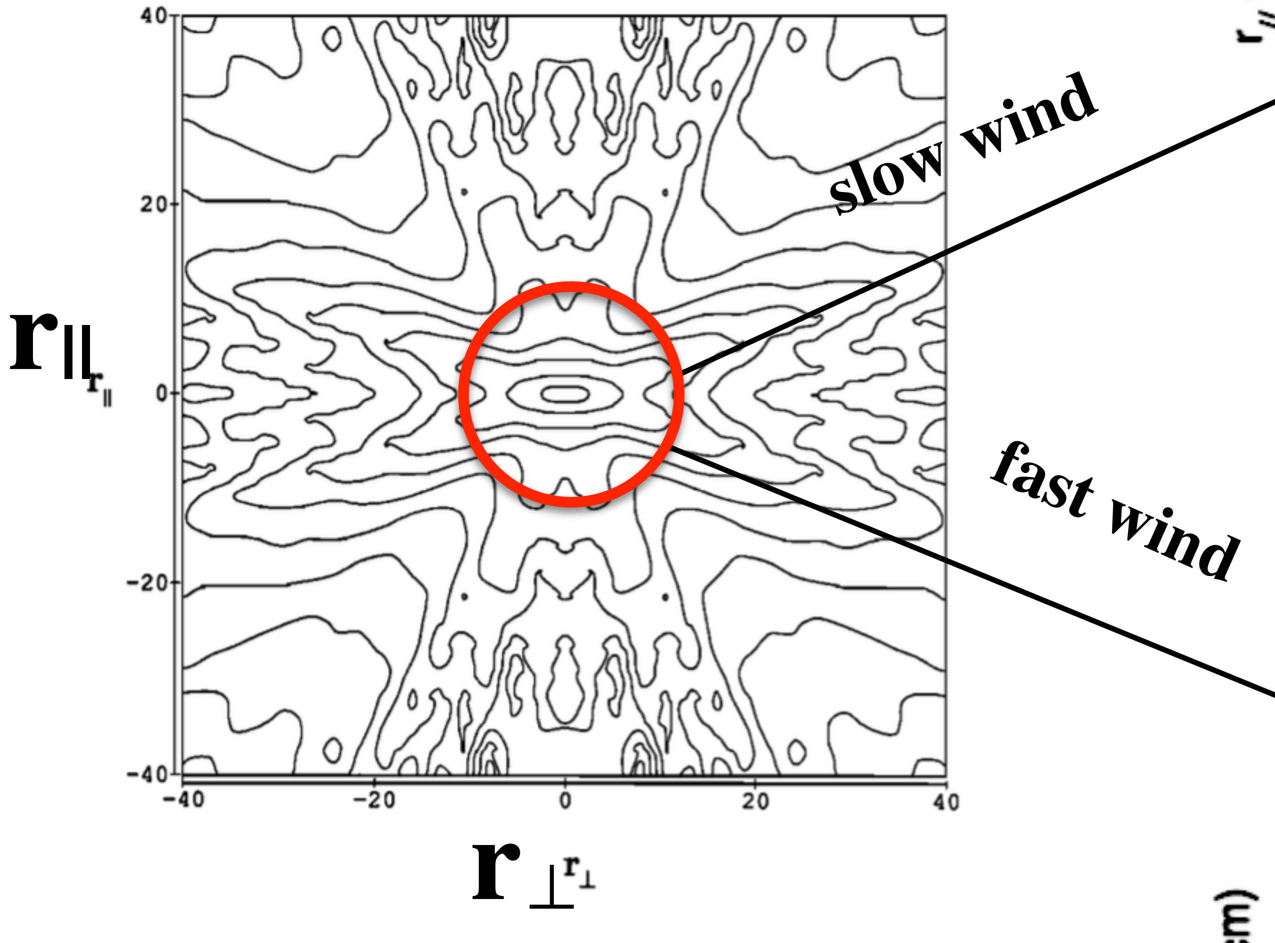
Slow is mainly 2D

Energy in $k_\perp$

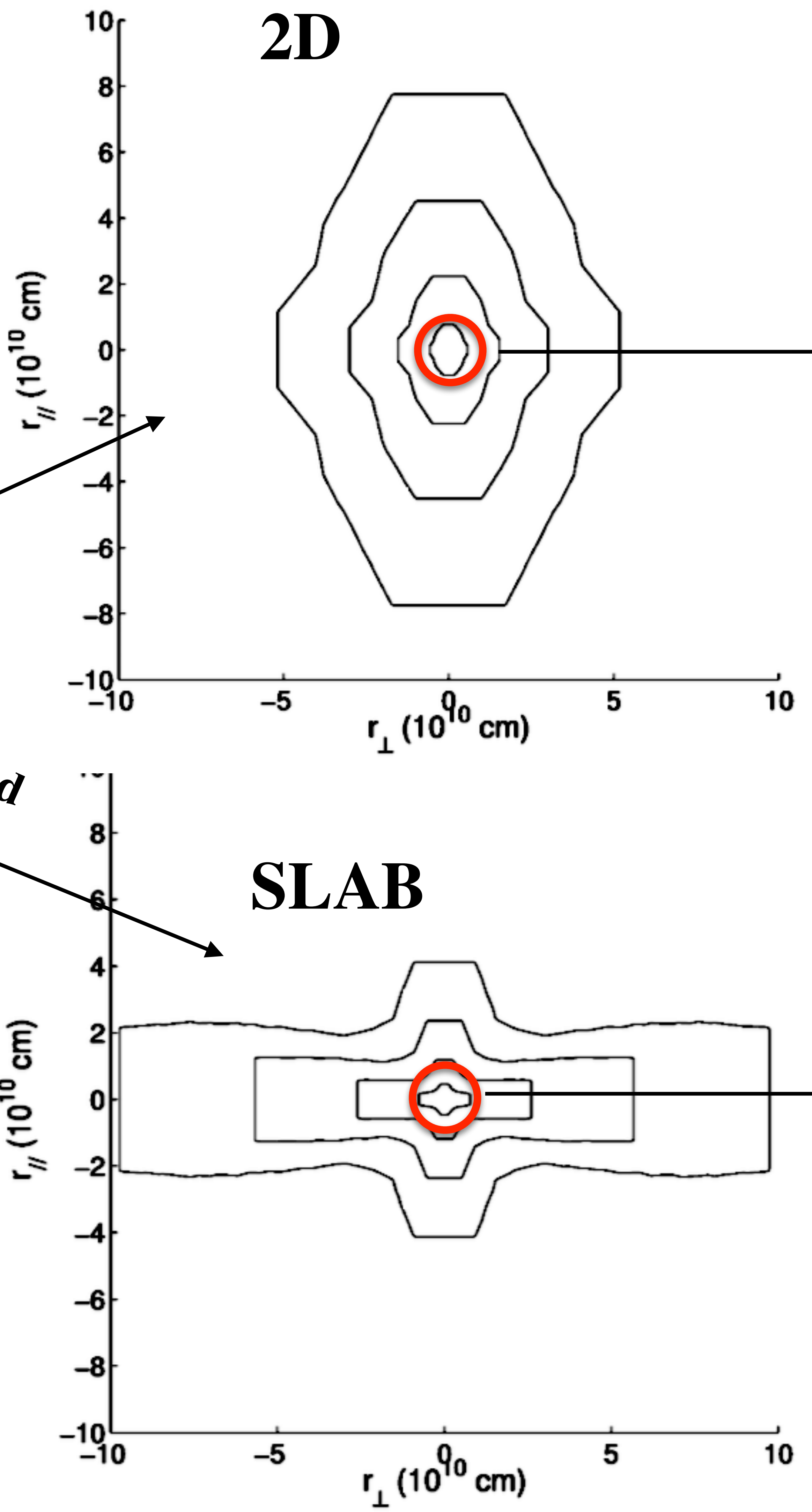
Fast is mainly Slab

Energy in $k_\parallel$

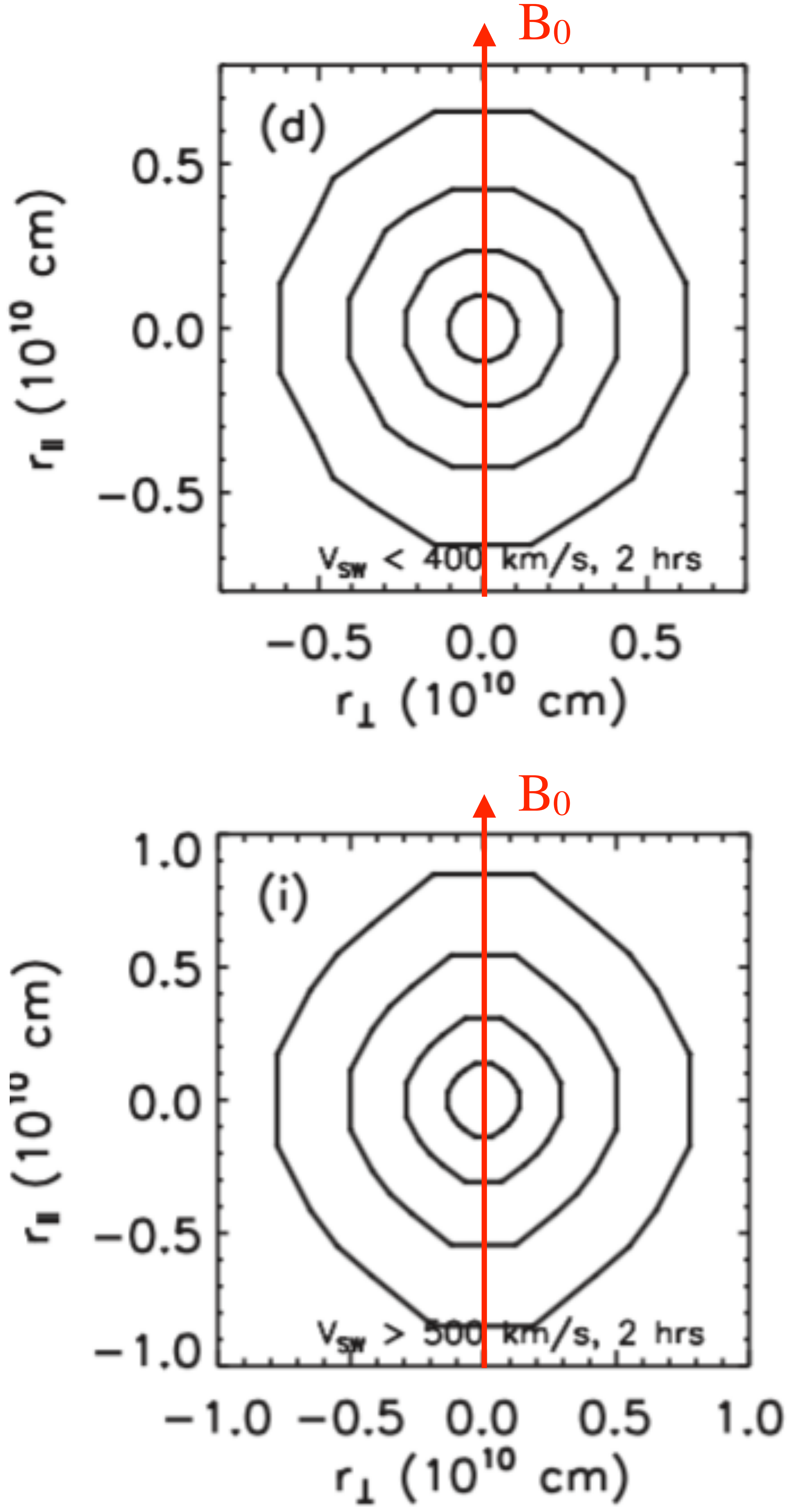
# Anisotropy at smaller scale




Matthaeus Goldstein Roberts 1990



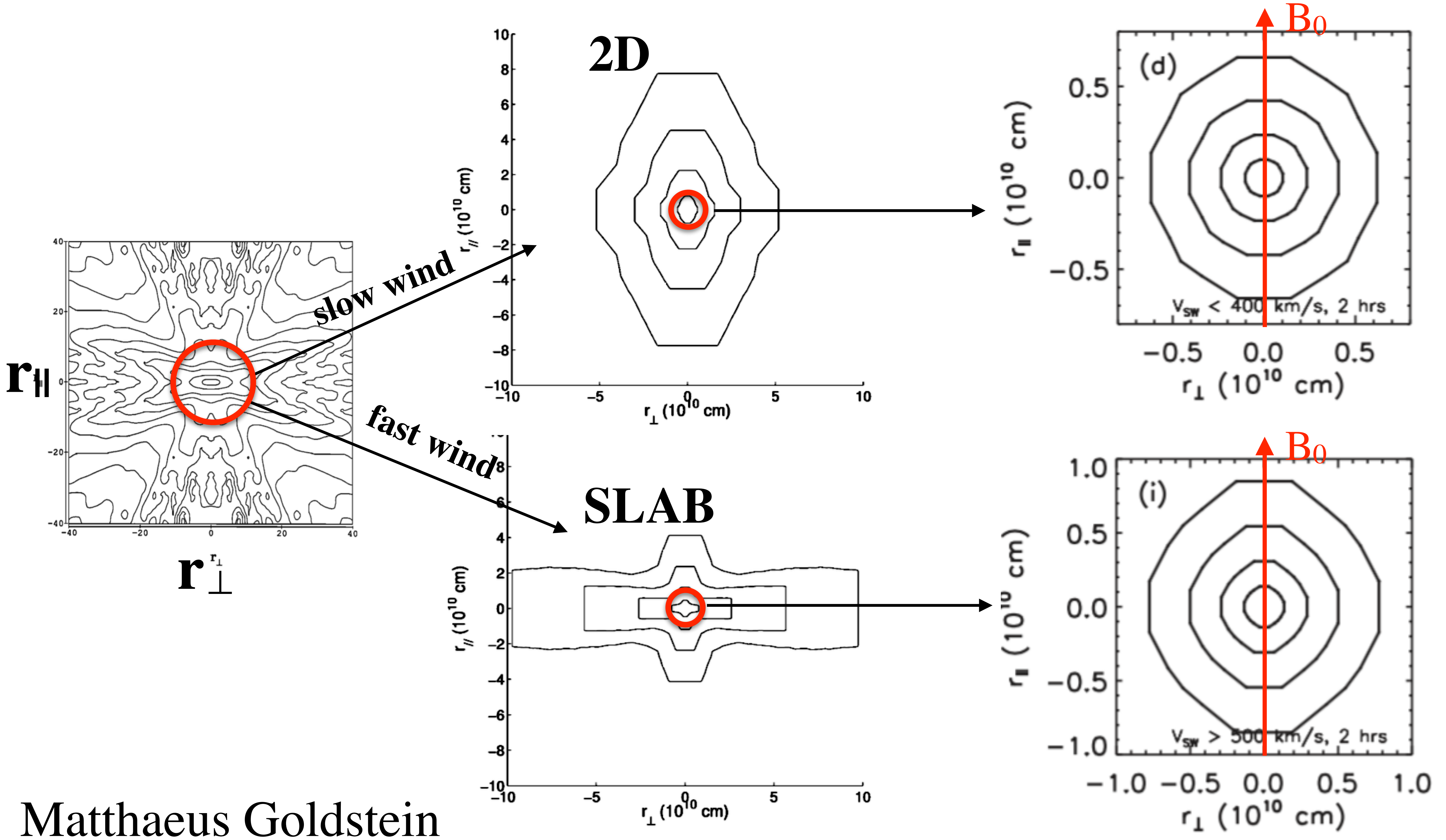

Dasso et al 2005




Wang Tu He 2019

Recover of isotropy at small scales?

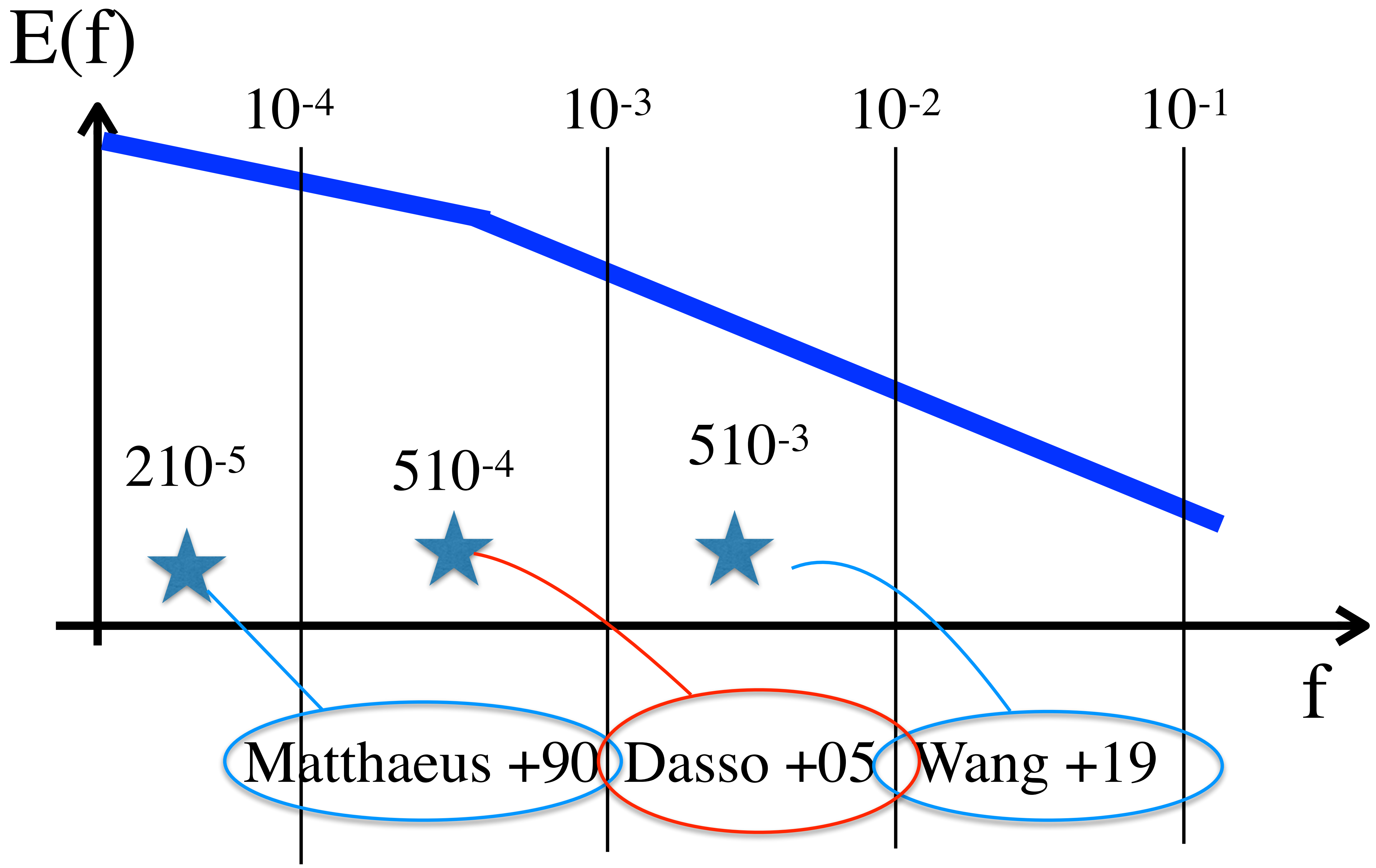
# Observed Anisotropy in the SW




Matthaeus Goldstein Roberts 1990

Dasso et al 2005

Wang Tu He 2019



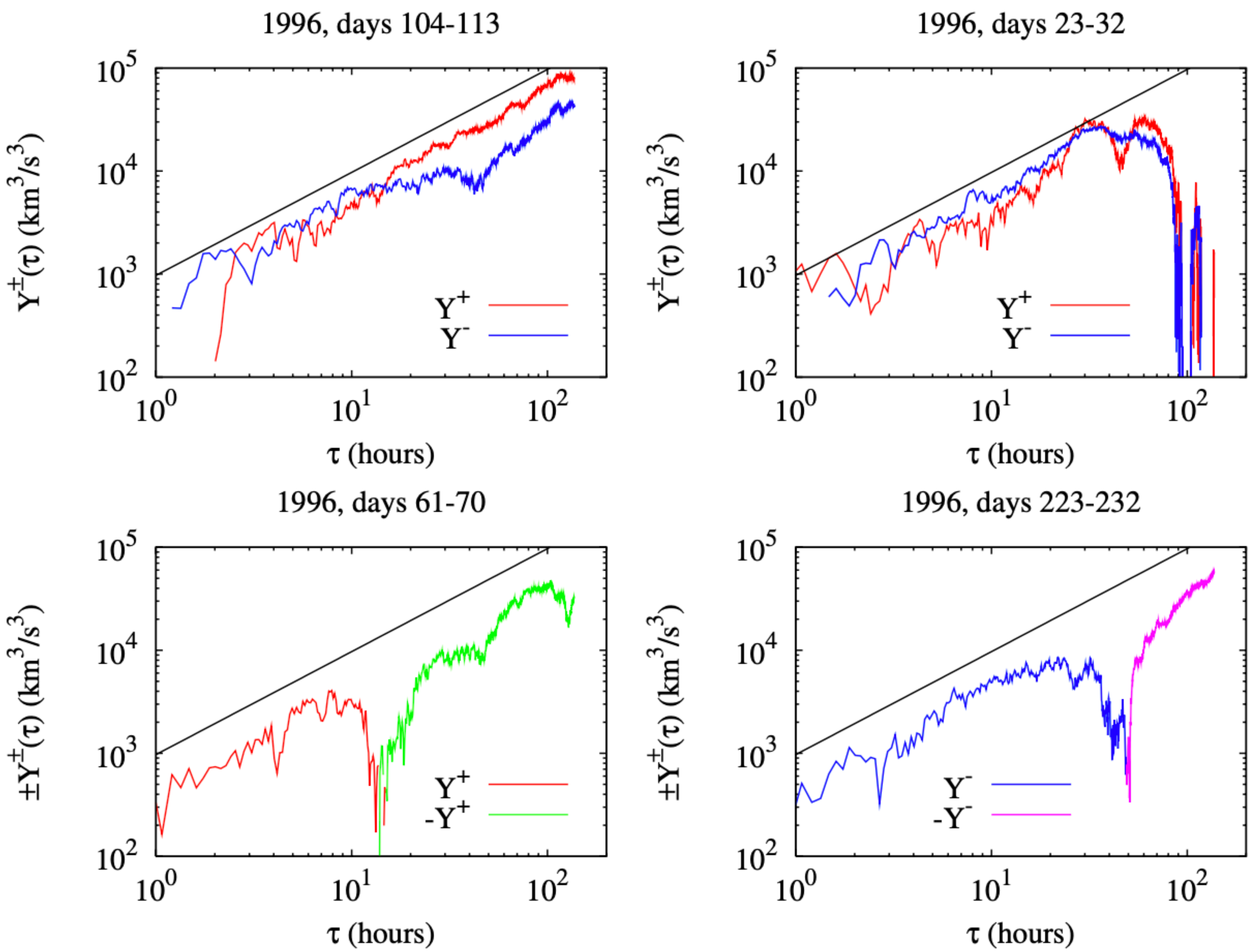
# Measurement of Cascade

$$\nabla_\ell \cdot \mathbf{Y} = -4\epsilon$$

with isotropy

$$Y_\ell = -\frac{4}{3}\epsilon\ell \qquad Y_\ell = \mathbf{Y} \cdot \hat{\ell}$$

Search for linear scaling of projected Yaglom vector




Change of sign

Linear scaling not found in all intervals

Sorriso Valvo + 2007 PRL

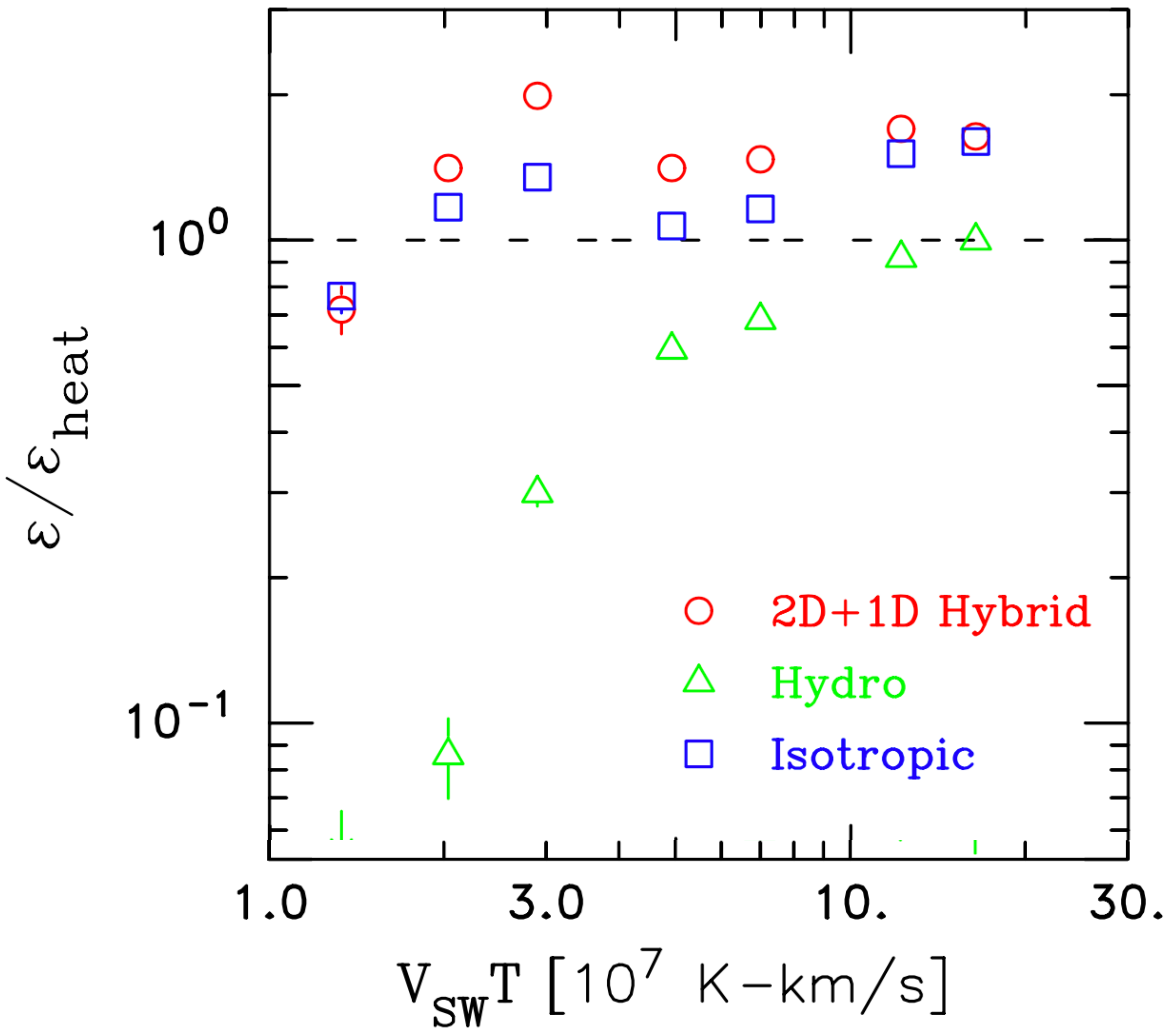
# With Anisotropic Geometrical Model

$$\nabla_\ell \cdot \mathbf{Y} = -4\epsilon$$

$$Y_\ell = -\frac{4}{3}\epsilon\ell$$

with 2D + 1D geometry

$$\epsilon^{2D} = -\frac{Y_{\ell_\perp}(\ell_\perp)}{2\ell_\perp}, \qquad \epsilon^{1D} = -\frac{Y_{||}(\ell_{||})}{4\ell_{||}}.$$



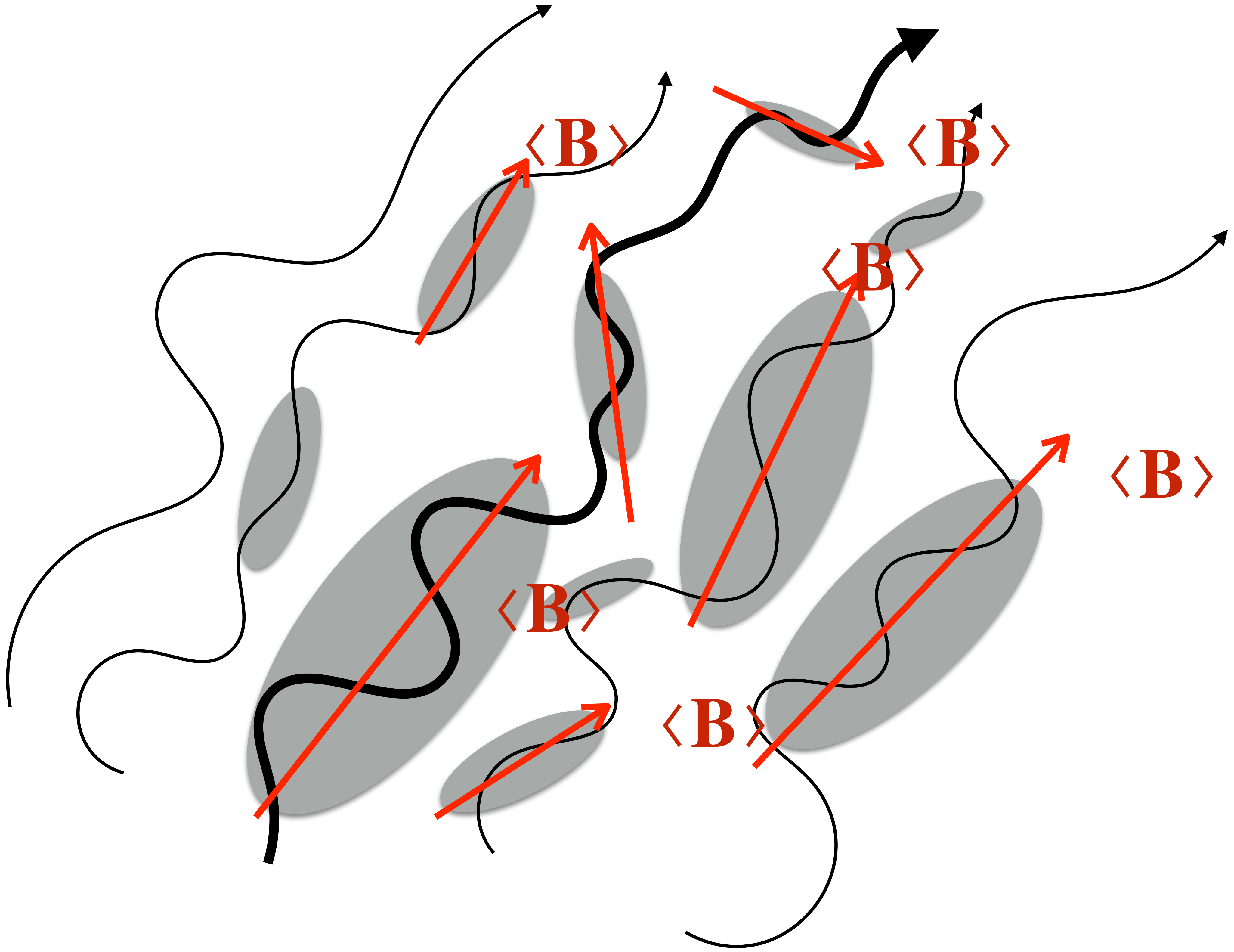

Cascade is larger in slow/cold solar wind turbulence

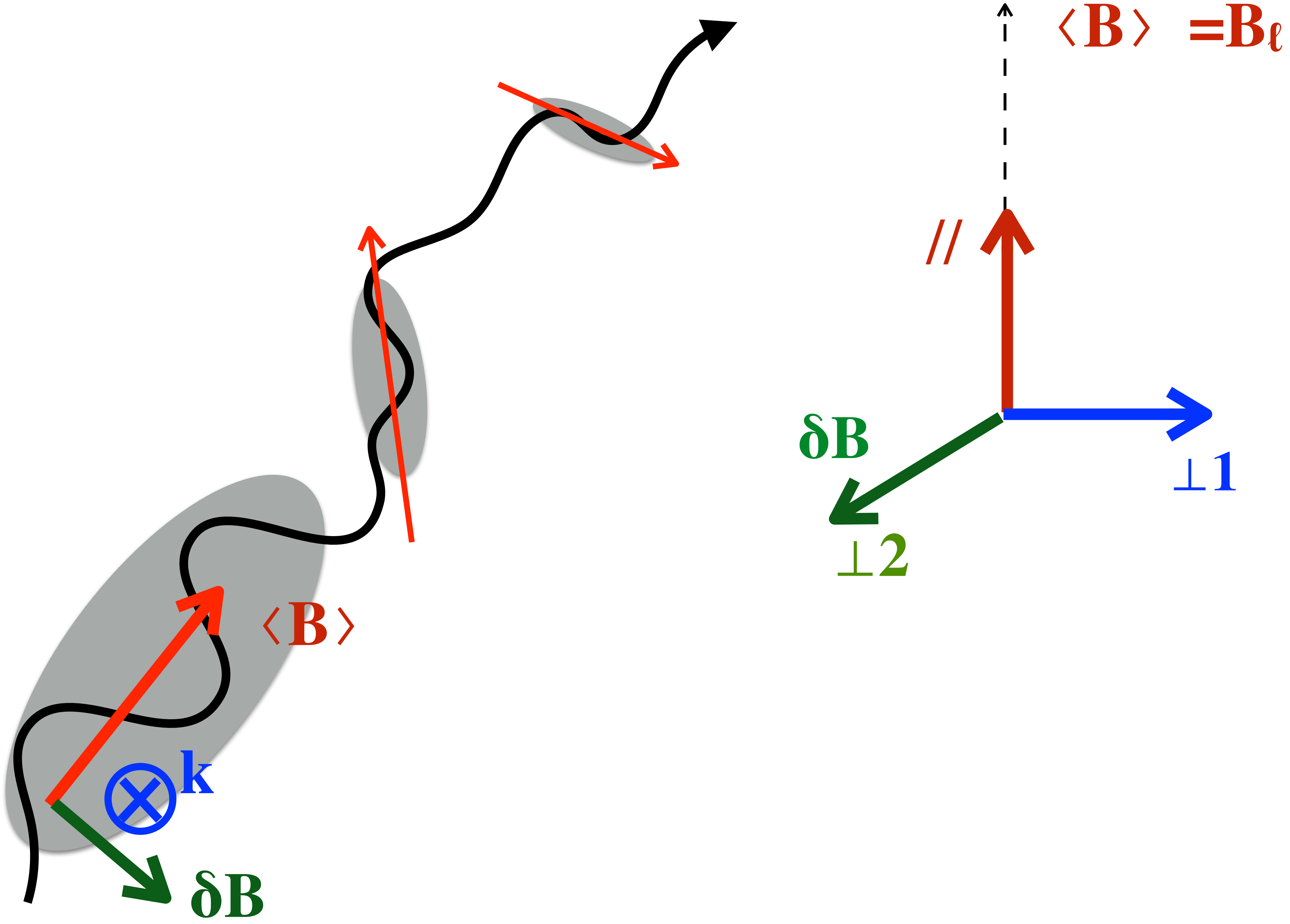
Stawartz+ 2009 ApJ

# Scale Dependent Anisotropy

Compute anisotropy with repect to a scale dependent definition of mean magnetic field

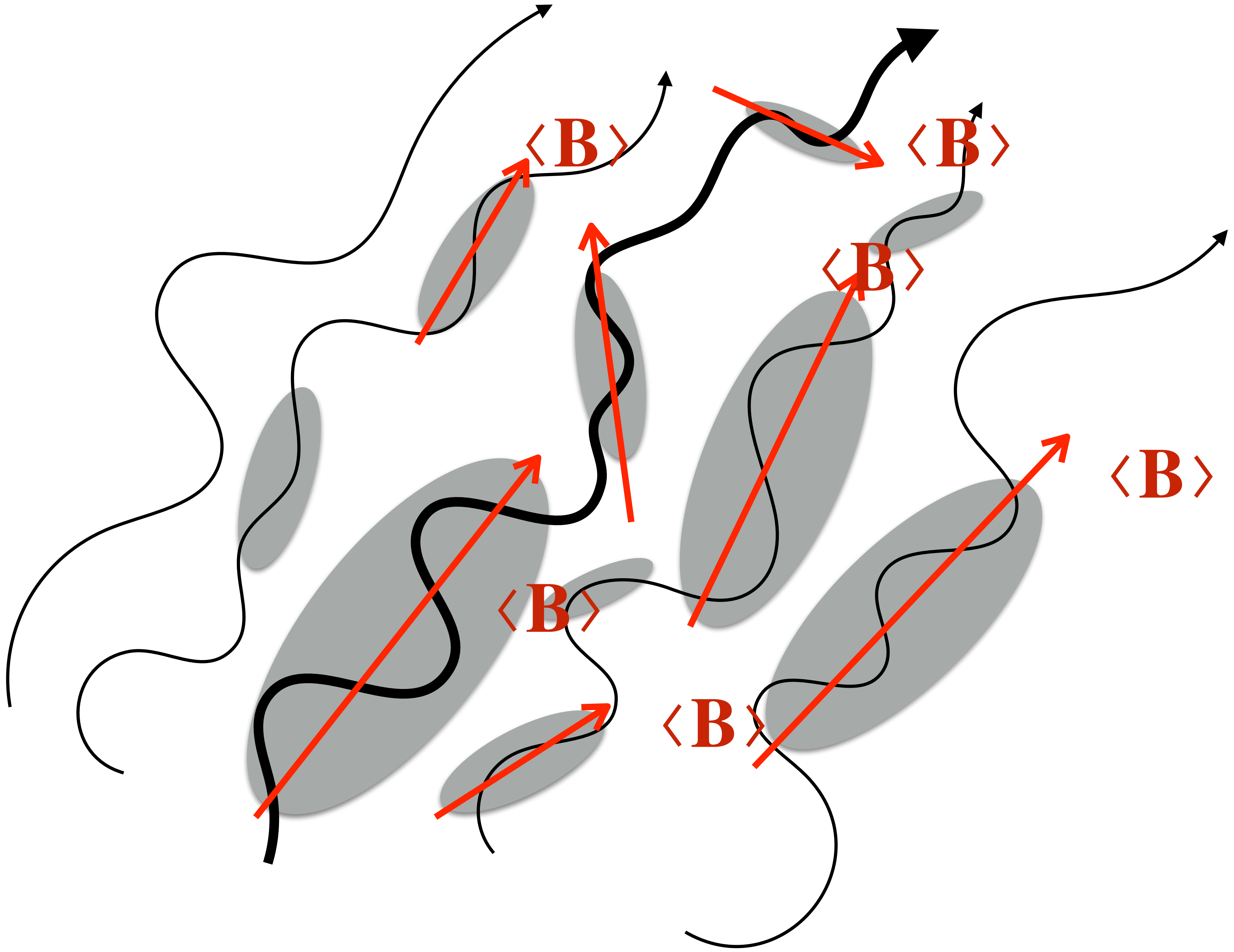
Not a II-order quantity!



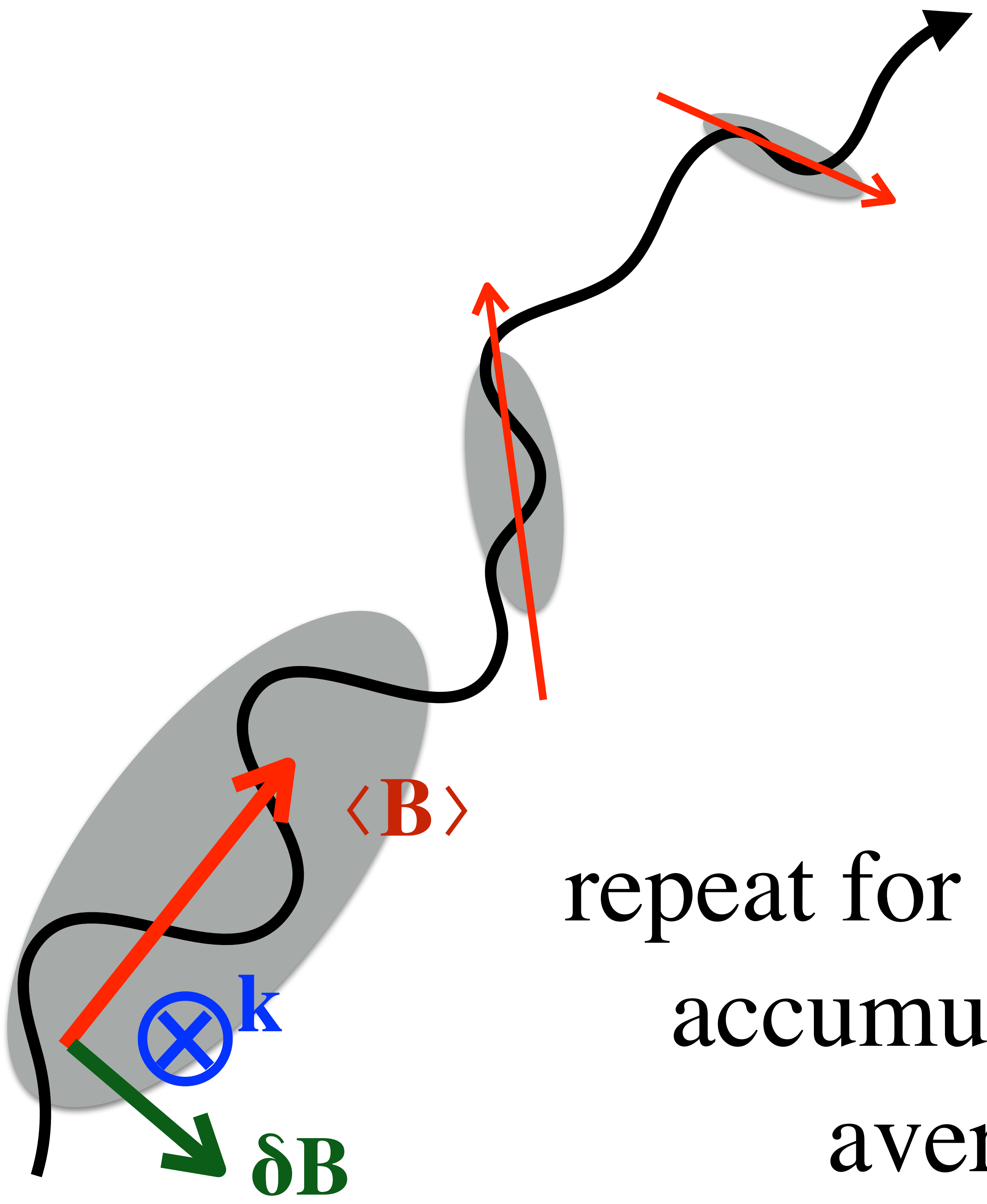



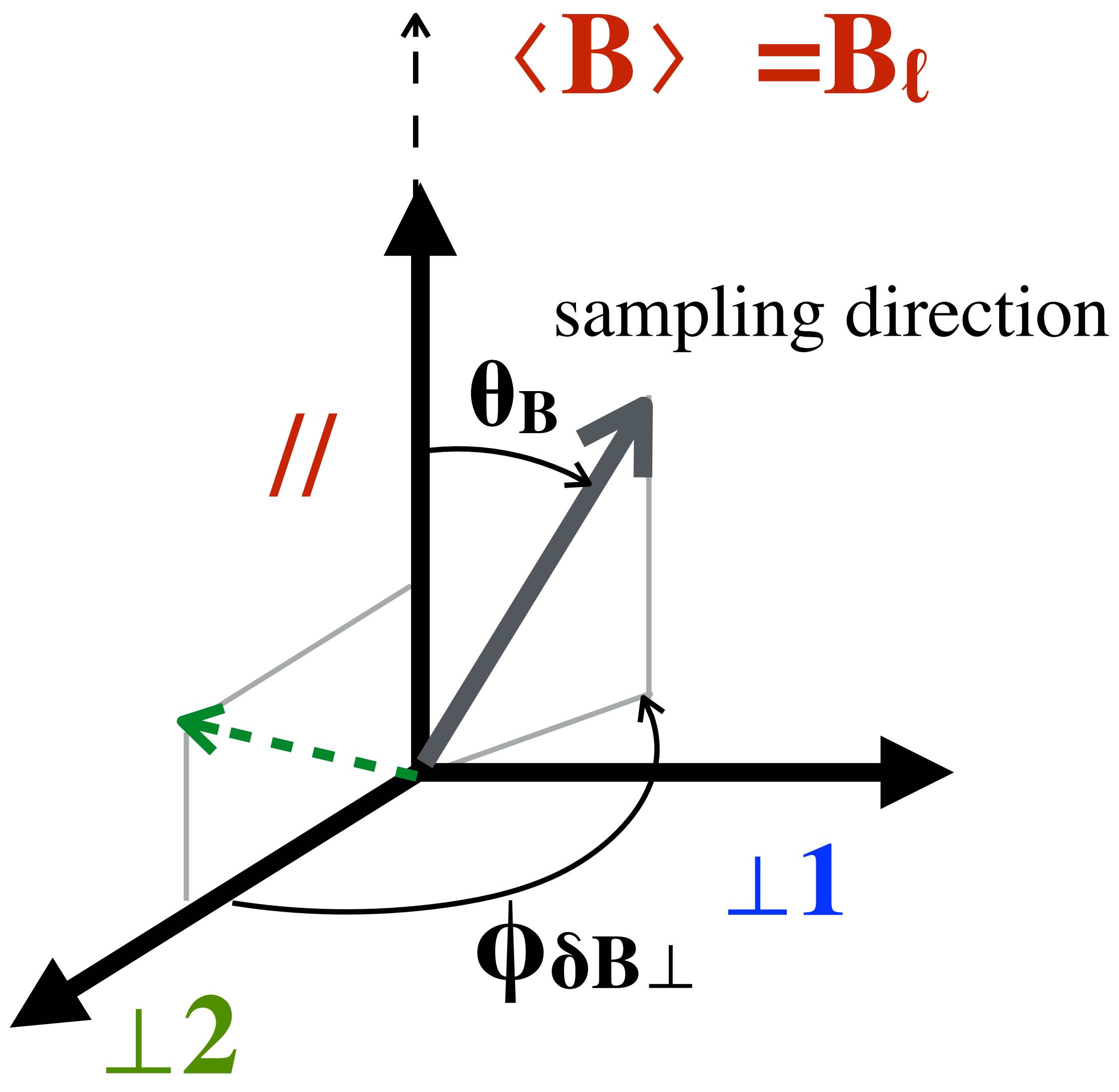
# Scale Dependent Anisotropy

Compute anisotropy with repect to a scale dependent definition of mean magnetic field



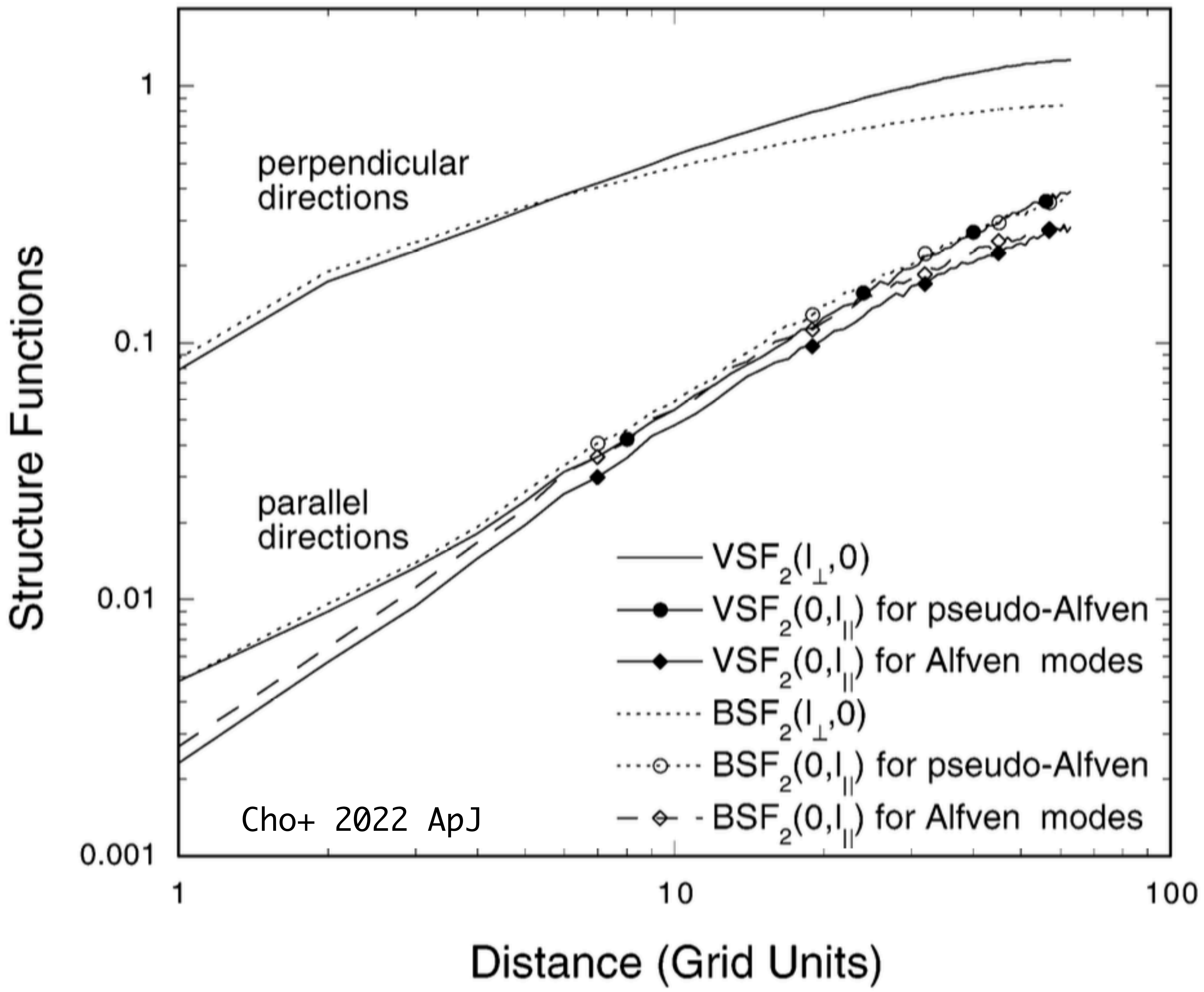

## Not a II-order quantity!

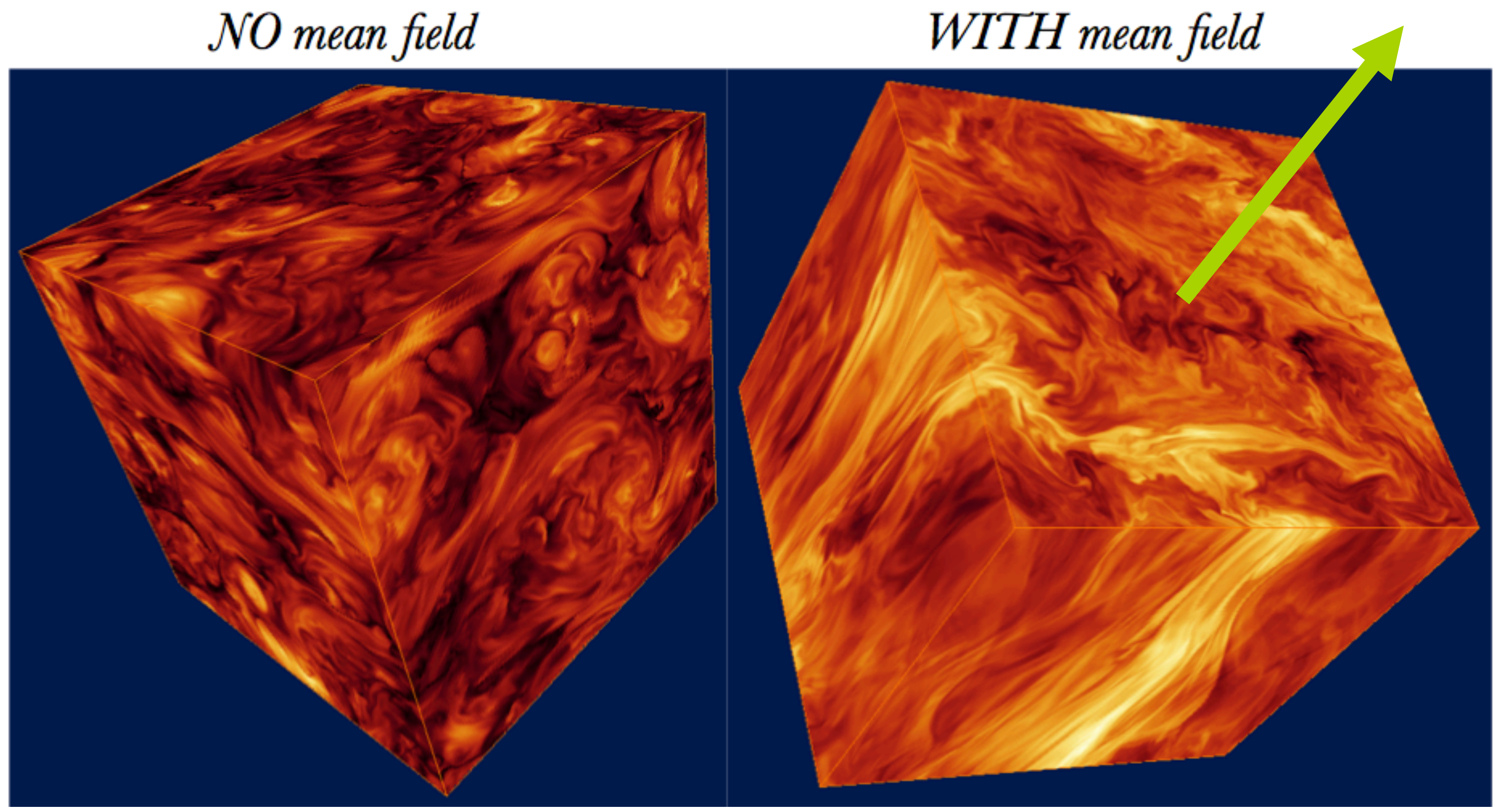


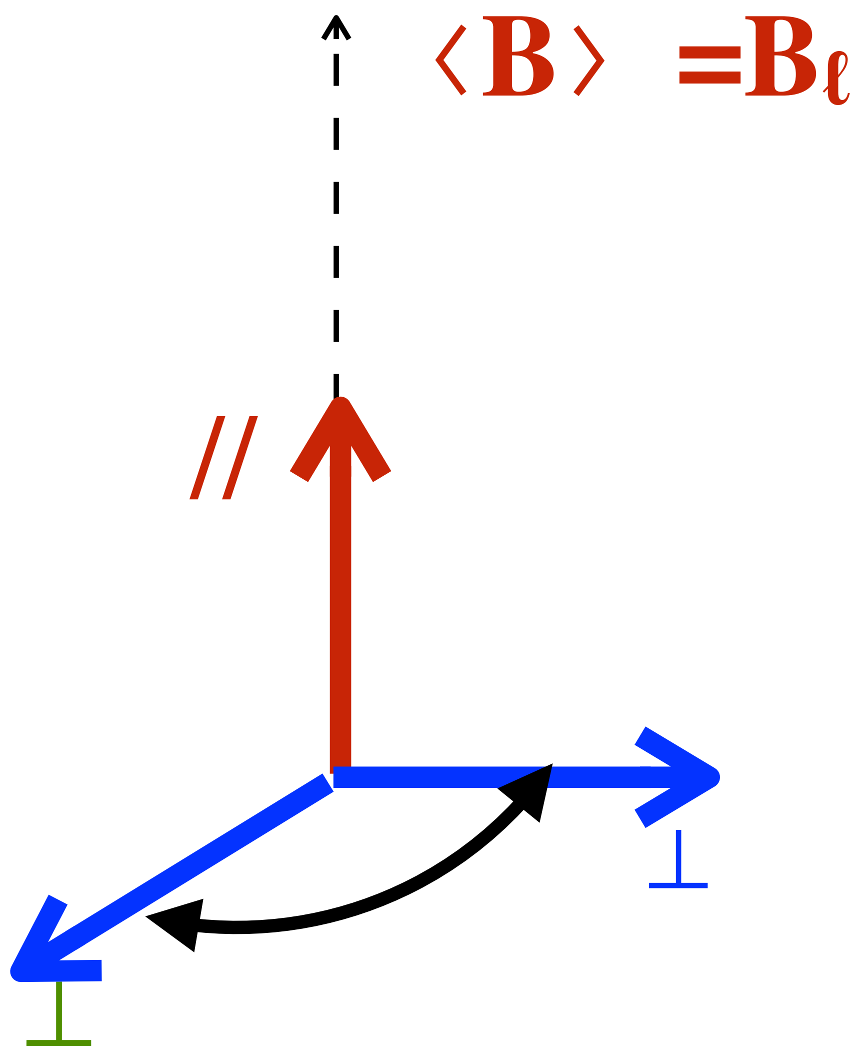

repeat for each point in space, accumulate statistics and average to obtain $SF(\ell_{\parallel},\ell_{\perp 1},\ell_{\perp 2})$

# Scale Dependent Anisotropy




2D



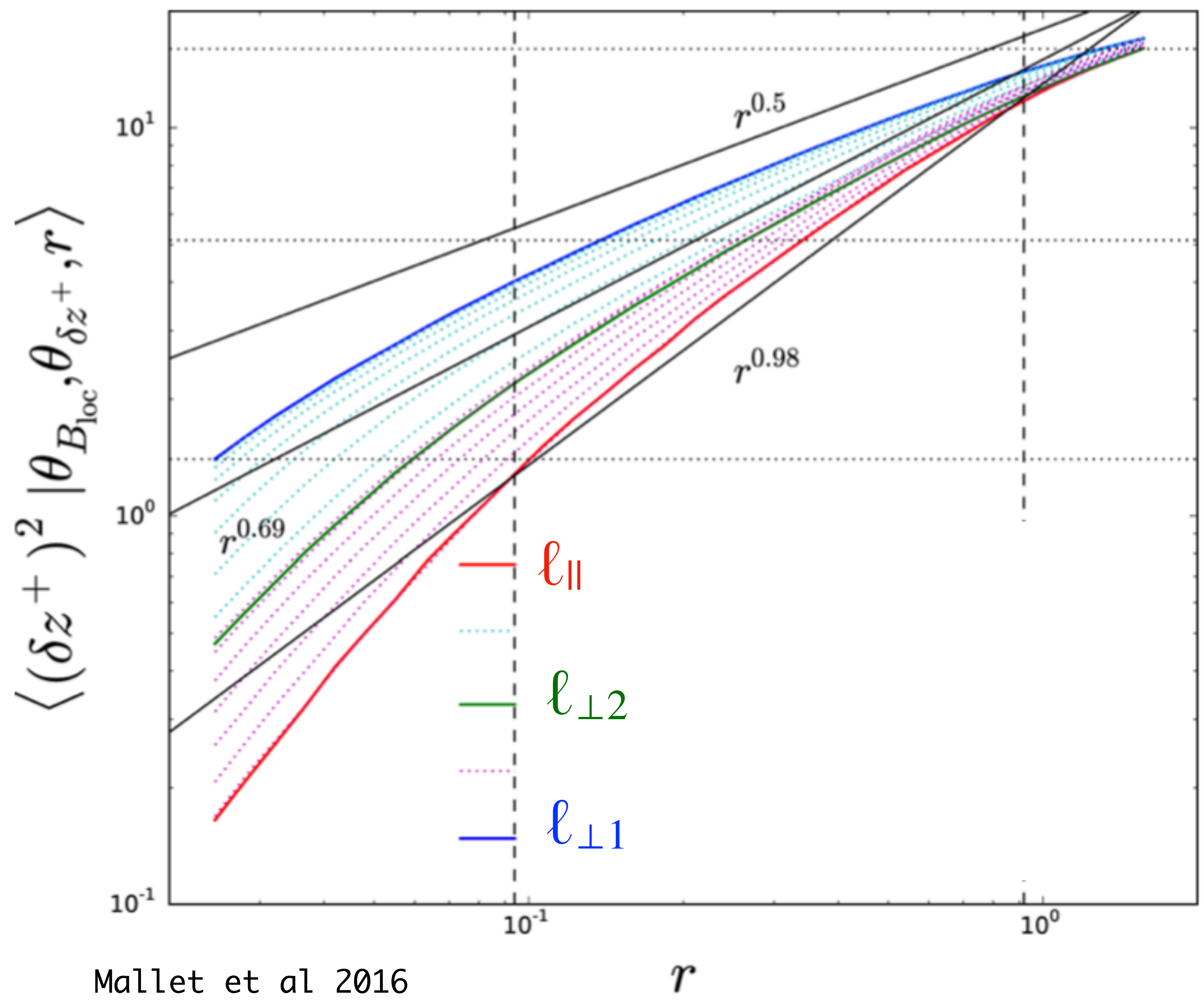

also w/o mean field you get an anisotropic scaling

Origin:
critical balance

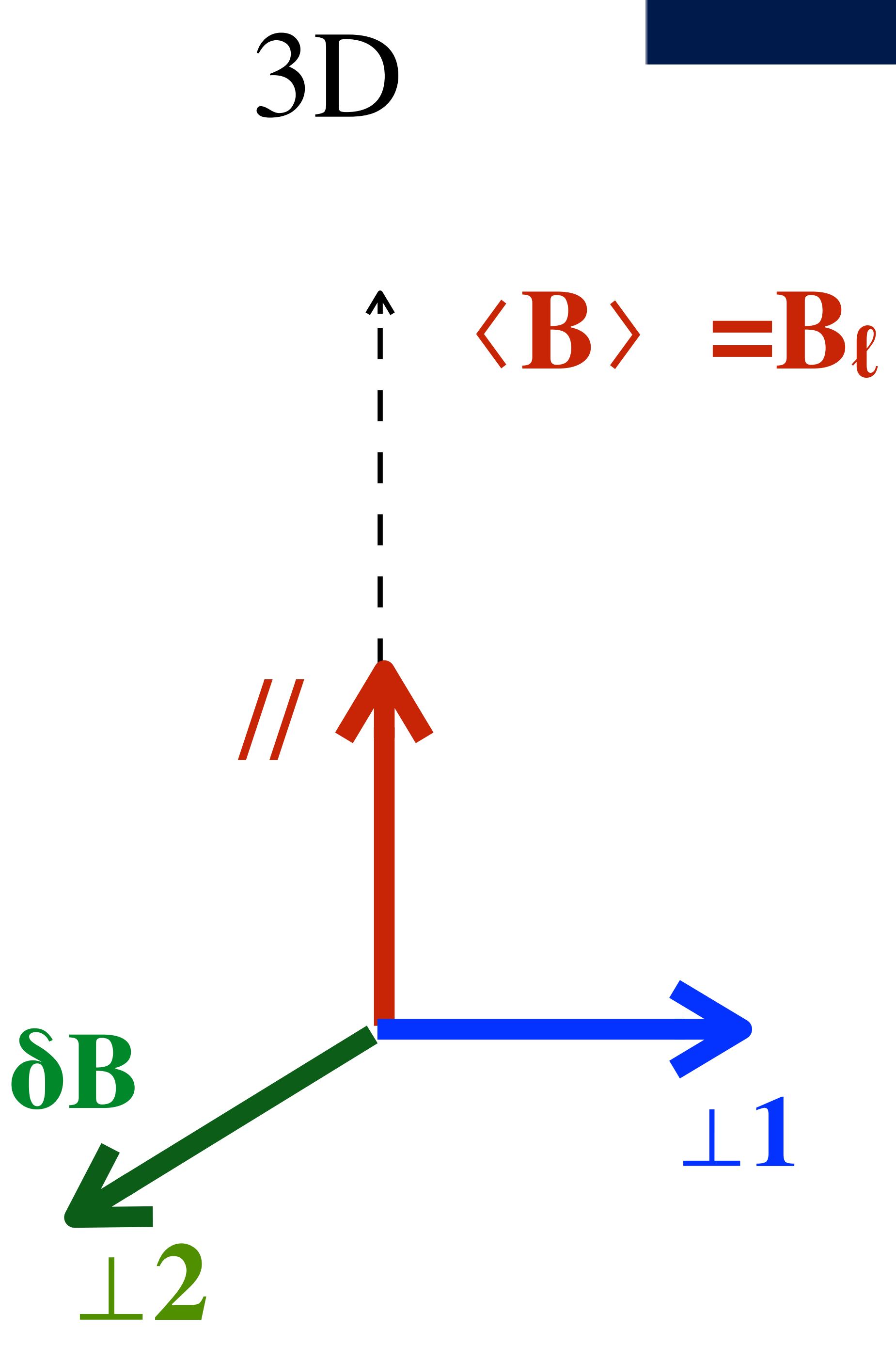
Goldreich & Shridar 1995 ApJ

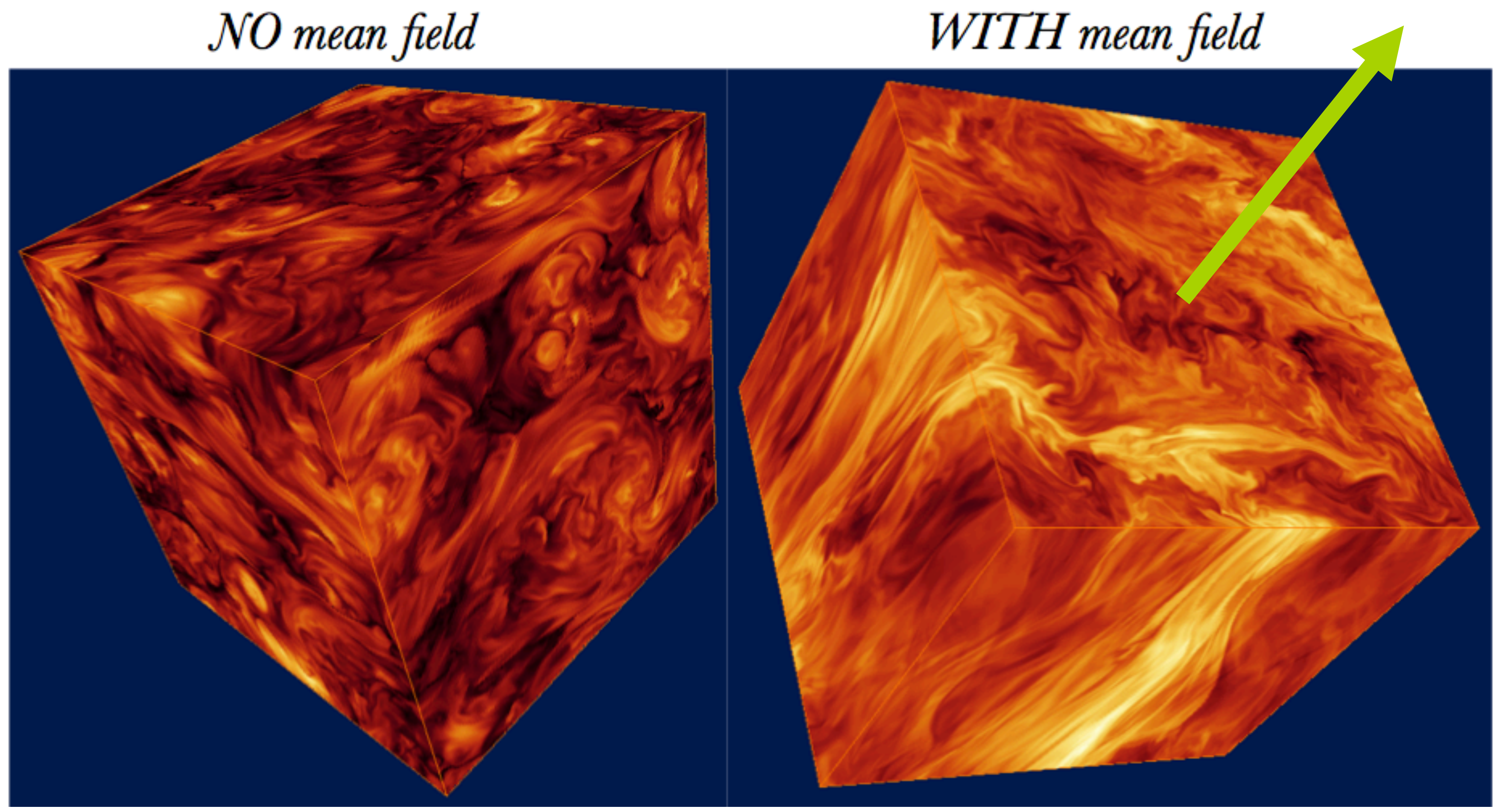
# Scale Dependent Anisotropy

Scaling of SF($\ell_{\parallel}$,$\ell_{\perp 1}$,$\ell_{\perp 2}$)




Mallet et al 2016



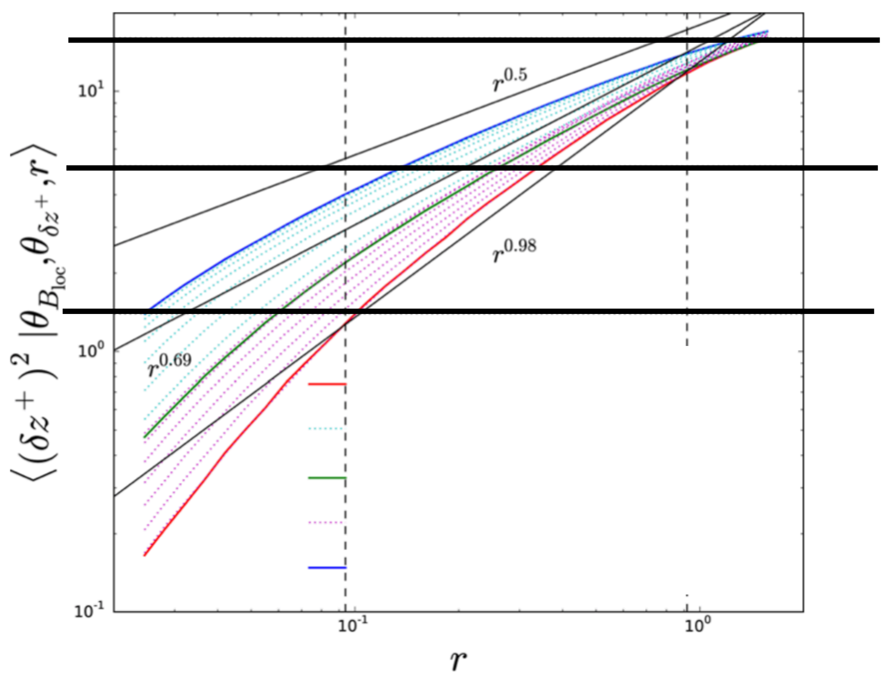




also w/o mean field you get an anisotropic scaling

Origin:
critical balance +
alignment

Boldyrev 2005 ApJ

# Scale Dependent Anisotropy

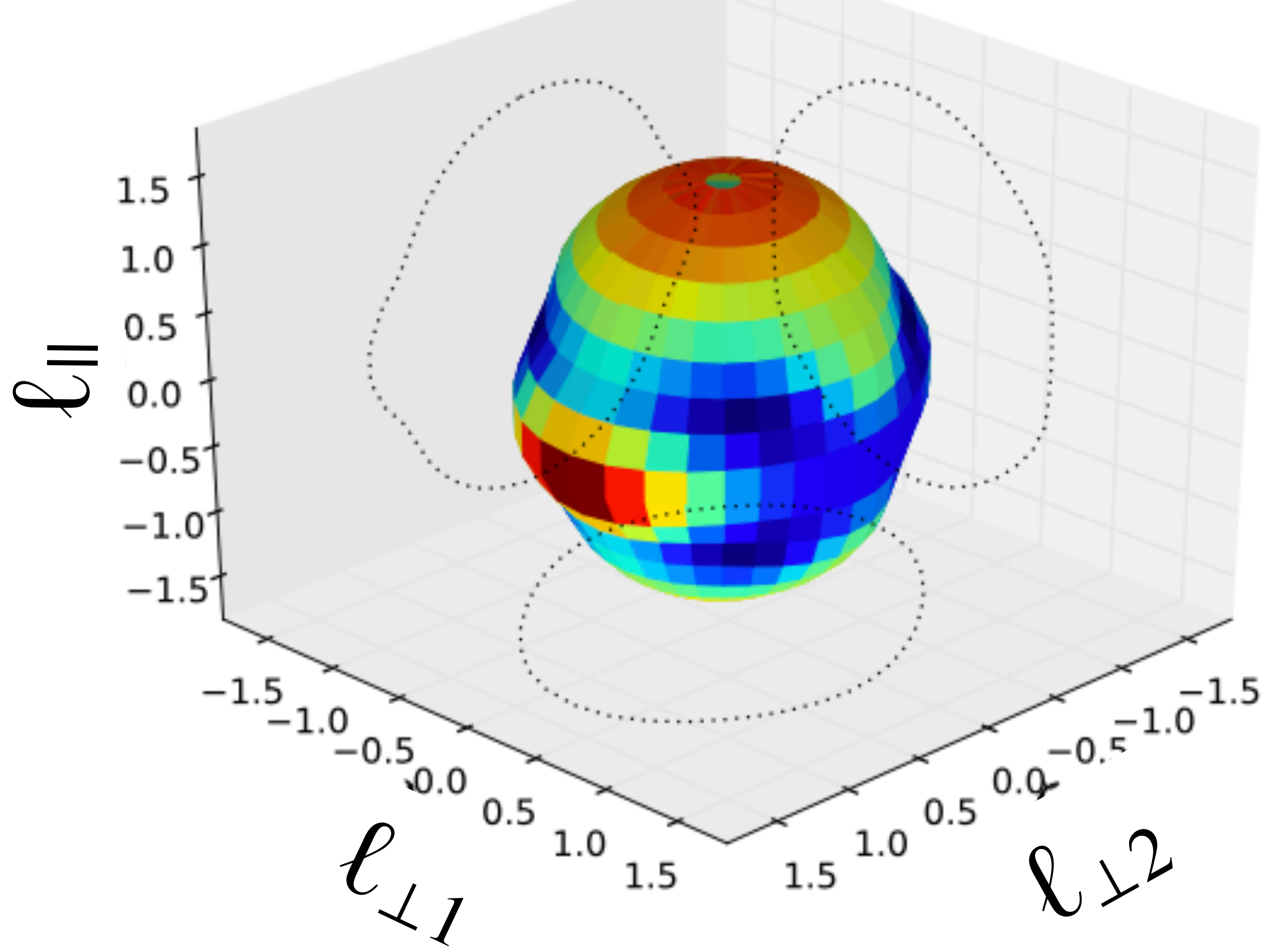



(a)

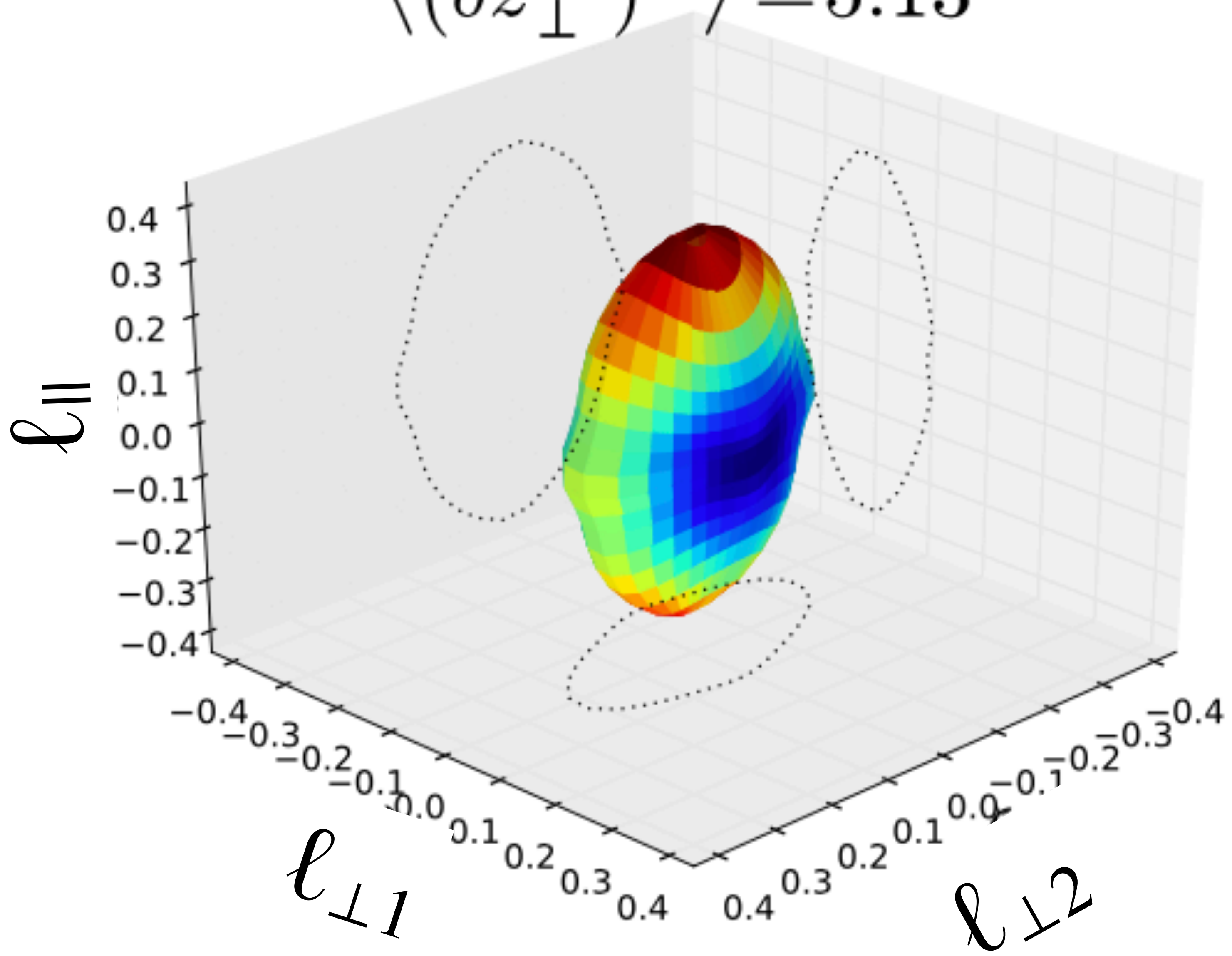
Statistical representation of turbulent eddies at different scales

**isotropic**

$\langle(\delta z_\perp^+)^2\rangle = 15.98$



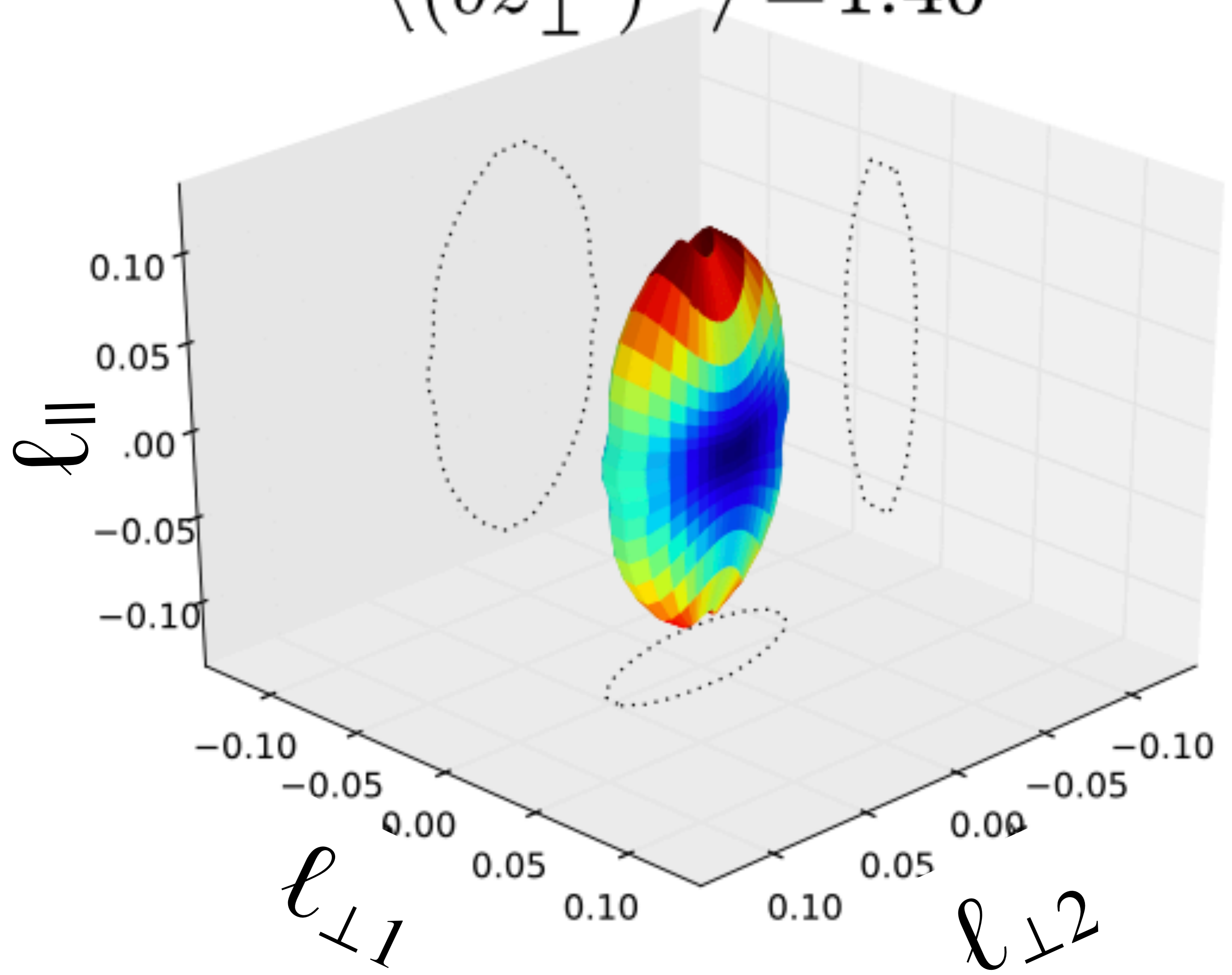

$\langle(\delta z_\perp^+)^2\rangle = 5.13$




**sheet-like**

(not tube-like)

$\langle(\delta z_\perp^+)^2\rangle = 1.40$

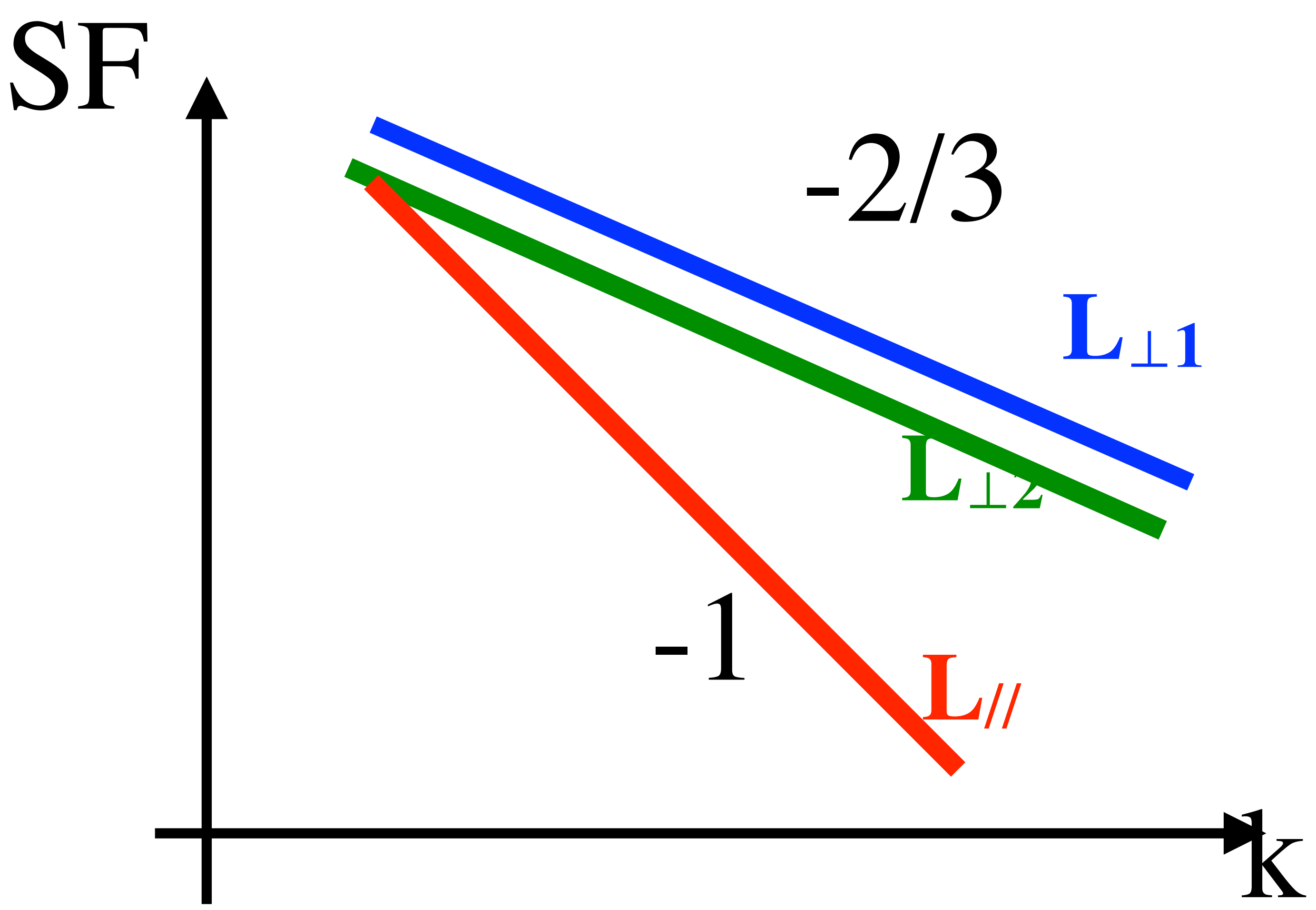



smaller energies (smaller scales)

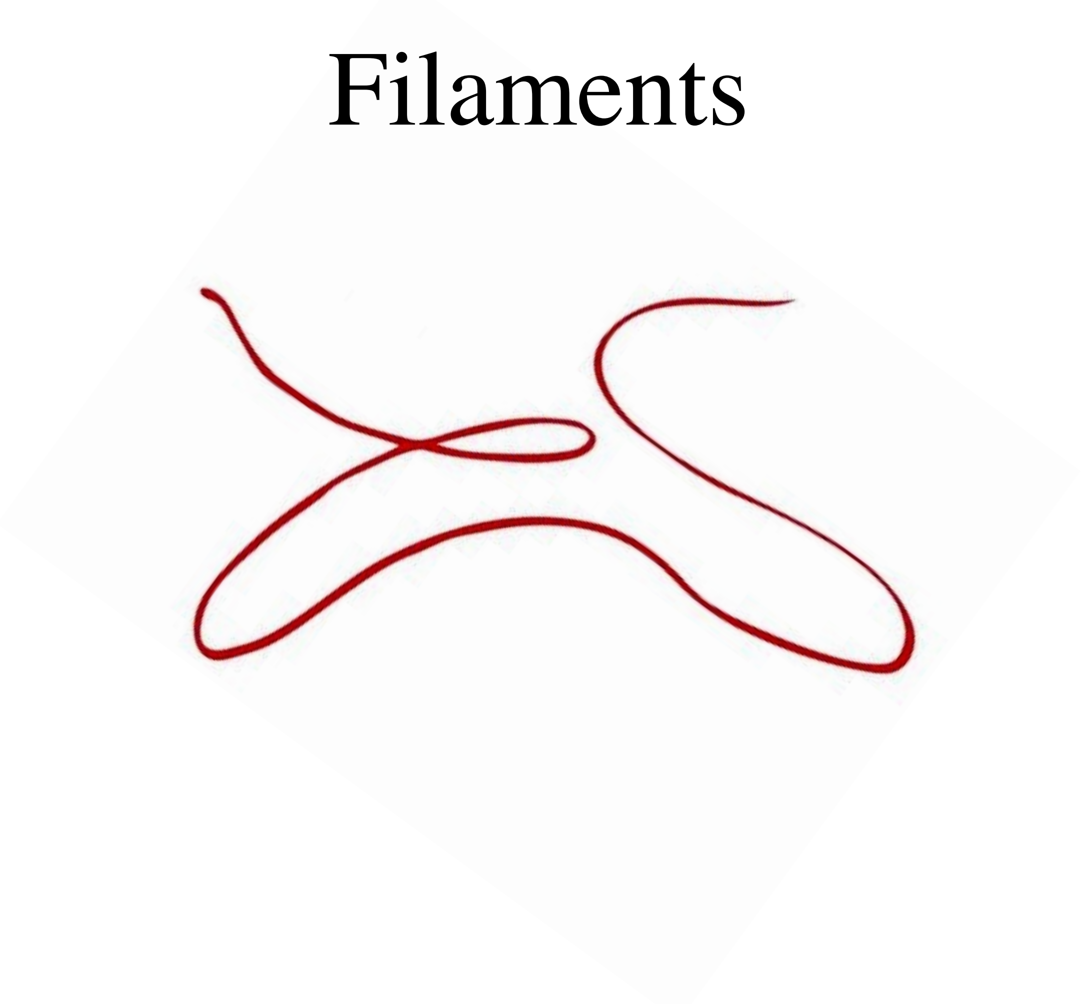
Mallet et al 2016

# Different small-scale structures

Critical Balance
(Goldreich-Shirdar 1995)

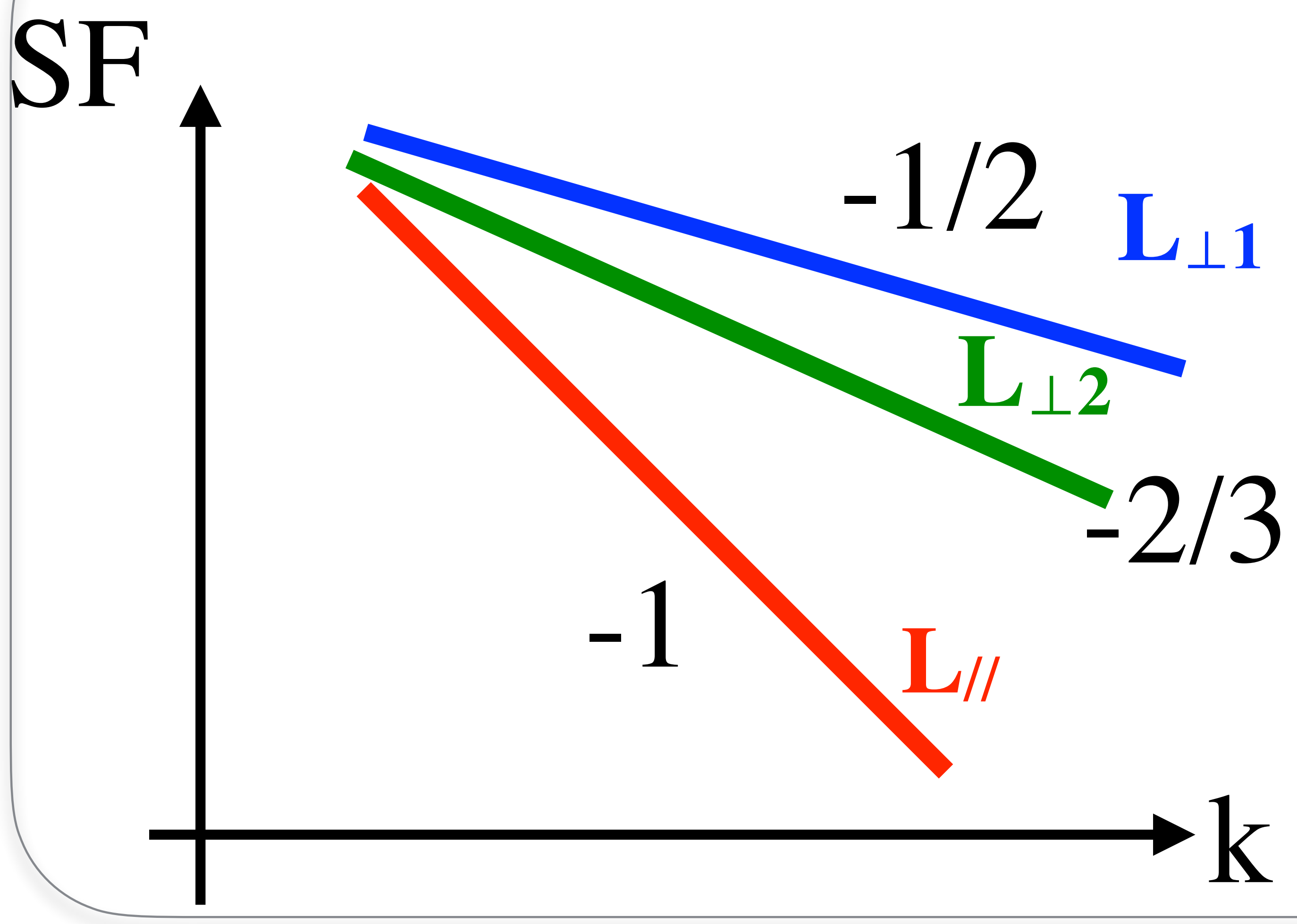



Filaments



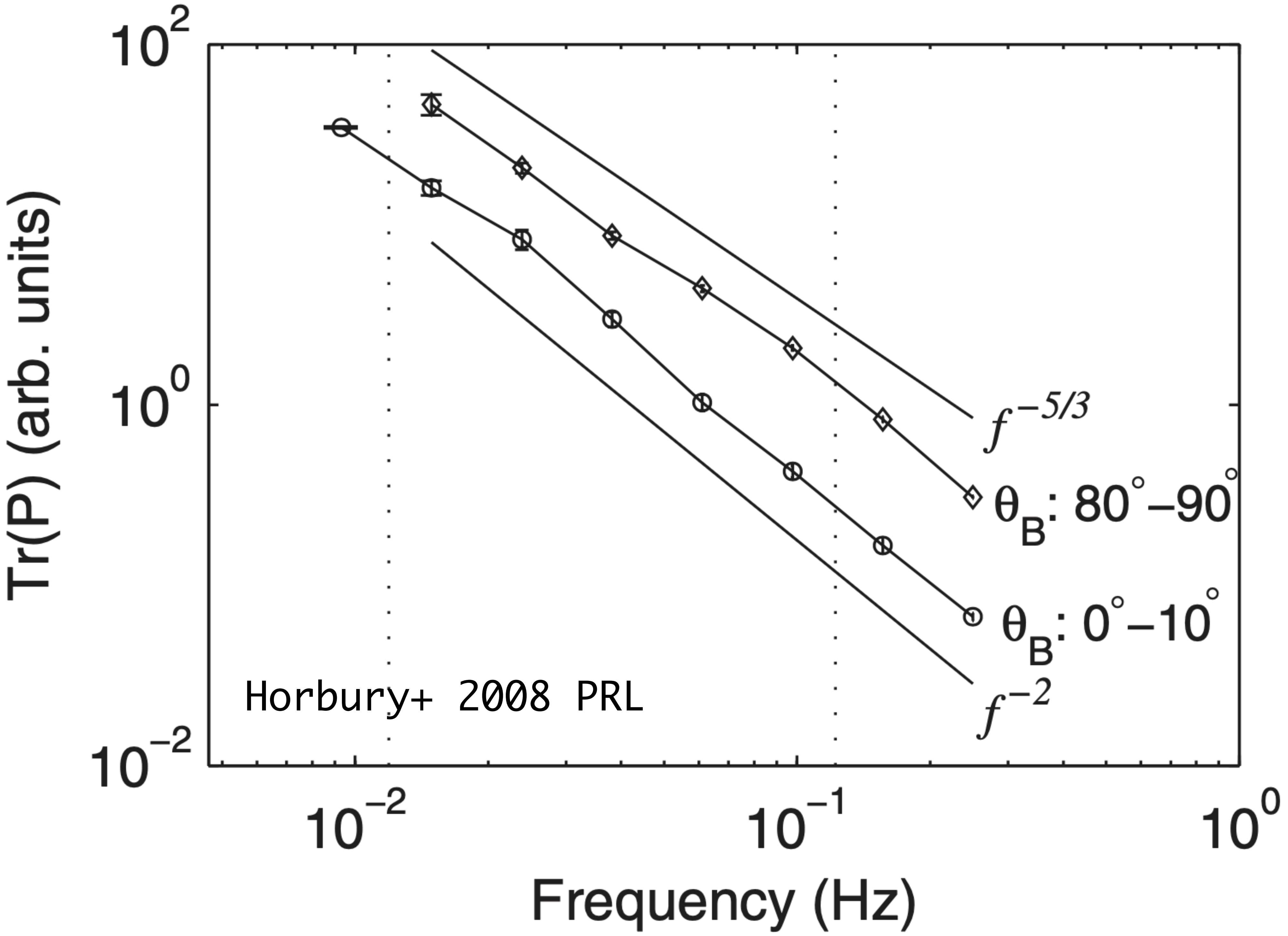
or

Critical Balance
& δV-δB alignement
(Boldyrev 2005, 2006)

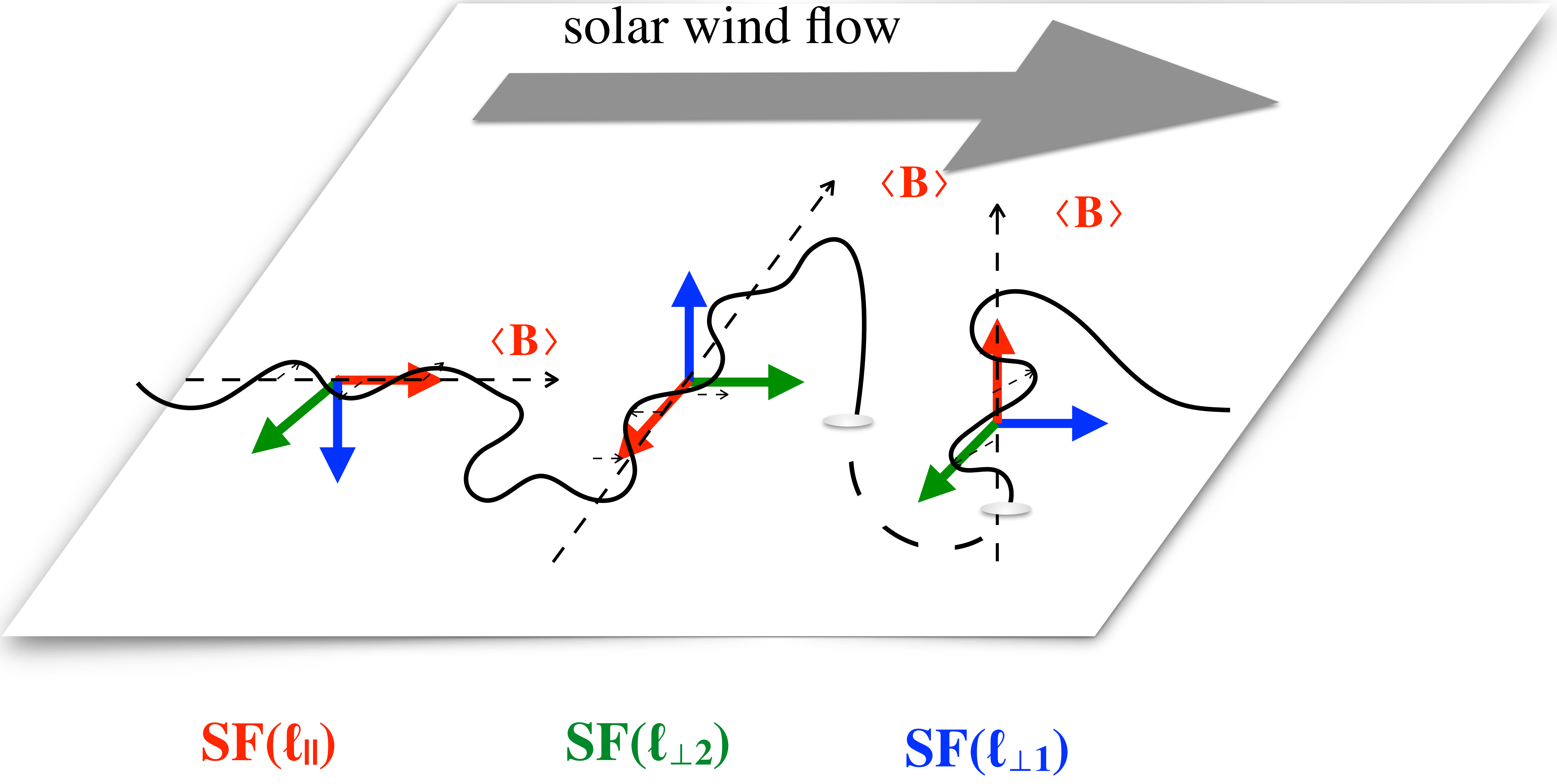



Ribbons

# Measurements in the SW

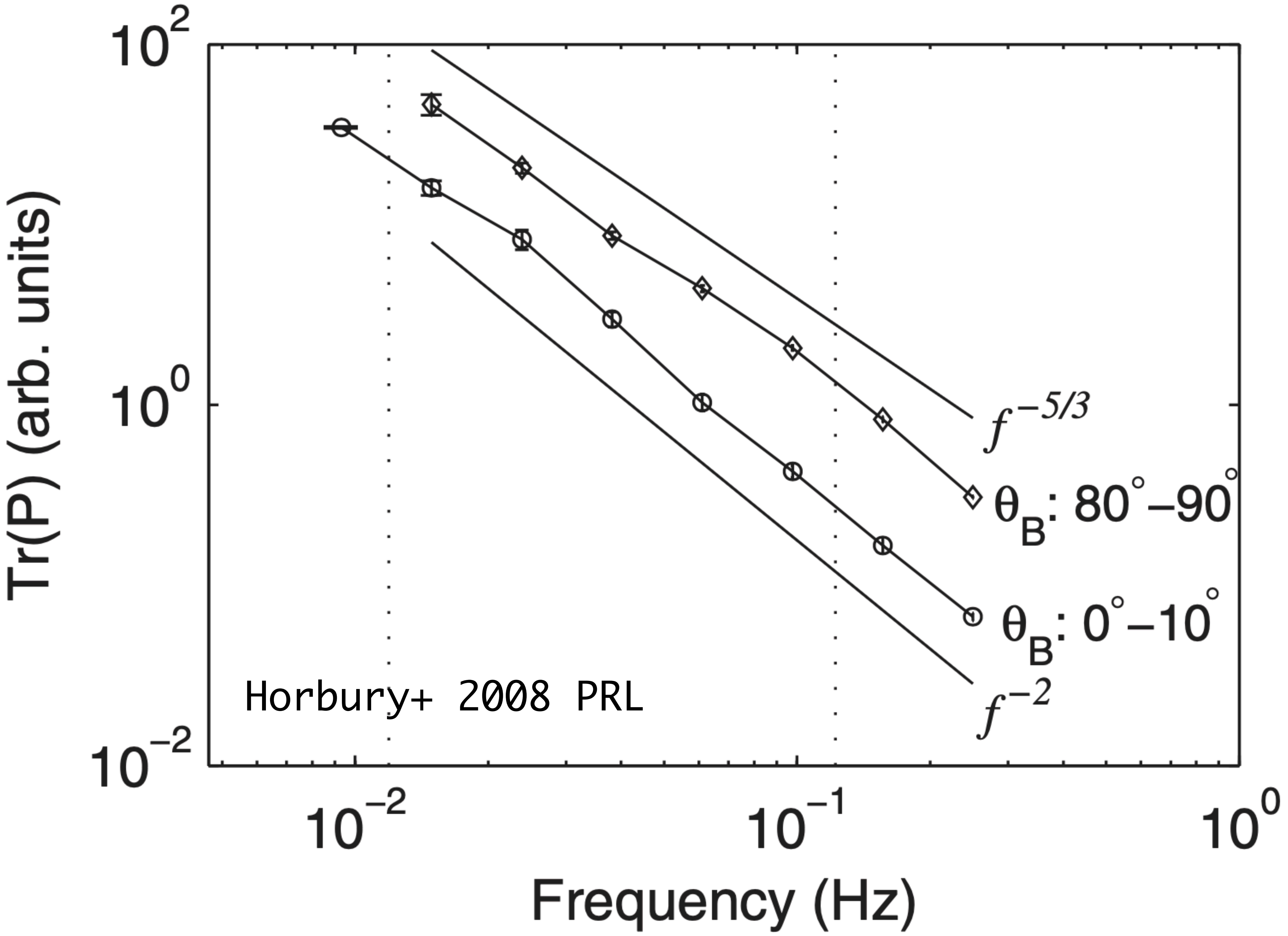



wandering of field lines allows measuring all directions

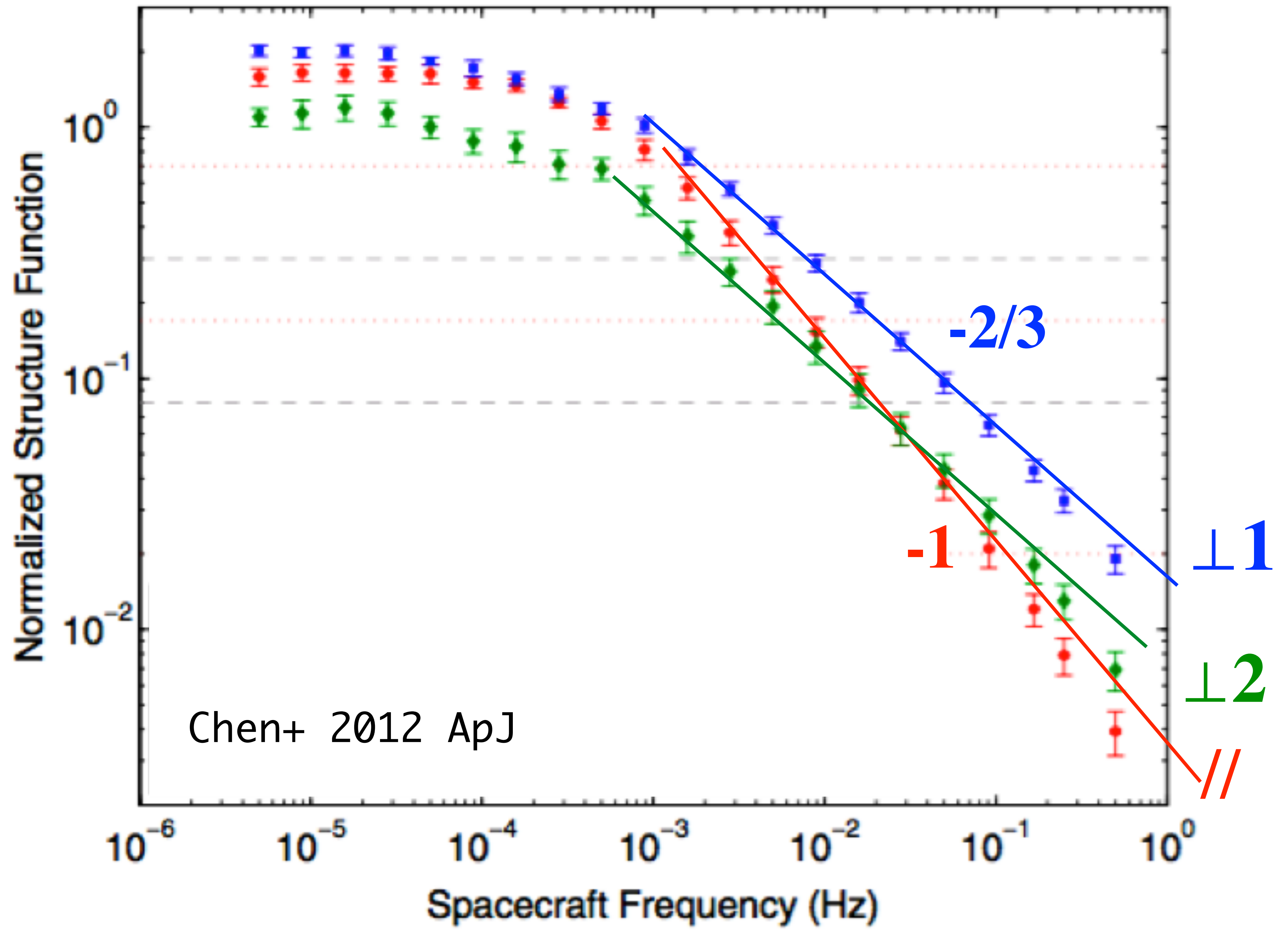



Different Parallel and perpendicular scaling
(with power-law indexes matching theory)

# Measurements in the SW



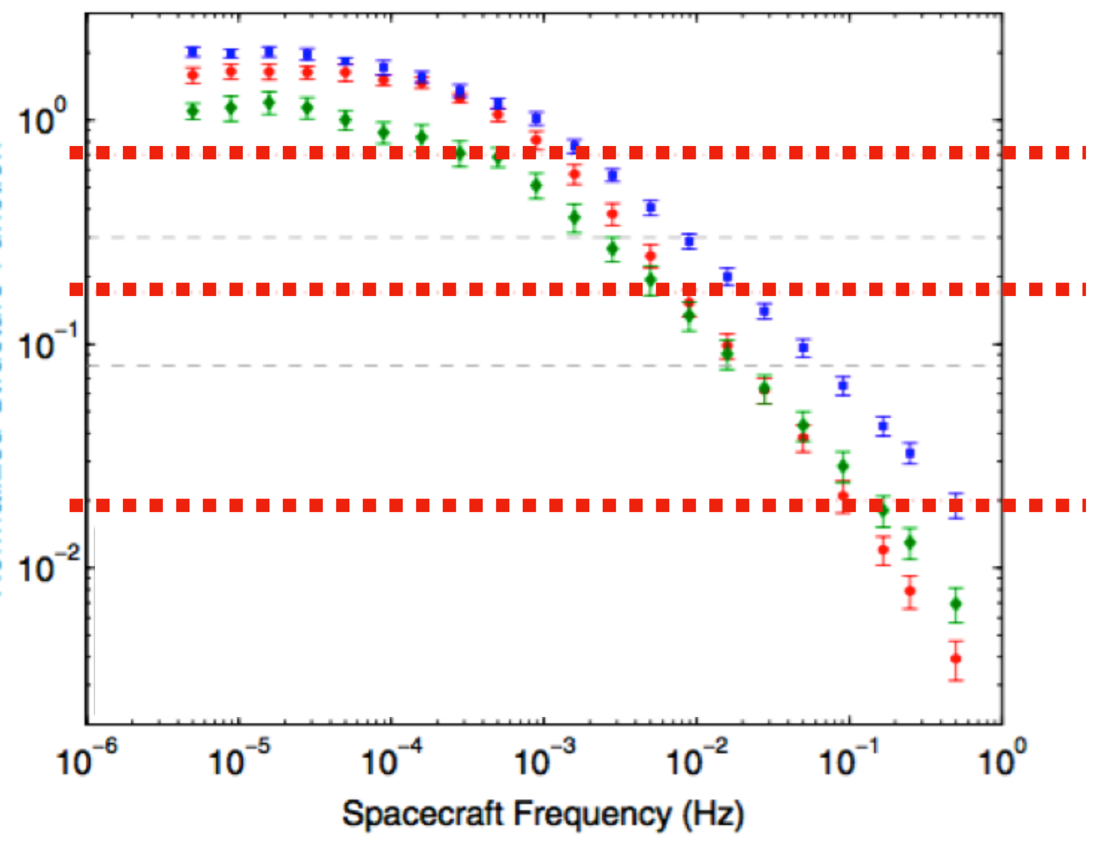

Different Parallel and perpendicular scaling
(with power-law indexes matching theory)




The two perpendicular direction have the same scaling

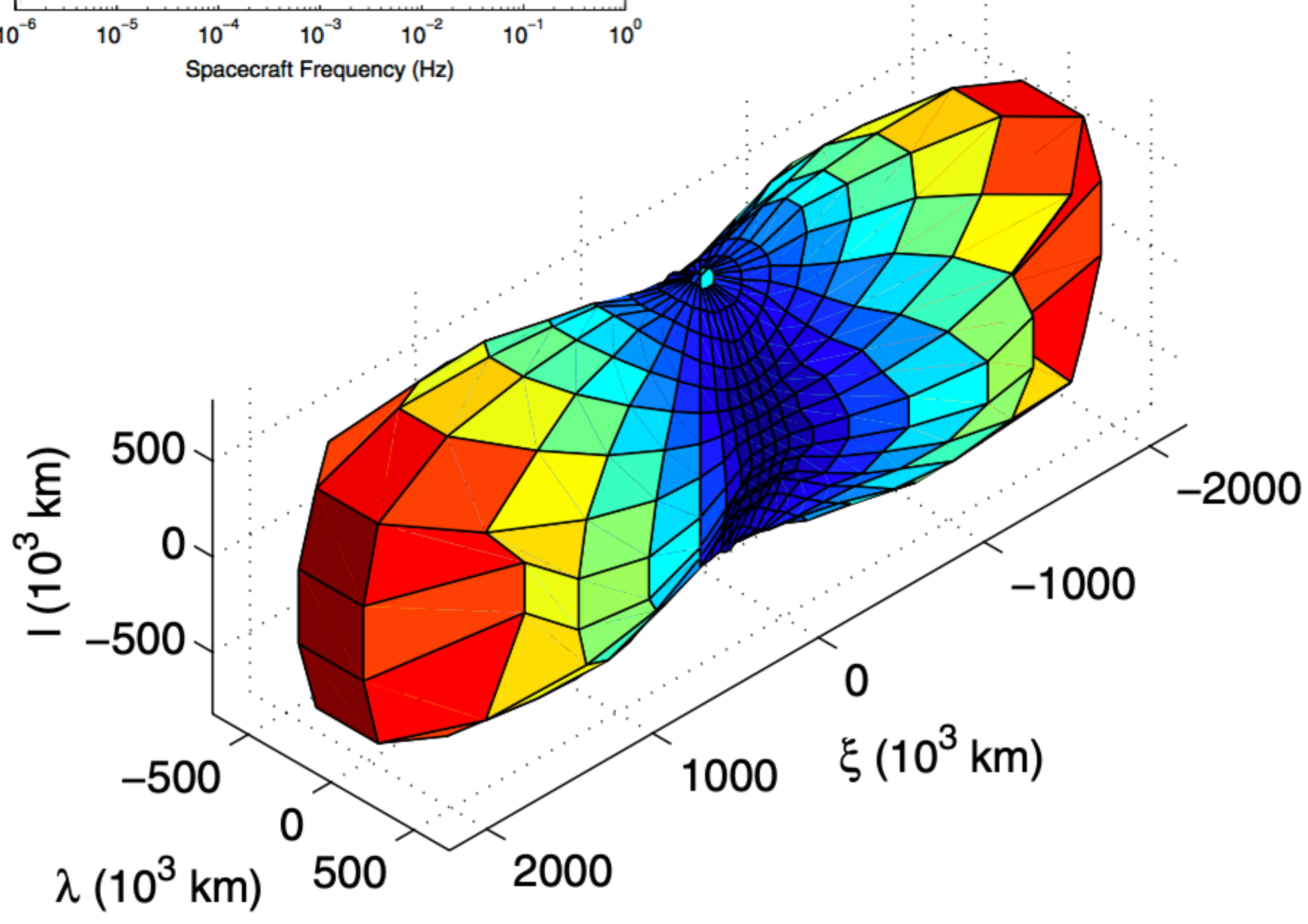
**tube -like structures at small scales**

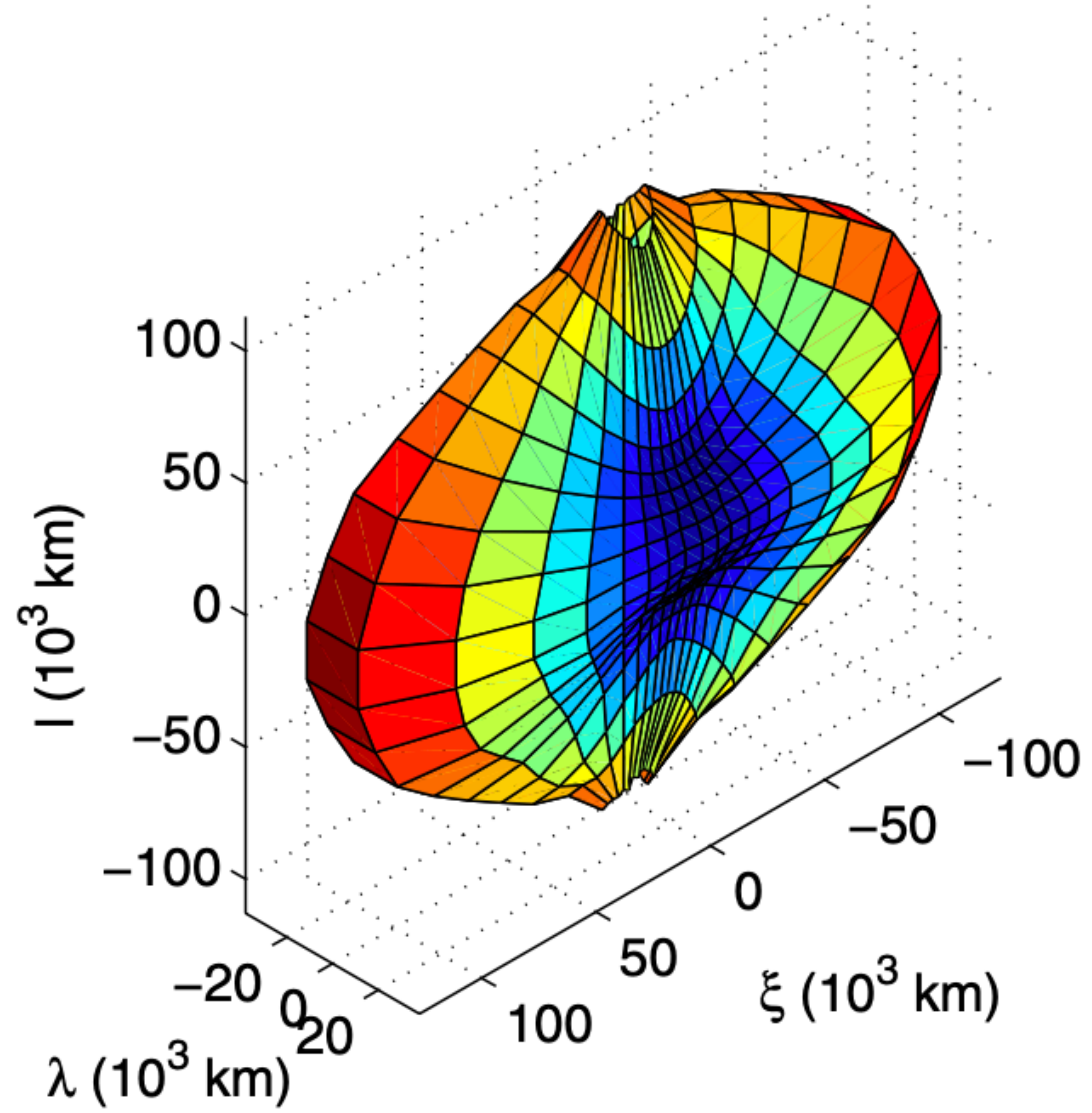
# Measurements in the SW

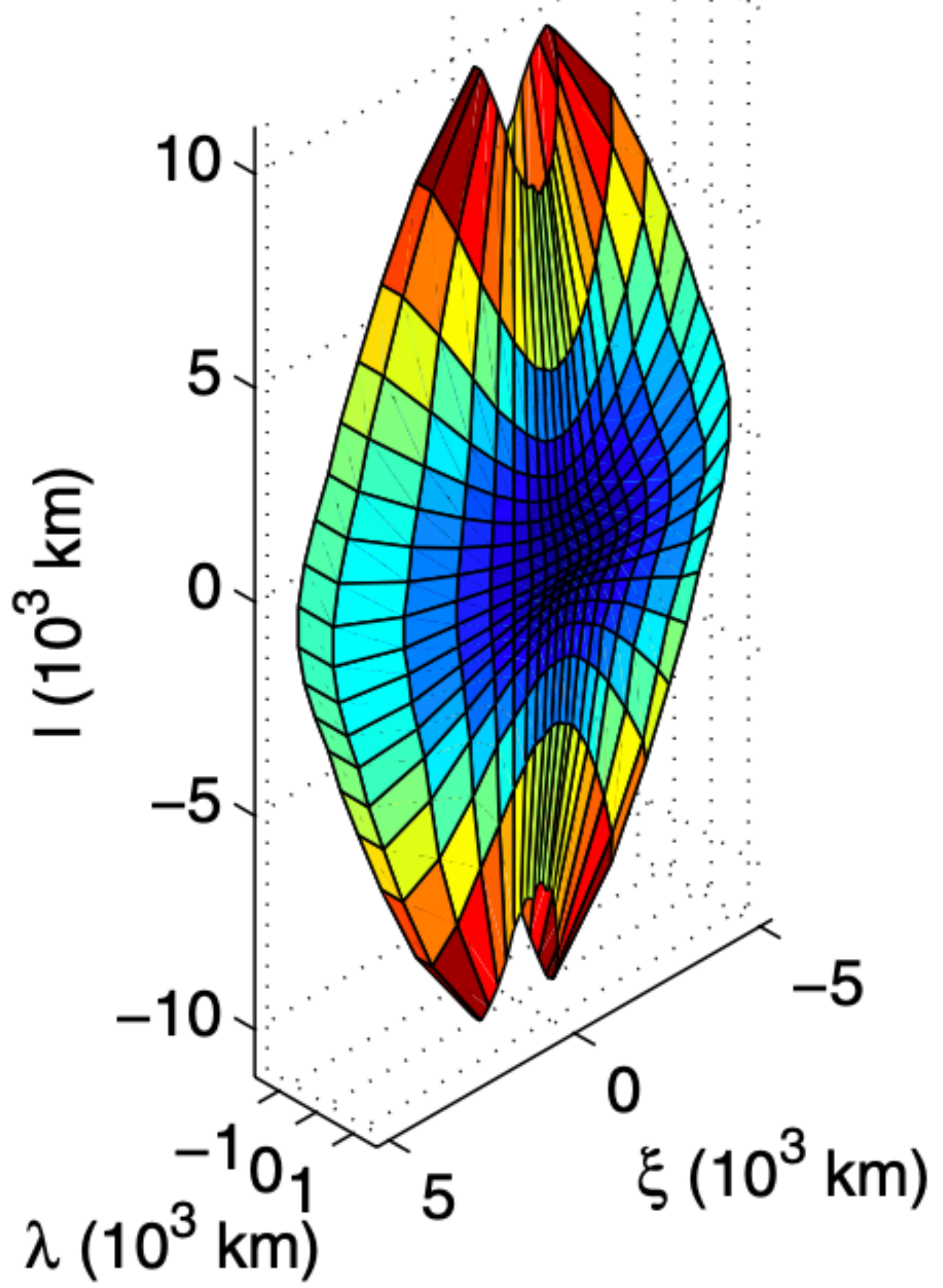



Anisotropic

Two perp direction conserve aspect ratio




smaller energies (smaller scales)